[[1]](#footnote-1) **An Effective Interpolation Scheme for Multi-Resonant Antenna Responses: Generalized Stoer-Bulirsch Algorithm with Adaptive Sampling**

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*Abstract— Multiple analysis calls and the high computational demand for each analysis present themselves as important bottlenecks in conducting practical electromagnetic design optimization studies particularly for complex material designs in metamaterial studies. To enable efficient reanalysis in such studies, in this paper we develop an algorithm for rational interpolation by generalizing the Stoer-Bulirsch algorithm to allow for non-diagonal Neville paths rather than the known standard diagonal path to enhance its interpolation capability. The algorithm is then integrated to an adaptive sampling strategy that exploits the non-diagonality and finds an optimum Neville path that provides more efficient and reliable fittings for multi-resonance antenna response functions. The resulting technique is applied to return loss responses of antenna models with textured material substrates and complex conductor topologies. Interpolation results are compared to the performance of a standard Stoer-Bulirsch algorithm. Results show that the proposed generalized scheme outperforms the existing Stoer-Bulirsch technique in terms of computational accuracy by detecting resonances while still maintaining minimum number of support points. To demonstrate the capability of the proposed generalized algorithm, it is adapted to a large-scale antenna design optimization example achieving significant bandwidth performance enhancements for a novel antenna structure within practical timespans.*

*Index Terms—Approximation theory, frequency response, microstrip antennas, genetic algorithms, Stoer-Bulirsch algorithm, adaptive sampling.*

# **INTRODUCTION**

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ESIGN optimization has been a difficult, demanding but necessary task for the development of new wireless applications. It is reasonable to expect that designs resulting from global design optimization studies that allow for full design space exploration will lead to novel configurations with enhanced performance. For instance, such designs include antenna shape, size, feed location and material variations in 3D. However, global large-scale volumetric antenna synthesis via heuristic techniques is a challenging task due to the need for fast and accurate re-analysis and proper definition of the optimization model. Such a synthesis framework is applied to novel metamaterial designs with complex material variations and conductor topologies particularly. Therefore, unless antenna design studies are limited to only a few number of design variables [1] such as in conventional substrate designs, global large-scale studies can become impractical, hence novel designs cannot be achieved. To address this issue, an approximation scheme suitable for frequency response functions of electromagnetic systems such as return loss curves of multi-resonance antennas with novel material and conductor topologies is proposed in this paper.

Rational functions offer an attractive solution for providing a global approximation taking into account the entire band of the frequency response into consideration. In addition, they are well suited for approximating resonances due to their inherent pole predicting behavior. As a result, their use has resulted in various representations of resonance type curves with reasonable number of support points [2-11].

Solving for the coefficients of the rational function is known as the Cauchy method and is first introduced in [12]. It has been employed for the extraction of a circuit model [13], the response of which fits the microwave device reflection and transfer functions of non-lossy systems. Lamperez et al. [14] extended the Cauchy method in [13] to reduce the model order of systems which may be lossy. In addition to the complicated mathematics needed for deducing the resulting expression or equivalent circuits, this technique is restricted to electromagnetic optimization of devices such as filters and multiplexers. Moreover, the solution within a standard Cauchy based method with techniques such as direct inversion becomes more prone to numerical errors as the number of support points increases (by producing a system of linear equations which is ill-conditioned).

A solution to overcome ill-conditioned systems via the Cauchy method is presented in [9] This technique has the potential of accurately predicting wide-band response utilizing narrow-band information via both interpolation and extrapolation. The computations have been automated later in [8]. In both of these studies, the order of the optimum rational function is chosen such that the number of unknown coefficients is less than or equal to the rank of the corresponding stiffness matrix for interpolation. However, a multi-resonance frequency response cannot be approximated via the rank constrained strategy as it limits the order of the rational model. Also, since for each newly added point the system is re-solved, it becomes computationally expensive when integrated with adaptive sampling techniques. An alternative approach to overcome these issues is a support point recursive technique initially introduced by Stoer and Bulirsch [15].

Stoer-Bulirsch (S-B) technique is a recursive method that adds one support point at a time and solves for the unique rational interpolator that passes through all existing support points. This recursive strategy in turn enhances its suitability to adaptive sampling techniques where each new support point is determined based on a certain error norm and its interpolator is found using the available data set. When compared with direct inversion solutions of rational interpolations, Stoer-Bulirsch technique is significantly less prone to numerical errors and is not constrained by the rank of the system.

