Distribution planning of bulk lubricants at BP Turkey

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Abstract: We address the distribution planning problem of bulk lubricants at BP Turkey. The problem involves the distribution of different lube products from a single production plant to industrial customers using a heterogeneous fleet. The fleet consists of tank trucks where each tank can only be assigned to a single lube. The objective is to minimize total transportation related costs. The problem basically consists of assigning customer orders to the tanks of the trucks and determining the routes of the tank trucks simultaneously. We model this problem as a 0-1 mixed integer linear program. Since the model is intractable for real-life industrial environment we propose two heuristic approaches and investigate their performances. The first approach is a linear programming relaxation-based algorithm while the second is a rolling-horizon threshold heuristic. We propose two variants of the latter heuristic: the first uses a distance priority whereas the second has a due date priority. Our numerical analysis using company data show that both variants of the rolling horizon threshold heuristic are able to provide good results fast.

Keywords: Distribution, large scale optimization, heuristics, OR in energy, multi-compartment vehicles.

1. Introduction

The efficiency in transportation and distribution planning is a key success factor in the petroleum industry. Petroleum (crude oil) is processed in oil refineries to derive different products such as fuel oil, gasoline, diesel fuel, kerosene, liquefied petroleum gas (LPG), petrochemicals, lubricating oils (lubes), etc. The refined products are categorized as light/white products like gasoline and heavy/black products like lubes. Ronen (1995) classifies the distribution of petroleum products into four categories: light products from refineries to tank terminals, light products from tank terminals to industrial customers, bulk lubes from lube plants to industrial customers, and packaged lubes from lube plants to industrial customers.

Petroleum products are mainly transported to the international markets by maritime transportation: approximately 60% of total petroleum produced is transported via sea lines (Rodrigue et al., 2009). The other modes of transportation are pipelines, trains, and trucks. Table 1 summarizes several properties of different transportation modes in the petroleum industry. In general, trucks are used to transport the end products to industrial customers or to petrol and service stations.

Table 1. Modes used in the transportation of petroleum products (Rodrigue et al., 2009).

	Pipeline	Marine	Rail	Truck
Volumes	Large	Very large	Small	Large
Materials	Crude/Products	Crude/Products	Products	Products
Scale	2 ML+	10 ML+	100 kL	5-60 kL
Unit costs	Very low	Low	High	Very high
Capital costs	High	Medium	Low	Very low
Access	Very limited	Very limited	Limited	High
Responsiveness	1-4 weeks	7 days	2-4 days	4-12 hours
Flexibility	Limited	Limited	Good	High
Usage	Long haul	Long haul	Medium haul	Short haul

In this study, we address the distribution planning of bulk lubes at BP Lube Division in Turkey. With its specific characteristics and elements of the distribution system the problem differs from many of the transportation problems addressed in the literature. Although the oil industry has been a major source of applications, white papers and reports on those applications and the academic research in the field are rather scant. Ronen (1995) provides a review of operations research (OR) applications in dispatching petroleum products and compares several applications in the oil industry. Among those, Brown and Graves (1981) address the gasoline transportation problem with direct deliveries (one stop only) from a single bulk terminal at Chevron U.S.A. Brown et al. (1987) generalize this problem for Mobil Oil Corporation by considering multiple deliveries during a trip. Bausch et al. (1995) propose an elastic set partitioning technique to solve the same problem. The candidate schedules are obtained by generating trips using the sweep algorithm. Franz and Woodmansee (1990) develop a rule-based semi-automated decision support system for a regional oil company to determine the daily schedule of the drivers and the dispatching of the tank trucks. Nussbaum and Sepulveda (1997) address the distribution problem for the biggest fuel company in Chile. The delivery plans are obtained using a heuristic approach and a planning, execution, and control system is developed employing a knowledge-based approach that utilizes a graphical user interface which mimics the mental model of the user. In a different setting, Day et al. (2009) implemented a three-phase heuristic for cyclical inventory routing problem encountered at a carbon dioxide gas distributer in Indiana. Their heuristic determines regular routes for each of three available delivery vehicles over a 12-day delivery horizon while improving the delivery labor cost, stockouts, delivery regularity, and driver-customer familiarity.

Ben Abdelaziz et al. (2002) model a mathematical program in a single period setting and apply a variable neighborhood search approach for dispatching the tank trucks for fuel delivery. Malépart et al. (2003) present four heuristics to solve the fuel delivery problem for servicing Esso stations in Eastern Canada. Taqa allah et al. (2000) propose heuristics to address the multi-period extension of this problem. Avella et al. (2004) address a daily petrol delivery problem using a heterogeneous fleet of tank trucks. They assume that the tanks are either completely full or completely empty and develop a fast heuristic and an exact method based on the set partitioning formulation and branch-and-price algorithm. To test the performance of their approach they use real-world data consisting of 25 customers and 6 tank trucks of 3 different types.

Recently, Cornillier et al. (2008a) tackle the petrol station replenishment problem (PSRP) where the quantities to deliver are decision variables that are allowed to vary within a given interval. They assume that the trucks make at most two stops during a trip, which considerably simplifies the problem. They develop an exact algorithm which decomposes the PSRP into a truck loading problem and a routing problem. The routing problem reduces to a polynomial matching problem since each truck visits at most two stations. The optimal solution to the truck loading problem can be obtained using a heuristic procedure or by solving an integer linear program. Cornillier et al. (2008b) extend the PSRP to a multi-period setting and develop a multi-phase heuristic with look-back and look-ahead procedures. Basically, the heuristic first determines the stations to be serviced in each period. Then, the problem reduces to solving multiple PSRPs where the exact algorithm of Cornillier et al. (2008a) is utilized. An iterative procedure is applied since the resulting solution may not be feasible with respect to the maximum workload constraints. Cornillier et al. (2009) address the PRSP with time-windows. In this case, the limit on the number of stops is relaxed to four. They develop two heuristics based on the mixed integer linear programming formulation of the problem. The first heuristic is designed to solve small instances. It makes a preselection of promising arcs and solves the arising mathematical model to optimality. The second is a decomposition heuristic based on route preselection and can be used to solve larger instances.

Vehicles with multiple compartments are also used in the transportation of food and grocery items. Chajakis and Guignard (2003) address the delivery scheduling problem with a homogeneous fleet of multiple compartment vehicles using Lagrangean approximation algorithms. They experiment four different Lagrangean relaxations, a Lagrangean substitution, and a Lagrangean decomposition technique to find lower bounds and develop a Lagrangean heuristic to obtain feasible solutions. Eglese et al. (2005) use simulated annealing to solve a similar real-world vehicle scheduling problem in the U.K. Because of the loading/unloading sequence dependency of the products they allow multiple visits to stores. Recent articles that address the routing and scheduling of tank trucks/multi-compartment vehicles include El Fallahi et al. (2008), Mendoza et al. (2010), Muyldermans and Pang (2010), and Knust and Schumacher (2011).

Our study considers the distribution of bulk lubes from a single lube production plant to industrial customers. The fleet is heterogeneous and consists of multi-compartment vehicles, i.e., tank trucks, where each compartment can only be assigned to a single product and a single customer. The objective is to minimize the distribution related costs. The problem basically consists of assigning the customer orders to the tanks of the trucks and determining the routes of the loaded tank trucks simultaneously. The orders have ± 2 days delivery flexibility; i.e. they can be delivered two days before or after their planned due dates. So, the planning horizon is 5 days and the problem is solved every day on a rolling horizon basis. Since the company is not charged for the return trip of the trucks to the plant the routing problem is an open vehicle routing problem (OVRP). In OVRP, the vehicles either do not return to the depot or are assumed to return to the depot by revisiting the customers in the reverse order. The research on OVRP has recently gained momentum and various methods have been proposed to efficiently solve it. We refer the reader to Repoussis et al. (2010), Emmanouil et al. (2010), Salari et al. (2010) for recent developments and Li et al. (2007) for a detailed review on OVRP.

