Abstract—The humanoid bipedal structure is suitable for an assistive robot functioning in the human environment. However, the bipedal walk is a difficult control problem. Walking just on even floor is not satisfactory for the applicability of a humanoid robot. This paper presents a study on bipedal walk on inclined planes. A Zero Moment Point (ZMP) based reference generation technique is employed. The orientation of the feet is adjusted online by a fuzzy logic system to adapt to different walking surface slopes. This system uses a sampling time larger than the one of the joint space position controllers. The average value of the body pitch angle is used as the inputs to the fuzzy logic system. A foot pitch orientation compensator implemented independently for the two feet complements the fuzzy controller. A 12-degrees-of-freedom (DOF) biped robot model is used in the full-dynamics 3-D simulations. Simulations are carried out on even floor and inclined planes with different slopes. The results indicate that the control method presented is successful in enabling the robot to climb slopes of 8.5 degrees (15 percent grade).

Keywords—Humanoid robots, bipedal walk, inclined plane, fuzzy systems, orientation controls

I. INTRODUCTION

The humanoid bipedal structure has certain advantages for a robot working in the human environment in assistive roles. A bipedal humanoid robot can avoid obstacles typical in the human environment via the locomotion on legs. There other advantages too. However, bipedal walk control is a challenge. This is due to the many degrees of freedom to be controlled under coupling effects and nonlinear dynamics. The last four decades witnessed intensive research on humanoid robot walking control. A number of successful projects and results are reported [1-6].

One of the most difficult problems in this field is the robust balance of the walk, not only on even floor but on surfaces with irregularities and slopes too. Studies on bipedal walking on inclined or uneven surfaces are reported [7-11]. An inclined plane presents a very typical floor condition encountered in human daily life. Though such planes are mostly part of the city and outdoor environments, since the indoor floors are not perfectly even, the inclined planes can be found at our homes and offices too. This paper presents a fuzzy logic parameter adaptation system for walking control on inclined planes. A smooth walking trajectory is generated based on the ZMP stability criterion and the Linear Inverted Pendulum Model (LIPM) as in [12-14]. Independent joint PID controllers are employed to track joint position references obtained via inverse kinematics from the ZMP based Cartesian center of mass (CoM) and foot references. The robot is assumed to consist of two legs and a body connecting them. The angle of the body with respect to a vertical axis is termed the “body pitch angle.” The angle of the foot soles with respect to the body is called “foot pitch angle” in this paper. The average body pitch angle computed over an history with finite number of samples is used as the input of a fuzzy logic system which computes the foot pitch angle online, to be applied as a walking reference modification. A simple rule base is constructed in such a way to introduce a deadband. This deadband inhibits the change of the foot pitch angle reference when the body is close to be straight, in order not to excite the robot dynamics unnecessarily. A complementary controller computes a reference modification in the joint level for the ankle pitch axes individually. This controller tries to minimize the ankle pitch torques with the aim of establishing proper contact with the support surfaces. Full dynamics 3D simulation studies with a 12 DOF robot model are carried out in order to test the proposed fuzzy parameter adaptation system.

The rest of this paper is organized as follows. The next section defines the problem of walking on planes with changing grades and the above mentioned pitch angles. Section 3 describes the fuzzy control system and the complementary ankle controller. Section 4 introduces the simulation platform and presents the simulation results of walking on changing slopes. The paper is concluded in Section 5.

II. PROBLEM DEFINITION

Figure 1 shows a typical biped robot walking on flat floor and on inclined planes with changing slopes. The motion of the robot is defined in a fixed coordinate frame called the world frame. The robot walking direction coincides with the x-direction of the world frame. Another frame is attached at the body of the robot. This body fixed frame (or the body frame) is shown in Fig.2 on the left. The body can be considered as a “central” link one since the legs are connected to it. Initially (before the start of the walk), the body frame axes are aligned parallel with the corresponding world frame
axis. However, this parallel alignment changes during the walk due to various effects including gravitational forces, foot to ground interaction, changing slopes and coupling effects between the links. The body pitch angle which is the angle the z-axis of the body frame makes with a vertical line is an indicator of the balance of the walk. This angle denoted by $\beta$ is shown in Fig.2 on the right hand side. Assuming that a gait with zero body pitch angle is planned in the reference generation, the online measurement of this angle can provide feedback indicating whether the robot is following this reference. It can also detect a falling forward or backward.

There many control actions which can enhance the stability during the walk working on the body pitch angle. For robots with an independent pitch joint which divides the body into lower body and upper body sections (equivalently into pelvis and trunk links, respectively), the motion of this joint can directly be used for stability enhancement. In [15], Park uses this joint angle as a balance control variable for a 2D (sagittal plane) biped model. [16] introduces the “virtual pelvis pitch angle” and shows that the body pitch motion can be applied as a control action for robots without an independent body pitch joint too.