It is known that the data set used for interpolation determines the quality of the resulting fitted curve. Therefore, adaptive sampling of a frequency response constitutes a key aspect in interpolation, hence, careful selection of these informative support points to serve as input data to the interpolation technique should result in a more successful interpolation [2, 3, 16, 17]. In addition, different numbering of the support points can define different interpolators with different accuracies [10]. One widely used interpolation strategy integrated with adaptive sampling is an iterative least-squares pole-residue modeling technique known as Vector Fitting [18]. It starts with an initial set of poles and in successive steps, the poles are relocated and the residues are calculated to optimize the fit. Each stage of the algorithm reduces to a linear problem. Nevertheless, the solution requires matrix inversion which is time consuming and depends on piecewise interpolation [19] for wide broadband modeling. In the method proposed here frequency data sets have been adaptively constructed and integrated to the proper choice of the rational function within Stoer-Bulirsch technique to deliver an efficient interpolation scheme as was applied in [20, 21] with standard Stoer-Bulirsch. Yan et al. [20] proposed a similar adaptive sampling through rational functions based on alternative diagonal Neville paths created by incrementing the numerator and denominator by one after a specific switching grid is chosen, where these paths remained essentially parallel to the main diagonal. Thus, existing Stoer-Bulirsch interpolations constructed based on a pre-determined standard diagonal path (where the order of numerator and denominator of the interpolator are equal or different by one) are not guaranteed to be optimum, and therefore resulting interpolations even if adaptively constructed could be further improved for accuracy and computational efficiency.

In this paper, we present the detailed steps of how to improve the standard Stoer-Bulirsch algorithm by generalizing the Neville algorithm that allows for an off-diagonal path as suggested in [22] and integrate it with a novel adaptive sampling technique. This technique not only selects the support points but also determines the optimized path used in constructing the interpolator. Consequently it enhances the potential of detecting resonances and minimizing the interpolation error. Also, to further enhance the performance of an adaptive Neville path for multi resonance wide band responses, the proposed algorithm is integrated to an initial sampling scheme. The resulting Generalized Stoer-Bulirsch algorithm with Adaptive Sampling (G-SBAS) and initial sampling (G-SBAIS) are analyzed and compared with the standard Stoer-Bulirsch (S-B) technique. Results demonstrate the capability of the optimized method G-SBAIS to overcome common problems of existing methods such as premature convergence and missing significant resonances. As a result, the proposed algorithm leads to approximations with enhanced accuracy norms and less number of support points. The capability of the method is demonstrated by integrating it to a large scale design optimization problem of a microstrip patch antenna with a complex topology.

# **Interpolation Scheme: Generalized Stoer-Bulirsch Technique with Adaptive Sampling**

Occurrence of nulls/poles in the frequency response of antennas such as return loss, gain, and efficiency motivate the use of rational functions to approximate these quantities. In its general form, a rational function can be described via a fractional polynomial with numerator and denominator of orders  and , respectively, as

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|  | (1) |

The solution of coefficients  and  of a frequency response function in the above form , known as the Cauchy method, requires  conditions in the form of function values and/or their derivatives.

Instead of standard known techniques such as direct inversion and total least squares, which suffer from disadvantages as addressed in the introduction, here we utilize an alternative technique; Stoer-Bulirsch algorithm. In what follows we introduce the Stoer-Bulirsch technique and update it to follow a generalized Neville path that is in its most general form not diagonal.

Letting  denote the  support point and  define a rational function with  for  where  and  are polynomials of degrees  and , respectively. The Neville path is used recursively to generate rational interpolators according to the following formulas

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|  | (2) |

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|  | (3) |

where . For details and derivation the reader is referred to [15].

 By following a pre-determined diagonal path as shown in , calculation of successive interpolators, , required for each newly added support point becomes straightforward as shown by diagonal branches of the interpolator construction tree in . As an example, consider the addition of the third support point; the algorithm starts by assigning the support point functional value  to  (a constant interpolator of order 0,0 passing through the third support point), followed by  using (order 0,1 passing through support points 2 and 3), and finally  using (order 1,1 passing through the support points 1, 2, and 3) as demonstrated graphically in . All required  with negative  and/or  used in the calculation are assigned zero value. Newly constructed interpolators at the third level, and , in addition to interpolators at lower levels are available for the calculation of the last interpolator  as a result of following the diagonal Neville path. However, if the interpolators are formed following a non-diagonal path, (e.g. dashed line in ), the calculation of the final rational function becomes a difficult task since necessary interpolators may not be available. More specifically, based on the recursive scheme as suggested by formulas and , additional intermediate interpolators, ‘*intermediates*’[[2]](#footnote-2), are needed which can be evaluated recursively via the use of old support points. It is noted that these intermediates also depend on how far the path is from the diagonal.