Our aim is to improve the bulk lubes distribution operations of BP Turkey using OR techniques. The problem seems similar to that of Cornillier et al. (2008b) since they both attack a multi-period delivery problem using multi-compartment vehicles. However, there are some key differences: Firstly, Cornillier et al. (2008b) restrict the trucks to make at most two visits during a trip, which significantly simplifies the problem. Moreover, their delivery quantities are variable whereas in our problem the distributors and service stations have already placed their orders but the company has two-day delivery date flexibility. Also, they assume a fixed planning horizon while our problem is solved on rolling horizons. In addition, their aim is to minimize total regular working time/overtime costs and distance related travel costs whereas in our case since the trucks are outsourced the objective function includes the costs associated with the number of visits that the trucks make and the travel cost on open routes.

We first formulate a mathematical programming model of the problem and then present two heuristic approaches to solve it. The first approach is a linear programming (LP) relaxation-based heuristic while the second is a rolling-horizon threshold heuristic. Two variants of the rolling-horizon threshold heuristic are developed: the first uses a distance priority whereas the second has a due date priority. To further improve the solution a simple local search procedure is applied to the results of both heuristics. The remainder of the paper is organized as follows: In Section 2, the description of the problem and its 0-1 mixed integer programming formulation are provided. Section 3 describes the heuristics proposed for efficiently solving this problem. The numerical investigation and results with regard to the performances of the proposed heuristics are given in Section 4. Finally, conclusions and future research directions are provided in Section 5.

2. Problem description and formulation

The problem is a multi-product, multi-period, heterogeneous fleet distribution planning problem involving the assignment of customer orders to tank trucks and routing of tank trucks. The elements of the distribution system can be classified into four categories: (i) the fleet, which consists of multi-compartment tank trucks; (ii) the distribution network, which includes a single lube production plant where the trucks are loaded and the cities where the customers are located at; (iii) the products with their specific properties; and (iv) the scheduling system, which has different constraints and flexibilities specific to the problem. In what follows, we provide further details on these elements of the problem and then formulate the mathematical model.

2.1. Elements of the problem

2.1.1 Tank trucks

The Lubes Division at BP Turkey uses a third party logistics (3PL) service provider for the distribution of the bulk lubes. It estimates the fleet type and size it will need throughout the year and makes an annual contract with the 3PL provider based on a pre-determined fleet dedicated to its delivery services. Therefore, accuracy in the estimation of the appropriate fleet size and type is very important. In the case the contracted capacity is insufficient in any day the company hires trucks from the spot market at an additional cost. Hence, the truck capacity can be considered as a loose constraint in that sense.

The tank trucks have 4 or 5 tanks (compartments), each with a different capacity. In addition to the tank capacity, the trucks have a weight restriction imposed by the regulations of the General Directorate of Highways. The maximum tonnage in a truck is determined according to its technical properties such as its number of wheels and engine power. Since the trucks in the fleet have different weight restrictions and tank capacities, the problem is a heterogeneous fleet distribution problem. Furthermore, the trucks are classified as big- and small-size trucks where small-size trucks have a total capacity of approximately 7 tons and are used to serve the customers whose unloading area is not large enough to accommodate the big-size trucks. We refer to this type of customers as “small customers” whereas the customers that can be served with any truck are called as “large customers”.

A tank in a truck can only be filled with the order of one customer only since the trucks are not equipped with a flow-meter. The flow-meter is the device used to measure the volume of the lube unloaded. If a truck does not have a flow-meter the whole capacity of each of its tanks is dedicated to one single order no matter how large the order size is. For instance, a tank truck with 5 tanks can be loaded with at most 5 orders and hence can at most serve 5 customers.

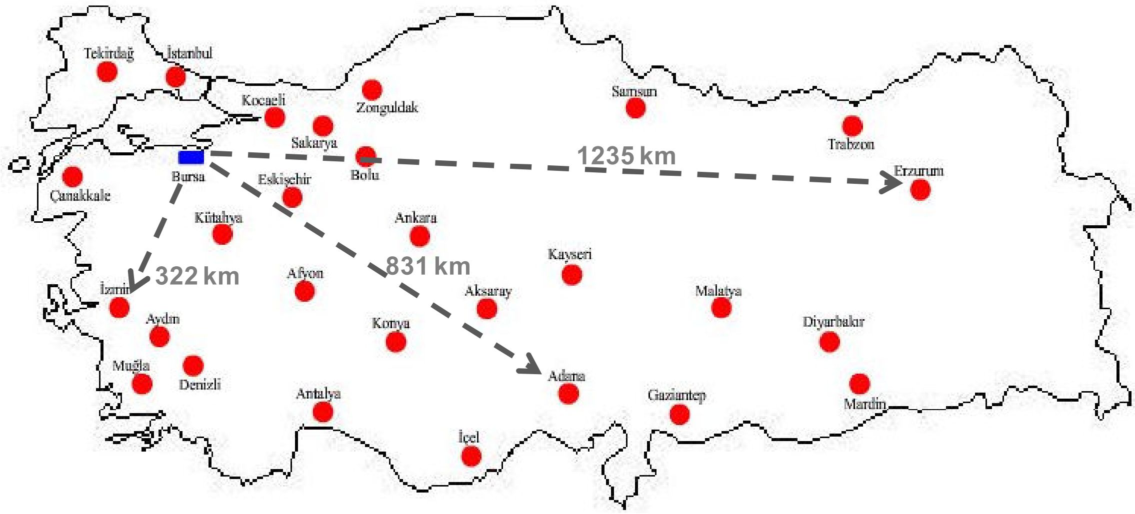


Figure 1. The distribution network (■ plant, ● cities)

2.1.2. Distribution Network

The distribution network consists of one plant in Bursa (North-West Turkey) and 180 customers dispersed in 28 cities located in different regions of Turkey (see Figure 1). The tank trucks are loaded at the plant according to the planned deliveries and visit the customers using a route such that the total distance until the last customer along the route is minimized. Once the loading decisions are made, the routing problem is easy to solve since a truck can at most visit five customers (or four depending on the truck). We observed that a truck usually visits at most three customers but as opposed to Cornillier et al. (2008a, 2008b) we do not impose any limit on the number of stops. The routing is only made for the city-to-city network and the distances between the customers located in the same city are not taken into account[1]. This is due to the fact that the company is charged for the long distance trips per kilometer basis and pays a fixed-cost for each additional customer serviced in the same city. For example, if a truck is loaded to service five customers located in two cities (e.g. two customers in the first and three in the second), it first visits the closest city and makes the deliveries of the two customers, and then moves to the next city to service the remaining three customers. At the end of its trip, the truck returns to the plant. The total cost to the company is determined according to the distance of the first city to the plant and the additional customer serviced in the same city plus the distance between the first and second cities and the two extra deliveries in that city. The company does not pay for the return trip to the plant, which

[1] Two towns distant from their city centers are considered as cities due to the fuel costs.

makes the routing problem an OVRP. In this paper, we refer to the distance-related variable cost as the *routing cost* and the cost per each additional customer visited in a city as the *visiting cost*.