In some approaches the body pitch angle $\beta$ of Fig. 2 actively modified with the benefit of increasing the robot moment of inertia about the world frame vertical axis. Yaw moment compensation techniques using this fact are studied by Fujimoto and Kawamura [17].

Keeping the body pitch angle at a certain reference value is also a common in many applications, especially for object manipulation and human-robot cooperation.

In this paper, the control objective is walking on changing slopes without losing balance. In contrast to [15] and [16] an upright body pitch orientation ($\beta = 0$) reference is employed. We believe that the upright posture has its own virtues on slopes. The minimal motion of the (presumably heavy) upper body induces a only low amount of interlink coupling torques and hence eases the walking control problem. However, this ease is at the expense of one degree-of-freedom in the control actions: The upper body rotary motion (via an independent joint between the pelvis and trunk, or via the “virtual pelvis pitch angle” in [16]) should be avoided. In this paper we propose the modification of the walking plane by the rotation of the foot references about a pitch axis at the ground level. The location of this axis in the body coordinate frame is defined by the $h_{body}$ and $x_{offset}$ parameters in Fig. 3. $h_{body}$ is the constant body height reference parameter employed in the ZMP and LIPM based reference generation method in [7], [21] and [22]. The same even floor reference generation method is employed in this paper too. $x_{offset}$ is the x-directional coordinate of the central point of the right and left foot frame references, as expressed in the body coordinate frame. The role of this offset parameter is to place the center of the support polygon right below the center of mass of the robot, shown as a circle in Fig. 3. The lefthand side drawing in this figure shows the standing posture and the right hand side drawing depicts the robot while walking on even floor.

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Fig. 1 Bipedal robot walk on changing slopes

Fig. 2 Body coordinate system and the and the body pitch angle

Fig. 3 A coordinate system is attached to the central point of the foot references on the ground level. This coordinate system serves in defining the rotation of the foot references to change the walking plane.
Placing a coordinate system at the foot trajectory central point with an orientation identical to the body coordinate frame, we define the angle $\theta$ in Fig. 4 as a rotation angle about the negative $y$-axis of this frame. (We term this angle “foot pitch angle.”) The foot reference coordinates (for any time), as expressed in this frame are rotated into new references. The result of this rotation is the change of the walking plane, without changing the body orientation. It should be noted that the rotational transformation of the foot references is more than a modification in foot orientations. This transformation also modifies the direction of foot motion. After the transformation, the foot motion originally defined parallel to the even floor becomes parallel to a plane wight slope angle $\theta$.

In the next section we develop a fuzzy logic system which adjusts the angle $\theta$ online by evaluating the body pitch angle $\beta$. This fuzzy system, supplemented by an additional foot pitch angle compensator with ankle torque input, is used as the walking controller on changing slopes.

III. THE PROPOSED CONTROL SYSTEM

Our simulation and experimental results [11,18-19] suggest that the body pitch angle is oscillating during the walk. Even in a steady and stable walk the body pitch angle trajectory can be in the form of a periodic signal with peak values of a few degrees. Therefore, a single measurement of the angle can be misleading in deciding upon the balance condition of the robot. In order to infer whether the robot goes on with a steady walking pattern or is in the trend of falling, the average rather than the instantaneous value of the pelvic pitch angle is more suitable. Averaging can be done in many different ways. One question to be answered is related to the length of the averaging window. Another concern is the sampling period for the data to be averaged. Usually, joint space controller sampling times of one to ten milliseconds are employed in robotic applications. Using a long averaging window from the current sampling instant back with a low sampling period in the order of the joint control sampling period would require large storage space and consume online computational resources. There is a trade-off in the accuracy of the averaging computation and the efficient use of the computational power.

Employing the number of samples $N$ used in the average computation and the body pitch angle sampling period $T_p$, as design parameters, the average value $\overline{\beta}$ of the body pitch angle $\beta$ in Fig. 2 is computed as

$$\overline{\beta}(kT_p) = \frac{1}{N} \sum_{i=0}^{N-1} \beta((k-i)T_p)$$  (1)

where $k$ is the sampling index.

The following guidelines describe the role of $\overline{\beta}$ in the body pitch angle adjustment used in this paper:

(i) When the average value of body pitch angle is positive we can infer that this is a posture inclined to a fall towards front, to the walking direction. In this case, the foot pitch angle should be decreased (and even made negative) to push the the ground by the toes and compensate for the gravity effect of the forward leaned body.

(ii) In a similar way to the guideline (i) above, when the average value of body pitch angle is negative, the foot pitch angle should be increased to compensate for the gravity effect of the backward leaned pelvis link by pushing the ground with the heels.