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| fig1.tif |
| Fig. 1 Neville path used in constructing a rational function of order  and  following diagonal (solid) and non-diagonal (dashed) paths |

Once Stoer-Bulirsch technique attains an  order interpolator with  sample points depending on a specified error criterion, the new direction is chosen as either or . Consequently, a new sample point is added and the recursive iteration of the newly added point continues. Starting with ,  and  the adaptive path is calculated until the criteria of ,  and , or alternatively , and  are met. It is noted that according to recursive algorithm stated by and , the calculation of the last interpolation function  or  requires all intermediate interpolation functions such as , , and ( or ) to be known a priori. Regardless of the path chosen,  and  are always available but  and/or are in general not available unless a diagonal path is followed. The path norm , which is the maximum allowed search distance from the minimum of  and , used in determining these intermediates is defined as

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|  | (4) |

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| Fig. 2 Construction tree for interpolators  based on diagonal Neville path algorithm; each branch corresponds to a newly added support point. |

The missing intermediates are evaluated at the recursive iteration level  by evaluating the terms ,,, using if  is part of the adaptive path or evaluating , ,,  using if  is on the path. As the search distance  increases the search space for the adaptive path increases but gives rise to an increase in the number of intermediate computations as well. To achieve a tradeoff between these two effects, the value of  was restricted to 4 according to computational experiments based on trial and error. The intermediates calculated are shown graphically in for the special case of . Following such an arbitrary trajectory of the Neville path leads to numerical difficulties not encountered when a conventional diagonal path is followed. Along the horizontal axis  at  (increase of numerator order) or along the vertical axis  at  (increase of denominator order), pure polynomial Neville’s formulas, given in [15], are employed with support points  and  in order to avoid this type of instability. Also, during the calculation of the intermediates with a comparatively high order numerator and low order denominator (e.g. for  and ) fittings of pure polynomials presents a challenge for fitting sharp resonances. This leads to degradation of accuracy when high and low intermediates are being subtracted in and . Therefore, a computationally optimized path is followed throughout instead of a purely adaptive path for calculating the intermediates. This computationally optimized path is constructed by following a diagonal path until the minimum value of numerator or denominator order is attained and then a horizontal or vertical line movement to the final desired interpolator is used. It is noted that this computationally optimized path is not considered in determining the order of the final rational function and hence is not subject to the path norm constraint.

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| Fig. 3 Representative interpolators and orders necessary for following alternative paths with path norm . Paths to be followed are associated with points and corresponding interpolators with the same type (dashed or solid) |

In the following two sections we discuss an adaptive sampling technique using the proposed generalized Stoer-Bulirsch technique.

## **Initial Sampling Strategy**

Starting with good initial points enhances the convergence of the fitting scheme [19]. In this section we introduce the initial sampling algorithm employed within the generalized S-B algorithm. The algorithm divides the frequency domain into sub-regions based on information related to the occurrence of pronounced resonances measured via relative error norms of successive interpolator with the goal of confining each likely resonance in one region. During the initial sampling strategy pure polynomial interpolations are used since they tend to diversify the sampling until convergence based on a chosen error norm, , is satisfied.

The flowchart, shown in , demonstrates this initialization procedure. Here, the normalized error norm of two successive interpolators  and  is defined as

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|  | (5) |

Where , , and  and  are the numerator and the denominator order, respectively. This formulation is proposed by [23] and proved to give a good mature stopping criterion. Here, the iteration number or the number of support points  is . The normalized error norm  basically measures how much the new interpolator  differs from the previous one  and is therefore a relative convergence metric of the adaptive interpolation process.

Also, we select the next support point to be the argument that maximizes the second error norm  which is a function of  (which could correspond to frequency) as follows

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|  | (6) |

where . Hence, the next sampling point is chosen where the error of successive interpolator functions is maximum, possibly where a major resonance is likely to occur.

The algorithm starts with pure polynomial interpolation until the normalized error norm  drops below a specified threshold () that should guide towards diversity of initial support points. This threshold is a parameter that is tuned in order to sample diversely over the wideband range and captures resonances. It is noted that an increase of the chosen value has no significant effect on the number of total support points of the resulting interpolators but a decreasing effect on the overall interpolation RMS error at first up until values of 150 and an increasing effect afterwards. Therefore  is specified as 150. A suggested tuning range for  when fitting multi-resonance curves with at least 3 resonances below -2 dB is .