2.1.3. Products

The company produces and distributes 130 different products in total. There are eight basic product families: base oil, hydraulic oil, engine oil-fuel, engine oil-diesel, marine, gear oil, gear oil-color, and special products. Each product family consists of product groups which differ with respect to lube concentration and specification. Since the products are liquid two different products cannot be mixed within the same tank. In addition, when a lube is unloaded it leaves some residue inside the tank. This residue may affect the quality of the lube that will be loaded next. So, the tank may require a cleaning operation depending on the lube type last loaded in the tank. For example, changing from a darker (thicker) lube to a lighter (thinner) lube requires the cleaning of the tank. On the other hand, in the opposite case cleaning is not necessary since the lighter lube will not affect the quality of the darker lube. The cleaning is performed using plain water and the associated cost is negligible.

2.1.4. Scheduling

The Sales Department receives the orders on a daily basis and assigns each order with an estimated delivery date. However, the planned delivery date is finalized after an advanced payment from the customer has been confirmed. The company has a two-day flexibility in determining the delivery date for consolidation purposes, i.e. an order can be delivered two days before or after its planned delivery date. In this study, we refer to the latest day that the delivery must be made as the due date of the order. That is, a demand with due date 5 can be satisfied in any of the days 1, 2, 3, 4, or 5. Therefore, the distribution problem is a multi-period problem which is solved for each day on a rolling horizon basis.

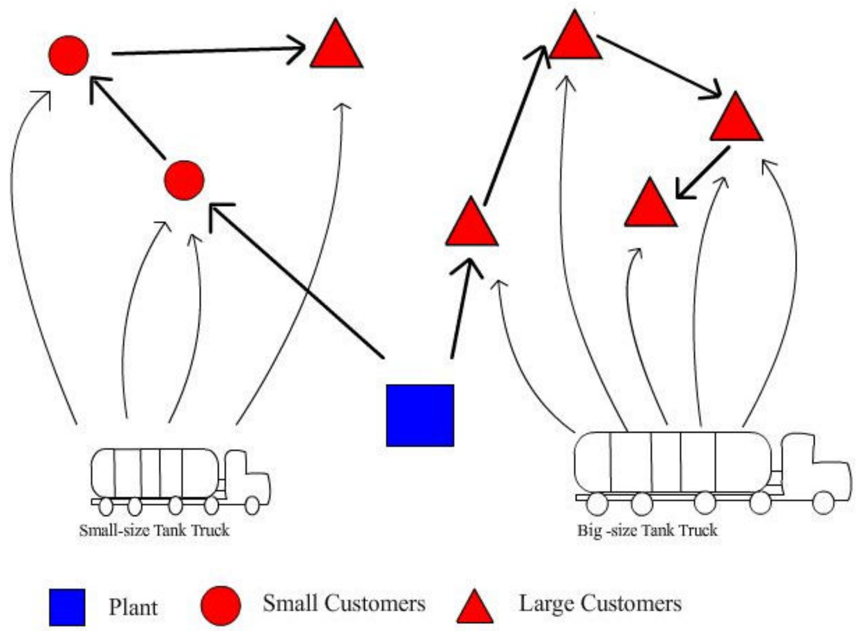


Figure 2. An illustrative example

In summary, the problem consists of two integrated problems: the assignment of customer orders to the tanks of the trucks and the routing of the trucks. Figure 2 depicts an illustrative example of a loading and routing scheme for two different truck types. The objective of the problem is to minimize the total distribution cost over the planning horizon. However, the realized total cost is calculated as the sum of the distribution costs of the first days in the planning period since the problem needs to be solved every day to finalize the delivery schedule of the next day only.

2.2. Model Formulation

In this section, a 0-1 mixed integer linear programming model is developed in an attempt to obtain optimal distribution plans. The planning horizon is one week, i.e. five days since no delivery is made during the weekends. Day 1 is the next business day when the trucks need to be dispatched. The model is solved every day for the 5-day planning period on a rolling horizon basis; however, only the distribution plan of the next day (i.e. the sub-solution involving day 1) is to be implemented and frozen. The input data are updated next day and the model is re-solved. The notation and the mathematical formulation are as follows:

Notation

T set of days

P set of products

K set of customers

J_t set of tank trucks available on day t

I_j set of tanks in tank truck j

R set of cities

J_t^B set of big-size tank trucks available on day t

K_r set of customers located in city r

K_s set of small customers

Q_j maximum weight restriction on truck j

Cap_{ij} capacity of tank i of truck j

D_{kpt} demand of customer k for product p with due day t

$d_{rr'}$ distance from city r to city r'

c_v cost of visiting an additional customer in a city (visiting cost)

c_r cost per km (routing cost)

Decision Variables

x_{ijkpt} fraction of tank i of truck j filled with product p ordered by customer k and due on day t

$$y_{ijkpt} = \begin{cases} 1, & \text{if } x_{ijkpt} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$q_{jkt} = \begin{cases} 1, & \text{if truck } j \text{ serves customer } k \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$z_{jrt} = \begin{cases} 1, & \text{if truck } j \text{ visits city } r \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$p_{jt} = \begin{cases} 1, & \text{if truck } j \text{ is in service on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$v_{jrr't} = \begin{cases} 1, & \text{if truck } j \text{ visits city } r' \text{ immediately after city } r \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

u_{jrt} sub-tour elimination variable

Mathematical Model

$$\text{Min} \quad \sum_{t\in T}\sum_{j\in J_t}\sum_{r\in R} c_v\left(\sum_{k\in K_r} q_{jkt} - z_{jrt}\right) + \sum_{t\in T}\sum_{j\in J_t}\sum_{r\in R}\sum_{\substack{r'\in R\\ r'\neq r}} c_r d_{rr'} v_{jrr't} \qquad (1)$$

subject to

$$\sum_{t=1}^{\tau}\sum_{j\in J_\tau}\sum_{i\in I_j} Cap_{ij} x_{ijkpt} \geq D_{kp\tau} \qquad \tau\in T, k\in K, p\in P \qquad (2)$$

$$x_{ijkpt} \leq y_{ijkpt} \qquad t\in T, j\in J_t, i\in I_j, k\in K, p\in P \qquad (3)$$

$$\sum_{i\in I_j}\sum_{k\in K}\sum_{p\in P} Cap_{ij} x_{ijkpt} \leq Q_j \qquad t\in T, j\in J_t \qquad (4)$$

$$\sum_{k\in K}\sum_{p\in P} y_{ijkpt} \leq 1 \qquad t\in T, j\in J_t, i\in I_j \qquad (5)$$

$$y_{ijkpt} \leq q_{jkt} \qquad t\in T, j\in J_t, i\in I_j, k\in K, p\in P \qquad (6)$$

$$q_{jkt} \leq z_{jrt} \qquad t\in T, j\in J_t, r\in R, k\in K_r \qquad (7)$$

$$q_{jkt} \leq p_{jt} \qquad t\in T, j\in J_t, k\in K \qquad (8)$$

$$\sum_{\tau=t}^{|T|} p_{j\tau} \leq 1 \qquad t\in T, j\in J_t \qquad (9)$$

$$y_{ijkpt} = 0 \qquad t\in T, j\in J_t^B, i\in I_j, k\in K_s, p\in P \qquad (10)$$