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| Fig. 4 Step I: Initial diverse sampling algorithm |

## **Adaptive Sampling Strategy**

The generalized interpolation algorithm that accounts for the non-diagonal Neville path was introduced in the beginning of Section 2. Having established the initial sampling scheme, this section focuses on a novel technique for integrating frequency adaptive sampling to the generalized Stoer-Bulirsch algorithm. It is noted that once a resonance is correctly interpolated, the proposed hybrid method aims to drive the search towards sampling data in regions which are likely to contain a new resonance.

The flowchart of the generalized Stoer-Bulirsch technique is shown in . The algorithm starts by constructing interpolators of previously found support points in Step I by following a diagonal path of the Stoer-Bulirsch technique. A diagonal path simply corresponds to a rational function the numerator order of which is equal to the denominator order or one less (first block of step II in ). The next sampling point, similar to Step I, is always selected according to , i.e. points maximizing the error  are selected.

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| Fig. 5 Step II: Adaptive sampling with generalized Neville path |

Regarding the generalized path selection, there are two alternative rational functions to be continued with along the non-diagonal Neville path. More specifically, the normalized error norms  given by of two rational functions  of order  and  passing through  points are compared against each other. The choice determines the next branch of the generalized path and corresponds to the one that minimizes the normalized error norm  between successive interpolators. This minimum error strategy has the effect of speeding up the interpolation convergence and consequently decreasing the required total number of support points towards meeting the criterion . Meeting the condition  successively more than once helps preventing local premature convergence. Here, the stopping criterion is reached when the condition  is met three times successively. Nevertheless, this convergence is still local, i.e. it predicts a region possibly encapsulating a major resonance to minimize the error, causing additional resonances of the overall response to be left out in the final interpolator. Following a maximum error path drives towards sampling at out-of-local regions that potentially possess resonances, Therefore, the proposed sampling strategy relies on a hybrid optimized route that is expected to effectively capture maximum number of resonances by allowing for jumps from regions where a local convergence is reached first by following a path that minimizes the normalized error $e\_{1}$ to other regions by following a path that maximizes the error . The convergence criterion for the maximum error path is reached when the error norm criterion  is satisfied for three successive interpolations. The value of  chosen as 4 based on trial and error study for this path. It is noted that within a typical range of possible  values of 3-10 a tradeoff exists between the effect of  on maximum accuracy and minimum number of support points. Therefore value of  is specified as the optimal value of 4 in the algorithm.

# **Results**

In this section, the proposed generalized S-B is first tested on return loss responses of various representative complex conductor topologies supported by heterogeneous material substrates as discussed in Section A. Then, in section B, a large scale design optimization study that relies on the proposed G-SBAIS technique of a complex antenna structure is presented.

## **Comparison of Interpolation Schemes**

Here, we present and compare results of the proposed interpolation strategy with two of its characteristic features, namely the initial sampling (G-SBAIS) and adaptive sampling strategy (G-SBAS) within generalized S-B algorithm as introduced in Section ‎II and the standard S-B. The error thresholds of all three methods are the same as given in Section II and corresponding flowcharts. Their short descriptions are given below.

1. Stoer-Bulirsch (S-B): Standard Stoer-Bulirsch algorithm following a diagonal Neville path is implemented.
2. Generalized Stoer-Bulirsch with Adaptive Sampling (G-SBAS): The adaptive sampling, explained in Section ‎II B, utilizing the generalized Stoer-Bulirsch algorithm is implemented.
3. Generalized Stoer-Bulirsch with Adaptive and Initial Sampling (G-SBAIS): The adaptive sampling (Section ‎II B) and the initial sampling (Section ‎II A) are integrated with the generalized Stoer-Bulirsch algorithm.

Above three strategies were used to construct interpolations to approximate the return loss response of complex microstrip patch antennas with arbitrary material distribution and patch conductor topologies. Comparative interpolation results for the return loss response of five antennas with different conductor and material topologies are shown in .

Each interpolation curve is compared to the original return loss response which is numerically simulated using a fine sampling rate of 1 MHz, i.e. 1001 uniformly distributed frequency points are sampled between 1-2 GHz. Since finer sampling does not improve the response further, it can be accepted as the original antenna simulation response.

To compare the performance of interpolations with respect to their capability in detecting resonances with minimum number of support points for return loss curves belonging to complex antenna designs, the proposed interpolations were compared for a set of return loss curves of five different design candidates with at least 3 resonances, where the standard Stoer-Bulirsch algorithm failed to provide a successful interpolation by missing at least one major resonance below -2dB. (a-c) depicts interpolation results with the same stopping criterion threshold of  for one of the chosen sample curves according to the three interpolation strategies. According to the results shown in (a), when a standard Stoer-Bulirsch algorithm is used, only a few major resonance of the response could be captured. Results adopting the proposed G-SBAS depicted in (b) show that additional major resonances (< -2dB) are successfully captured. Moreover, with the G-SBAIS technique, as shown in (c), all resonances are successfully captured. A full comparison of the three interpolation strategies for the five antenna design samples is conducted and provided graphically using the root mean square error (RMS) in (d). Based on the resulting accuracy norms, proposed G-SBAS and G-SBAIS techniques provide a better performance for all of the design cases when compared with the standard S-B technique. In order to evaluate each interpolation strategy’s overall performance, the average sum of error norms of all five designs is plotted in . Results show that the error in using the naive diagonal Stoer-Bulirsch algorithm has dropped by 29.3% and 67.3% when compared with the proposed G-SBAS and G-SBAIS methods, respectively. In order to compare the computational time performances of three interpolation strategies, an additional interpolation study is carried out to determine the average number of support points required to reach the same reference accuracy norm. Specifically, the number of support points to reach the same RMS error of that in G-SBAIS ( d) is determined and tabulated in . While the results of S-B and G-SBAS are almost identical they point towards a reduction in the number of support points by 13.9% when compared with G-SBAIS proving that the proposed G-SBAIS method outperforms existing S-B technique both in terms of accuracy and number of support points.

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| (a) Method 1: Standard Stoer-Bulirsch technique | (b) Method 2: Generalized Stoer-Bulirsch technique |
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| (c) Method 3: Generalized Stoer-Bulirsch technique with initial sampling | (d) Method 4: Average Error norm (Root mean square) of the three interpolation strategies for five different antenna design configurations |
| Fig. 6 Interpolation results via various interpolation strategies (a-c) and their corresponding average error norm |

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| TABLE IAverage sum of root mean square error of five designs obtained using three different interpolation strategies | TABLE IIAverage total number of support points of five designs obtained using three different interpolation strategies  |
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|  | Average RMS (dB) |
| S-B | 1.8275 |
| G-SBAS | 1.2920 |
| G-SBAIS | 0.5974 |

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|  | Average number of support points |
| S-B | 55.5 |
| G-SBAS | 56.3 |
| G-SBAIS | 47.8 |

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## **Large Scale Design Optimization via G-SBAIS**

The goal of the chosen design problem here is to improve the bandwidth of an initially unmatched miniaturized antenna structure via searching for its novel optimal material and conductor distributions in three dimensions. The problem is chosen to be symmetric around perpendicular axes passing through the center of the antenna. This symmetry is chosen merely to reduce the number of unknowns in the Genetic Algorithm (GA) encoding. The proposed generalized G-SBAIS algorithm is not only applicable to GA but can as well be used to speed-up optimization problems solved using other type of optimization solvers. The design volume corresponds to a substrate with a specified size of  and a conductor surface printed on the top of the substrate. The irregularly shaped conductor is probe-fed at  from the lower left corner of the substrate. The substrate is divided into 3 layers with 1.06 mm thickness. Each substrate layer of the antenna is discretized into  design cells with 800 finite triangular prism elements used in the Finite Element Analysis simulations. Each cell’s relative permittivity and permeability can range from 1 to 30 allowing for novel material distributions and is represented by a design variable in the design space. Also, the radiating conductor’s topology can vary via the on or off (conductor or no conductor) nature of design cells allowing for non-intuitive radiating conductor shapes. Consequently, another 400 cells are added to determine the optimal topology of the conductor. Therefore, total number of design variables  in the design example can be evaluated via  turning the design problem into a large scale one that could be solved using a micro-GA Algorithm typically running for 100 generations with an average population of 40 individuals. The final optimum result that the optimizer has converged to is a random mixture consisting of certain number of material shades that were specified in the design process for the design variables to be selected from. The number of material shades specified here is 30 corresponding to the integer values of 1-30 for permittivity and permeability values. Similar antenna design studies with complex topologies were conducted in literature using gradient based techniques [24, 25] and possible fabrication strategies for complex antenna substrates were also proposed [26] demonstrating the potential impact and realization possibilities of the complex antenna designs presented in this paper. Computational time requirement for a single layer geometry using a full wave finite element analysis such as the Fast Spectral Domain Analysis (FSDA) algorithm [27] and standard linear interpolation through 101 uniformly distributed frequency points within 1-2 GHz corresponds to approximately 1 week using a micro-GA with a population size of 40 individuals evolving within 100 generations. A GA based optimization tool is used here in order to avoid local optima. More specifically, the total computational design time required can be calculated using below formula:



Here, the computational time of the design process becomes prohibitively long as the scale of the problem increases with added number of layers and the increase of number of support points.