$$z_{j0t} = 1 \qquad t\in T, j\in J_t \qquad (11)$$

$$\sum_{r'\in R} v_{jrr't} = z_{jrt} \qquad t\in T, j\in J_t, r\in R \qquad (12)$$

$$\sum_{r'\in R} v_{jr'rt} = \sum_{r'\in R} v_{jrr't} \qquad t\in T, j\in J_t, r\in R \qquad (13)$$

$$u_{jrt} - u_{jr't} + 5v_{jrr't} \leq 4 \qquad t\in T, j\in J_t, r\in R, r'\in R, r\neq r' \qquad (14)$$

$$x_{ijkpt} \geq 0 \qquad t\in T, j\in J_t, i\in I_j, k\in K, p\in P \qquad (15)$$

$$y_{ijkpt} \in \{0,1\} \qquad t\in T, j\in J_t, i\in I_j, k\in K, p\in P \qquad (16)$$

$$q_{jkt} \in \{0,1\} \qquad t\in T, j\in J_t, k\in K \qquad (17)$$

$$z_{jrt} \in \{0,1\} \qquad t\in T, j\in J_t, r\in R \qquad (18)$$

$$p_{jt} \in \{0,1\} \qquad t\in T, j\in J_t \qquad (19)$$

$$v_{jrr't} \in \{0,1\} \qquad t\in T, j\in J_t, r\in R, r'\in R, r\neq r' \qquad (20)$$

$$1 \leq u_{jrt} \leq 5 \qquad t\in T, j\in J_t, r\in R \qquad (21)$$

The objective function (1) minimizes total routing and visiting costs. Here, if a truck services more than one customer in a city $\left(\sum_{k\in K_r} q_{jkt} - z_{jrt}\right)$ counts the number additional customers visited and incurs a visiting cost. The cost of traveling from any city to plant is set to zero to have an OVRP environment. Constraint set (2) makes sure that a customer demand is satisfied on or before its latest delivery date. Constraints (3) link the binary variables y with the continuous assignment variables x: if tank i of truck j is filled with demand p of customer k due on day t ($x_{ijkpt}>0$) then that tank is utilized ($y_{ijkpt}=1$). Constraint set (4) ensures that total load on a truck does not exceed the maximum weight restriction. Constraints (5) make sure that only one product is loaded on a tank. Constraint set (6) assure that if tank i of truck j is used for servicing customer k on day t ($y_{ijkpt}=1$) then the tank truck j must serve customer k on that day ($q_{jkt}=1$). Constraints (7) make sure that if customer k is served by truck j on day t ($q_{jkt}=1$) then that truck visits the city where that customer is located at on the same day t ($z_{jrt}=1$). Constraints (8) determine the days during which the trucks are on service. Since the returns of the trucks during the planning horizon are not considered constraint set (9) ensures that a tank truck is dispatched only once during the planning horizon. Note that the expected return days of the trucks on the road are taken into account when solving the problem the next day with the updated data. Constraint set (10) makes sure that small customers are not serviced using big trucks. Constraints (11) set the plant as the origin of all available trucks. Constraints (12) link the visiting variables with the routing variables. Constraints (13) impose that the same tank truck enters and leaves a visited city. (14) are the Miller, Tucker and Zemlin (1960) sub-tour elimination constraints. Finally, constraints (15)-(21) define the decision variables.

Since this problem is intractable in real-life industrial environment, we propose in the next section a greedy LP relaxation-based algorithm and a heuristic approach with two variants in an attempt to obtain good solutions in reasonable computational time. Note that we also considered the following tighter sub-tour eliminations constraints:

$$w_{jrt} - w_{jr't} + Q_j v_{jrr't} \le Q_j - \sum_{i\in I_j}\sum_{k\in K_{r'}}\sum_{p\in P} Cap_{ij} x_{ijkpt} \qquad t\in T, j\in J_t, r\in R, r'\in R, r\ne r' \quad (14')$$

$$\sum_{i\in I_j}\sum_{k\in K_r}\sum_{p\in P} Cap_{ij} x_{ijkpt} \le w_{jrt} \le Q_j \qquad t\in T, j\in J_t, r\in R \quad (21')$$

where w_{jrt} is an additional continuous variable associated with each city r (Kulkarni and Bhave, 1985; Desrochers and Laporte, 1991; Kara et al., 2004). However, we observed that the performance of the LP relaxation-based algorithm using these constraints was poorer. This might be due to the fact that loose constraints result in a larger feasible region which, in turn, allows the algorithm to find certain solutions which are not achievable otherwise.

3. Solution Methodology

Our first solution approach is an LP relaxation-based algorithm and the second is a rolling-horizon threshold heuristic for which two different variants are presented. As mentioned earlier, the distribution plan is made daily and the plan of the following day is implemented. So, the proposed algorithms are also designed to finalize the delivery schedule of the next day by iteratively solving them every day.

3.1. Linear Programming Relaxation-based Heuristic (LPH)

The proposed LP relaxation-based heuristic (LPH) basically utilizes the LP relaxation with some rounding techniques and tries to find a good feasible solution for the original problem. Our initial experiments on the LP problem have shown that the existence of visiting costs in the objective function causes inefficiently utilized tank trucks in the solutions. For this reason, our LP relaxation-based algorithm is implemented by considering the routing costs only.

Step 0.	Initialize the LP problem and solve it.
Step 1.	Select a demand arbitrarily with due date 1 (D_{kp1}). If all demands with due date 1 are satisfied, go to Step 4.
Step 2.	Find the maximum y_{ijkp1} corresponding to D_{kp1} and set it equal to 1. If $D_{kpt} \geq Cap_{ij}$ let the corresponding x_{ijkpt} =1; otherwise set $x_{ijkpt} = D_{kpt} /Cap_{ij}$
Step 3.	Re-solve the LP problem. If the LP problem is infeasible, let previously set y_{ijkpt} and x_{ijkpt} equal to 0. Otherwise; if the selected demand D_{kp1} is satisfied go to Step 1, else go to Step 2.
Step 4.	Select a partially loaded truck. If there is none, go to Step 7.
Step 5.	For the selected truck, find the maximum $y_{ijkpt} < 1$ and set it equal to 1. If $D_{kpt} \geq Cap_{ij}$ let the corresponding x_{ijkpt} =1; otherwise set $x_{ijkpt} = D_{kpt} /Cap_{ij}$
Step 6.	Re-solve the LP problem. If the LP problem is infeasible, let previously set y_{ijkpt} and x_{ijkpt} equal to 0. Otherwise; if all y_{ijkpt} variables corresponding to the selected truck are 1 or 0, go to Step 4, else go to Step 5.
Step 7.	Terminate.

Figure 3. Description of LPH

In the original model, recall that the binary variables y are used for the assignment of the tanks of the trucks and the variables x are used to determine the utilization of the tanks. In this algorithm y's are the key variables because the algorithm first finds the loading scheme of the tanks with respect to the customer orders then routes the tank trucks with respect to the truck loads.

The basic idea in LPH is to satisfy the demands of the first day and then to assign the remaining orders to the available tanks of the partially loaded trucks to efficiently utilize their capacities. Firstly, the data of the LP model is initialized and the model is solved to optimality. Then, the algorithm selects a demand with due date 1 and assigns it to the tank for which the associated y_{ijkpt} value is the largest. After having satisfied all demands with due date 1, the algorithm attempts to load the remaining empty tanks of the partially loaded tank trucks with the waiting orders in the planning horizon. The steps of LPH are detailed in Figure 3.

Once the demands are assigned to tank trucks, the routes can be obtained by finding a Hamiltonian path originating from the plant. Furthermore, since a tank truck can visit at most five different cities the optimal route of each truck may easily be determined.

To determine the plan of the next day, the demand information and the set of available tank trucks are updated according to the solution of the previous day along with relevant additional data that may become available and the algorithm is re-run.

3.2. Rolling-horizon Threshold Heuristics

The primary objective in the rolling-horizon threshold heuristic approach is to find a minimum cost distribution plan by satisfying the demands with due date 1, as is the case in LPH. We propose two variants: the first uses the distance priority whereas the second has a due date priority in selecting the next customer order to be assigned.

3.2.1. Rolling-horizon Threshold Heuristic 1 (RHTH1)

Rolling-horizon Threshold Heuristic 1 (RHTH1) aims at assigning the demands of small customers first, starting with the customer that has a due date 1 and that is farthest to the plant. When all small customers have been served the algorithm assigns the demands of the large customers in the same way. The threshold parameter λ is used for controlling the insertion of a new customer demand into an existing tour. The RHTH1 procedure is depicted in Figure 4.

Step 0.	Initialize the data. Set the threshold parameter λ.
Step 1. a.	Select the small customer farthest to the plant with a demand due on day 1 (D_{kp1}) of. If none exists, go to Step 2a.
b.	Select an available small tank truck that has the maximum weight restriction.
c.	Assign the selected demand to the selected tank truck using the *PutDemand* (*PD*) procedure.
d.	Assign the selected tank truck to the not-yet satisfied demands of small customers using the *FillTruck* (*FT*) procedure.
e.	Assign the selected tank truck to the remaining not-yet satisfied demands using *FT*.
f.	Update the set of available the tank trucks and go to Step 1a.
Step 2. a.	Select D_{kp1} of the customer farthest to the plant. If none exists, go to Step 3.
b.	Select an available tank truck that has the maximum weight restriction.
c.	Assign the selected demand to the selected tank truck using *PD*.
d.	Assign the selected tank truck to the not-yet satisfied demands of large customers using *FT*.
e.	Update the set of available the tank trucks and go to Step 2a.
Step 4.	Terminate.

Figure 4. Description of Rolling-horizon Threshold Heuristic 1

PutDemand Procedure

The *PutDemand* (*PD*) procedure utilizes the well-known best-fit heuristic used for solving the bin packing problem in an attempt to maximize the tank utilization. The selected demand is loaded to the best fitting tank if the tank capacity is sufficient, i.e. to the tank that will provide maximum utilization. If the tank capacity is not sufficient, the tank with the maximum capacity is fully filled and the remaining portion of the demand is loaded to a second tank following the same best-fit logic.

FillTruck (FT) Procedure

Given a set of customer demands to be satisfied and an available tank truck, the *FillTruck* procedure (*FT*) iteratively assigns those demands to the tank truck using *PD*. If the given tank truck is completely empty, then *FT* assigns the demand of the farthest customer to the plant which has a due date 1. If the tank truck is partially loaded, then *FT* attempts to assign the order of the customer that is nearest to the previously assigned customer(s) by considering the increase in the routing cost. The additional cost of adding city *r”* to a route is calculated as follows:

$$\text{Insertion cost} = min\left\{C_{r'r''},\ \min_{\forall arc(r,r')}\left\{C_{rr''} + C_{r''r'} - C_{rr'}\right\}\right\}$$

where C_{mn} is the cost of visiting city *n* immediately after city *m*. The first term in the formula corresponds to appending city *r"* to the end of the route whereas the second term evaluates the insertion of city *r"* between all pairs of subsequent cities *r* and *r'* and selects the one giving the minimum cost. The demand with the minimum insertion cost is assigned to the tank truck using *PD* if its insertion cost is less than λ. The procedure is repeated until all orders have been assigned or all the tanks of the truck have been loaded. The steps of the FT procedure are given in Figure 5.

Step 1.	If the tank truck is completely empty, go to step 2; otherwise, go to step 3.
Step 2.	Select the demand with due date 1 and farthest to the plant and load it using *PD*. Go to step 3.
Step 3.	If there exists an order from a customer located in the same city as the previous demand assigned, go to step 4; otherwise, go to step 5.
Step 4.	Select the demand with the earliest due date and go to step 6.
Step 5.	Find the customer with the minimum insertion cost. If the minimum insertion cost is smaller than λ go to step 6; otherwise, go to step 7.
Step 6.	Load the order using *PD* and go to step 3.
Step 7.	Terminate.

Figure 5. Description of *FillTruck* Procedure

The parameter λ plays an important role in the performance of the heuristic. If λ is set too high then the utilization of the trucks are expected to increase; however, the total routing cost may increase as well due to the servicing of distant customers. If λ is set too low then more trucks may be needed due to the decrease in the utilization of the trucks, which in turn will increase the distribution costs as well.

3.2.2. Rolling-horizon Threshold Heuristic 2 (RHTH2)

Similar to RHTH1, Rolling-horizon Threshold Heuristic 2 (RHTH2) assigns the demands of the small customers first and satisfies the demands of large customers next. In RHTH2, a truck is loaded by the customer orders with due date 1 using the *FT* procedure, as is the case in RHTH1. Then, the remaining tanks of the truck are assigned with the waiting orders chronologically, i.e. the demands with due dates 2, 3, 4, and 5 in this sequence. This

Step 0.		Initialize the data. Set the threshold parameter λ.
Step 1.	a.	Select an available small tank truck that has the maximum weight restriction.
	b.	Assign the demands of small customers that have due date 1 using *FT*. If there exist a not-yet satisfied demand with due date 1, go to step 2a.
	c.	Assign the demands of small customers that have due dates 2, 3, 4, 5 using *FT*.
	d.	Assign the remaining not-yet satisfied demands due on days 1, 2, 3, 4, 5 using *FT*.
	e.	Update the set of available tank trucks and go to Step 1a.
Step 2.	a.	Select an available tank truck that has the maximum weight restriction.
	b.	Assign the demands of large customers with due date 1 using *FT*.
	c.	Assign the demands of large customers with due dates 2, 3, 4, 5 using *FT*.
	d.	Update the set of available the tank trucks and go to Step 2a.
Step 3.		Terminate.

Figure 6. Description of Rolling-horizon Threshold Heuristic 2

difference between RHTH1 and RHTH2 can be interpreted as RHTH1 has a distance priority while RHTH2 has a due date priority. The steps of RHTH2 are given in Figure 6.

To further improve the solution quality we perform a 2-opt local search (LS) procedure to the results of both heuristics. LS considers all pair-wise exchanges, both within a route and between different routes, and performs the one which provides the maximum improvement.