The antenna model created in FSDA constitutes of 800 triangular prism elements per layer. It uses finite element-boundary integral (FE-BI) technique for analyzing three-dimensionally inhomogeneous doubly periodic structures with in-layer periodicities. The boundary integral employs the FSDA for efficient analysis. The user supplies the triangular surface mesh modeling the different layer surfaces of the array unit cell. The program then grows the volume mesh along the surface normals. The building block of the resulting volume mesh is the triangular prism. Periodic boundary conditions are imposed on the vertical walls of the patch antenna and hence an infinite array of the analyzed antenna is simulated. The solver used here is BiConjugate Gradient. The default relative accuracy of 0.01 of the solver is used as a termination criterion.

The fitness function corresponds to the return loss response and is simulated using the proposed G-SBAIS interpolation technique integrated to the full wave FSDA solver. The design study converged to the return loss curve plotted in with a bandwidth performance of 11.4%. The interpolated return loss response using the proposed G-SBAIS technique is compared to the original response sampled via 101 support frequency points proving an almost exact match with correctly predicted bandwidth performance. However, the return loss response interpolated using the standard S-B interpolation technique is not able to capture the original response as depicted in .

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| Fig. 7 Return loss curve of the optimum design interpolated using G-SBAIS and S-B |

In addition to the miscalculated response of the final design’s return loss curve in due to missing resonances of fittest individuals, it is noted that the standard S-B was not able to deliver an improvement within 100 generations because of erroneous bandwidth calculations during the design iterations. The design study using G-SBAIS and the micro-GA converges to the optimum design with an overall improvement from 3.4% to 11.4% in 100 generations in about 10 days. With standard linear interpolation this example would not be practically feasible. The optimal conductor topology and three-dimensional material distributions in terms of permittivity and permeability distributions within each layer are given in (a), (b), and (c) respectively representing a non-intuitive complex topology. The average number of support points used by the G-SBAIS technique during the optimization process corresponds to 50 support points per individual. As expected, complexity of the design problem and the range of the design variables affect the behavior of the return loss curves in terms of number and depths of resonances within the design search and consequently influences the number of support points required for the entire process [22].

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| (a) | (b) |
| (c) |
| Fig. 8 Distribution of (a) conductor (white), (b) Permittivity and (c) permeability of lowermost to uppermost layers |
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# **Discussion and Conclusion**

In this paper a generalized Stoer-Bulirsch approach, namely G-SBAIS, for adaptively interpolating complex multi-resonant antenna response curves with reduced number of support points and improved accuracy was presented. The method relies on generalizing the standard Stoer-Bulirsch technique with a non-diagonal Neville path and makes use of this relaxation in shaping the best path that minimizes error and number of data samples needed. A hybrid technique following a path that minimizes the square error norm,  given by , allowing for reduced number of support points followed by a path maximizing  that enhances accuracy ensures a compromise between these conflicting merits and outperforms conventional adaptive sampling techniques. The proposed method has been implemented on return loss responses of various complex conductor shaped antennas with textured material substrates and resulted in an overall accuracy increase of 67.3% when compared with the naive Stoer-Bulirsch interpolation technique. A design optimization example to maximize bandwidth of a complex antenna with inhomogeneous material and conductor topologies was successfully integrated with the proposed G-SBAIS technique and resulted in a bandwidth improvement from 3.4% to 11.4% with an average number of 50 support points. The 3-fold bandwidth improvement achieved using the proposed G-SBAIS algorithm in this large scale design optimization example of a complex antenna motivates its use for novel antenna structures made of non-intuitive conductor and material compositions. By allowing for remarkable antenna performance improvements not possible via intuitive designs, this capability is expected to make a significant impact in many critical antenna applications.

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2. In this context, we refer to *intermediates* as any interpolator different from the final one $Φ\_{1}^{μ,ν}$that passes through all available support points [↑](#footnote-ref-2)