Table 2. Distance-based routing costs

	Plant	IST	KOC	SAK	BOL	ANK	ADA
Plant	0	875	475	572	982	1375	3000
IST	0	0	400	532	943	1630	3380
KOC	0	400	0	133	543	1231	2980
SAK	0	532	133	0	410	1098	2847
BOL	0	943	543	410	0	687	2437
ANK	0	1630	1231	1098	687	0	1764
ADA	0	3380	2980	2847	2437	1764	0

Table 3. Demand data

Customer Id	Product	Quantity (tons)	Due date
IST1	P1	3.0	1
KOC1	P3	2.0	1
BOL1	P2	2.8	1
IST2	P3	5.0	1
ANK1	P4	4.5	1
SAK1	P1	1.5	1
ADA1(S)	P5	3.5	1
IST3	P2	1.0	2
ANK2	P4	2.2	3
SAK2	P3	2.0	3
IST4(S)	P1	2.0	4
ADA2	P2	3.0	5
BOL2	P3	1.0	5

3.2.3. An Illustrative Example

To illustrate the working mechanism of RHTH1, we provide a small example with 6 cities and 13 orders to be planned. The distance-based routing costs between the cities are shown in Table 2 and the customer orders are given in Table 3. The first 3 letters of the "Customer Id" indicates the city where the customer is located in. The additional notation "(S)" denotes that the corresponding customer can only be serviced with a small-size truck. For instance, "KOC1" denotes the customer #1 in Kocaeli which can be serviced by either big- or small-size truck whereas "IST4(S)" denotes customer #4 in Istanbul which can only be serviced by a small-size truck.

Figure 7 illustrates the solution obtained using RHTH1 by setting threshold parameter λ=500. The step-by-step description of the procedure is as follows:

(1) ADA1 is selected as the small customer that has an order with due date 1 and that is farthest to the plant and its demand for P5 is assigned to the small truck (tank truck 1) with the maximum weight restriction using the best-fit approach. Since the order size exceeds the capacity of all the tanks of the truck, the order is loaded into two different tanks.

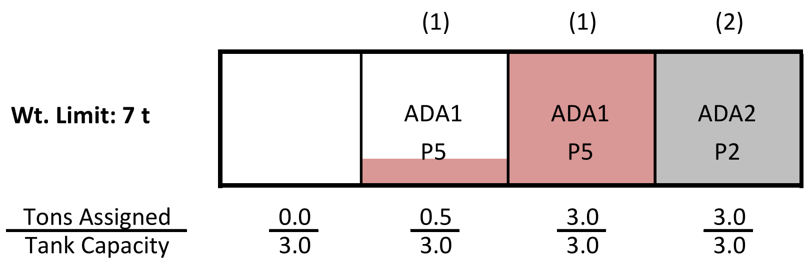
(2) IST4 is selected as the next small customer and its insertion cost is calculated as follows:

$$\min\{C_{rd}, (C_{rd} + C_{0d} - C_{0r})\} = \{3380, (3380 + 875 \text{-} 3000)\} = 1255$$

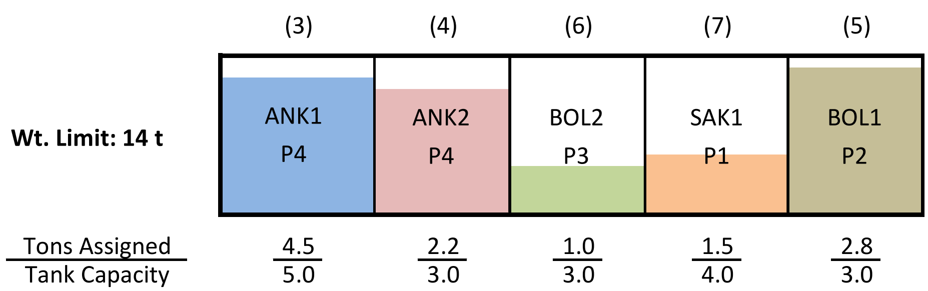
Since 1255>500 and the order of IST4 is due on day 4, it is not assigned.

Since no other small customer order exists, we select next the customer that is located to the closest customers who have already been assigned: P2 of ADA2 is assigned to the tank truck with best-fit. No more orders can be loaded to truck 1 due to its maximum weight restriction.

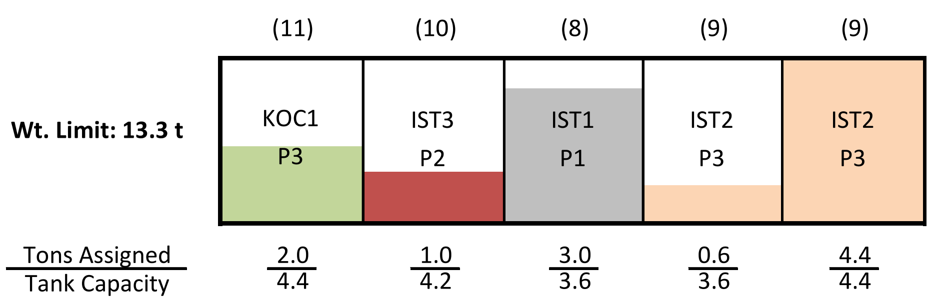
(3) Select the demand of the customer farthest to the depot which is due on day 1: P4 of ANK1 is selected and assigned to the available tank truck with maximum weight restriction (tank truck 2).



(a) Tank truck 1: 0 → ADA



(b) Tank truck 2: 0 → SAK → BOL → ANK



(c) Tank truck 3: 0 → KOC → IST

Figure 7. Solution obtained using RHTH1

(4) Select the customer nearest to ANK1: ANK2 is selected and its order for P4 is assigned.

(5) Select the customer nearest to ANK2: BOL1. Its insertion cost is min {687, (687+982-1375)} = 294 < 500. So, P2 of BOL1 is assigned.

(6) BOL2 is selected following BOL1 since it is located in the same city and its order for P3 is assigned next.

(7) SAK1 is selected as the nearest customer to BOL2. Its insertion cost is min {410, (410+572-982)} = 0, which means that SAK1 is on the way to BOL2. Hence, P1 of SAK1 is assigned next. Since all the tanks are filled, a new truck will be selected and loaded.

(8) Select the demand of the customer farthest to the depot which is due on day 1: P1 of IST1 is assigned to the available tank truck with maximum weight restriction (tank truck 3).

(9) IST2 is located in the same city as IST1: P3 of IST2 is assigned.

(10) P2 of IST3 is assigned next.

(11) KOC1 is selected as the nearest customer to IST3: min {400, (400+475-875)} = 0 and P3 is assigned last.

Since all demands due on day 1 are satisfied, the assignment phase terminates. Note that because IST4 can only be serviced by a small-size truck its order is not assigned and left for the planning the next day since its due date is 4.

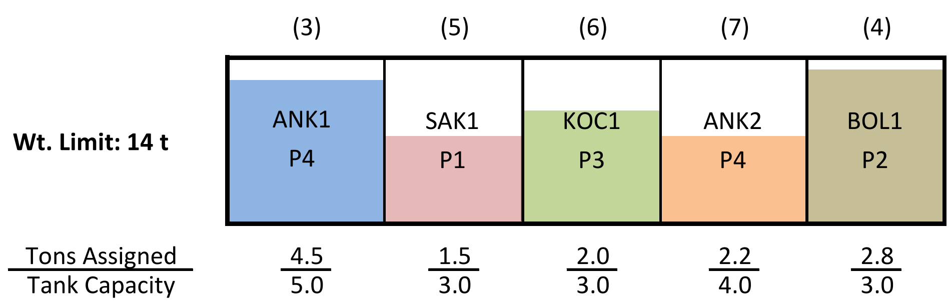


Figure 8. Loads on tank truck 2 using RHTH2: 0 → KOC → SAK → BOL → ANK

The difference between RHTH1 and RHTH2 procedures is shown in Figure 8 using the partial solution for the loading scheme on tank truck 2. Note that RHTH2 first loads truck 1 as depicted in Figure 7.(c). On truck 2, the order P4 of ANK1 is assigned first as in

RHTH1. Next, instead of considering all the orders in the list RHTH2 takes into account only the orders with due day 1. Hence, the nearest customer to ANK1 with an order due on day 1 is found as BOL1 and its order P2 is assigned next. Then, the orders of SAK1 and KOC1 are loaded, respectively. Finally, since none of the remaining orders due on day 1 is feasible with respect to maximum weight restriction order P4 of ANK2 due on day 3 is assigned to the last tank.

4. Numerical investigation

In this section, we test the performances of proposed heuristics using the real data of BP Lubes Logistics Operations. In our preliminary analysis, we use one-month data to first investigate the sensitivity of RHTH1 and RHTH2 to the value of threshold parameter λ and then to compare the numerical results given by RHTH1, RHTH2, and LPH as well as the upper bounds obtained by using IBM ILOG CPLEX 11.0. Next, we report the costs achieved by RHTH1 and RHTH2 in comparison with the costs realized by BP using a new data set spanning a quarter. The computational study is performed on a notebook computer equipped with Intel Celeron 1.6 GHz processor and 1 GB Ram. The algorithms are coded in Java programming language.

Table 4. Fleet composition

	Tank Capacities (in tons)					Truck Capacity	Max Weight Limit
Truck	**1**	**2**	**3**	**4**	**5**	**(in tons)**	**(in tons)**
1	6.0	3.0	3.0	3.0	5.0	20.0	13.3
2	5.1	4.4	4.9	4.3	5.2	24.0	19.1
3	5.0	4.4	4.8	4.2	5.0	23.4	19.1
4	5.0	3.5	3.8	3.5	4.0	19.8	13.5
5	5.0	3.5	3.8	3.8	4.0	20.0	13.3
6	4.5	3.7	3.6	3.7	4.5	20.0	13.3
7	4.4	4.2	3.6	3.6	4.4	20.1	13.3
8	5.0	3.0	3.0	4.0	3.0	18.0	14.0
9	5.0	3.0	3.0	4.0	3.0	18.0	14.0
10	3.0	3.0	3.0	3.0	-	12.0	7.0
11	3.0	2.0	2.0	2.0	3.0	12.0	7.8
12	2.0	2.5	2.1	2.4	2.0	11.0	7.8

The data consist of the cities where the customers are located at and the associated distance matrix, the order quantities with their due dates, and tank truck related information such as the maximum weight restriction, number of tanks and their capacities. The fleet of the

3PL dedicated to the distribution of bulk lubes consists of 10 tank trucks. As mentioned earlier, if more trucks are needed they are hired from the spot market. Therefore, we considered 2 more additional trucks for capacity flexibility. Out of the 12 tank trucks considered, 3 are small-size and 9 are big-size trucks. The details about the fleet are given in Table 4.

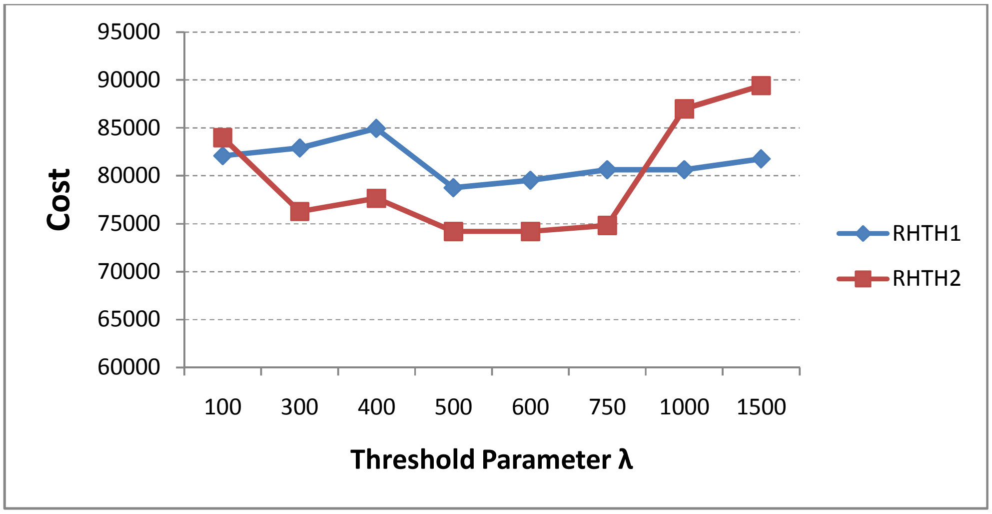


Figure 9. The effect of threshold parameter λ on the solution quality

We have noted earlier that the threshold parameter λ is an important and integral component affecting the performance of RHTH1 and RHTH2. To observe its role in the solution quality, we perform a sensitivity analysis by solving the problem on the monthly data for varying values of λ between 100 and 1500. Note that we did not perform the LS to better observe the effect of λ value. The total cost figures[2] are reported in Figure 9. The results show that RHTH2 is more sensitive to the threshold parameter. This is indeed an expected result since RHTH2 attempts to assign the demands in day 1 firstly until the utilization. We observe that both small and large λ values give high costs whereas intermediate λ values (500-750) provide better solution quality in both heuristics. Since λ=500 performs best in both RHTH1 and RHTH2 we utilize this value in the following comparative analysis.

[2] The cost figures are in "Monetary Units (MU)" that are kept fictitious for confidentiality reasons.

Table 5. Daily costs using the preliminary data

Day	RHTH1	RHTH2	LPH	CPLEX
1	14580	9463	14546	17069
2	4848	4492	6168	7768
3	2625	3607	5276	3643
4	475	2019	2372	1375
5	8516	7641	7850	7322
6	12635	13539	13520	18047
7	4000	4000	3500	5554
8	1750	2625	875	3769
9	875	3409	5097	4003
10	3100	2225	3372	3578
11	3733	1980	1375	3466
12	5421	5421	7307	4237
13	3214	3603	1979	17868
14	3413	2538	3412	6386
15	9529	7632	6814	6850
Total Cost	**78714**	**74194**	**83463**	**110935**

The preliminary analysis includes 15 instances corresponding to 15 consecutive business days. We implemented the plan of the first day only and re-solved the problem for the next day after updating the data accordingly. In Table 5 we report the daily costs obtained by RHTH1, RHTH2, LPH, and CPLEX and in Figure 10 we summarize the weekly costs. CPLEX upper bounds are obtained by setting the global time limit to 3000 seconds. The lower bounds are omitted because the optimality gap varies around 90% and they do not provide any meaningful information. Note that the LS is skipped to make a fair comparison. Note also that the cost figures reported in the table are the costs of the 1st days of the 5-day planning horizon obtained after 15 consecutive runs, updating the data after each run. Although we have monthly data the results include only the first three weeks of the month due to the fact that the problem is solved on a rolling horizon basis and the plan for the 4th week requires the data of the 5th week. We limit the data size with one month since performing 15 runs while updating the data manually for CPLEX is too time consuming. Besides, we believe that the data size in this preliminary investigation is sufficient to provide insights with regard to the performances of the algorithms proposed.

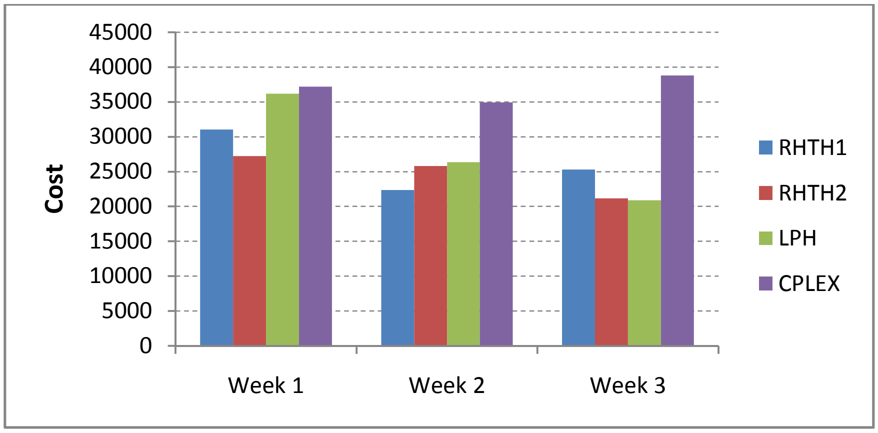


Figure 10. Comparison of preliminary results based on weekly costs

The results indicate that the relative performance of each algorithm differs from one day to another, even from one week to another. This is expectable due to the solution construction mechanisms and the criteria they include. Because of the rolling horizon nature, the results obtained in the first few days may be misleading and an overall cost analysis may be more meaningful. Firstly, we observe that both rolling-horizon threshold heuristics provide competitive results. Their performances are superior in particular when compared with the CPLEX. Although LPH has a comparable performance against RHTH1, RHTH2 outperforms it with a significant total cost margin (12.5%). Secondly, although RHTH2 seems to outperform RHTH1 with respect to the total cost figure, further investigation is needed according to the weekly results. Furthermore, when the 3rd week is being planned some demands from the 4th week are also considered since the time horizon is not frozen. For instance, the cost of day 15 may also include the delivery of some demands due on days 16 thru 19. Hence, a heuristic may assign some of the orders due in the 4th week to the distribution plan of the 3rd week, incurring a higher distribution cost in the 3rd week. Due to its heuristic nature, RHTH1 is more inclined to do so. As a matter of fact, when we analyze the not-yet satisfied demands at the end of day 15, we observe that remaining orders in the case of RHTH1 is less than that of RHTH2. Therefore, the results will not be conclusive if the time horizon is not frozen.

When we investigate the computational time efficiency of the algorithms, we see that both RHTH1 and RHTH2 can be solved in a negligible time (less than 1 second) and their CPU time does not increase much with the growing problem size. On the other hand, LPH requires significantly more computation time: the CPU time varied from 5 minutes to 1.5

hours in 15 different runs reported in Table 5. The size of the problem determined by the active variables and constraints affects the performance of LPH substantially. So, it is hard to justify the computational effort spent for LPH by its solution quality.

Table 6. The weekly results for the three-month data

Week	RHTH1	RHTH2	Current System
1	9978	9821	10000
2	8362	9193	11833
3	14949	14193	13542
4	13339	14444	15789
5	18643	15691	18591
6	6349	7357	9691
7	18550	20112	15206
8	18104	16974	17636
9	4367	5530	8229
10	21787	20365	22917
11	9579	10684	11613
12	7829	7753	8172
13	10503	11378	18345
Total Cost	**162339**	**163495**	**181564**

In the current system in practice, the company has some pre-determined routes or clusters of cities that can be serviced by the same tank truck. The dispatcher develops the daily delivery plans according to these routes manually using MS Excel. To further investigate the performances of the rolling-horizon threshold heuristics and compare them with the realized costs, we perform an extended computational study on a real data of 3 months. To better evaluate both algorithms fairly, we freeze the time horizon at the end of 13th week, i.e. the demands due thereafter are not considered. A total of 65 runs were performed for 65 consecutive business days (13 weeks * 5 days) by implementing the plan for first day only and updating the data at the end of each run according to this plan. The weekly costs are shown in Table 6 and a monthly comparison is given in Figure 11. We observe that RHTH1 and RHTH2 outperform the current system by 11.8% and 11.1%, respectively. Furthermore, the performance of RHTH1 is slightly better than that of RHTH2. These results are promising in the sense that both of the proposed rolling-horizon threshold heuristics are capable of improving the current distribution costs of the company substantially. It is also worth noting that the experimental data belongs to a period of economic downturn during which the customer orders slowed down. Therefore, the benefits of the proposed approach might be more substantial when the economy ramps up.

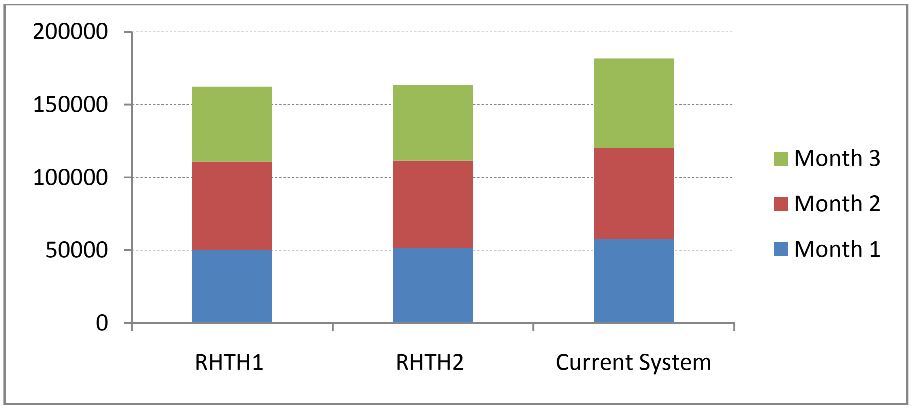


Figure 11. Results of extended experimental analysis

5. Conclusions and Future Research

In this study, we addressed the distribution planning problem of bulk lubricants of BP Turkey. We formulated this large-scale industrial problem as a 0-1 mixed integer program and proposed an LP relaxation-based and two rolling-horizon threshold heuristic approaches to efficiently solve it. The performances of the proposed heuristics were tested using the real data of the company. The numerical results revealed that rolling-horizon threshold heuristics RHTH1 and RHTH2 were efficient in terms of both computational effort and solution quality.

The advantages of using the rolling-horizon threshold heuristics are threefold: First, they are both cost efficient and easy to implement. Second, they can significantly reduce the efforts of the logistics planners who manually load and dispatch the trucks based on their experiences in the current practice. Third, they can standardize the planning operations. Since these heuristics do not require any commercial solver they can be implemented and integrated with the company's database system with little effort. The computation time they require and the ease of their usage would greatly facilitate the decision-making process in the company. Furthermore, the data and model parameters can be easily modified to conduct sensitivity analyses. The company is currently in the process of integrating their ERP system into the Lubes Division and evaluating the implementation of one of the two heuristics.

Future research on this problem may consider the cleaning (setup) costs of the tanks when switching from one lube type to another and the use flow-meter devices. Although the cleaning operation only consumes water and would have a minor effect on the total cost, the water consumption may arise as an important criterion from a "green logistics" perspective.

The impact of equipping trucks with a flow-meter requires detailed investigation. What-if type analyses may be performed to evaluate the benefit of installing the flow-meter to all or some of the tank trucks. The flow-meters would not only affect the routing costs but may also reduce the number of trucks needed, hence may affect the annual contract negotiations with the 3PL. Finally, although the company currently has ± 2-day delivery flexibility it does not know the possible implications of the early and tardy deliveries. If the related parameters are determined, the model can be extended to involve penalties associated with the early and tardy deliveries.

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