The two guidelines above can be used in various ways to construct an online adjustment mechanism for $\theta$. An analytic relation between $\overline{\beta}$ and $\theta$ could be one of the choices. The method proposed in this paper employs fuzzy systems for this control. (In the context of bipedal walking control, online modification of walking trajectories with sensor feedback are the most common closed loop control actions. Therefore, we call the online adjustment of $\theta$ “control” too.) Fuzzy systems are natural choices to exploit verbal descriptions (like the two guidelines above) of the plant or problem to obtain control or adaptation mechanisms.

Table 1 and Fig. 3 describe the three fuzzy rules used in the computation of $\theta$. In Table 1 the subscript “P” of the rule strength $\Delta \theta_p$ stands for positive. “Z” symbolizes “zero” and “N” is for “negative.” The numerical values of the rule strengths and the corner positions of the trapezoidal membership functions in Fig. 5 are tabulated in the simulation results section, in Table 4. These values are obtained by trial and error simulations. The rules are summarized in Table 1. An example for a rule is: “Rule 1: If $\overline{\beta}$ is negative, then increase $\theta$ with increment $\Delta \theta_p$.” The choice of the rule base and the membership functions satisfies the conditions (i) and (ii) above. The defuzzification is carried out by the expression

$$\Delta \theta = \frac{\mu_{\text{Negative}} \Delta \theta_p + \mu_{\text{Zero}} \Delta \theta_z + \mu_{\text{Positive}} \Delta \theta_e}{\mu_{\text{Negative}} + \mu_{\text{Zero}} + \mu_{\text{Positive}}}$$  (2)

At every computation cycle (with period $T_p$), the parameter $\theta$ is updated by

$$\theta_{k+1} = \theta_k + \Delta \theta_k$$  (3)

This command (reference) value used as an input for the inverse kinematics computations. The “integrating” nature of (3) is very useful to adapt to inclined planes with unknown slopes. The rule strength $\Delta \theta_p$ is chosen equal to zero and
hence the central trapezoidal membership function in Fig.5 represents a deadzone about the vicinity of zero body pitch angle. This deadzone helps the convergence of the foot pitch angle without control activity (change in $\theta$) in the steady state.

Although the control strategy above is quite successful in changing the the “reference” walking plane gradually and smoothly, an accompanying controller is necessary for the robot to survive the impact in the very moment of the change between planes. Unlike $\theta$ which is an input for the inverse kinematics routine, the additional control variable acts in the joint level, on the ankle pitch joint position references. The ankle pitch torques, independently for the right and left ankles are used to modify these joint position references. This control action is explained below.

In [4] landing orientation controllers are proposed for the ankle joints. This approach assumes that two joints with coinciding joint axes perpendicular to each other are present at ankles. The scheme computes joint angle reference modifications in such a way that the feet are aligned parallel to the ground when they are in contact with the ground. The reference modification law in [4] is the form of a first order filter applied on the foot to ground contact torques. We adopted this method for our control system too. For the pitch axis of the ankle, we employ the following reference modification law in the Laplace domain.

$$\theta_{\text{ankle}}(s) = \theta_{\text{ankle}}(s) + (K_{\text{ankle}} / s + \lambda_{\text{ankle}}) T_{\text{ankle}}(s) \tag{4}$$

where $s$ is the Laplace variable, $\theta_{\text{ankle}}$ is the ankle pitch joint reference angle computed from the inverse kinematics. $\theta_{\text{ankle}}$ is the reference ankle pitch angle after the reference modification. $T_{\text{ankle}}$ is the torque about the roll axis due to the interaction of the foot with the ground. This torque is assumed to be measured by torque sensors positioned at the ankle in an experimental work, ans is readily available in our simulation computations.

### TABLE I
THE FUZZY RULE BASE

<table>
<thead>
<tr>
<th>Rule</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative $\bar{\beta}$</td>
<td>$\Delta \theta_p$</td>
</tr>
<tr>
<td>Zero $\bar{\beta}$</td>
<td>$\Delta \theta_z$</td>
</tr>
<tr>
<td>Positive $\bar{\beta}$</td>
<td>$\Delta \theta_n$</td>
</tr>
</tbody>
</table>

$K_{\text{ankle}}$ and $\lambda_{\text{ankle}}$ are low pass filter constants which are determined by trial and error in our approach. In the digital implementation, the Laplace domain transfer function in (4) is approximated by a difference equation. We used Tustin’s approximation technique to obtain the difference equation. When the foot is in contact with the ground only with a corner or an edge, a torque is developed and with the application of (4), the joint angle references are modified in such a way to turn the ankle to achieve foot orientation parallel to the ground. When the foot touches an inclined plane with the toes only, the effect of this equation is to turn the ankle pitch joint until the foot is in contact with both planes, the original one and the newly touched inclined plane. When applied together with the fuzzy controller in (1-3), this motion has an impact absorbing nature, which enables plane to plane transitions without falling.

The next section presents simulation results with the controllers discussed above.

### IV. SIMULATION RESULTS

The biped model used in simulations consists of two 6-DOF legs and a trunk connecting them. Three joint axes are positioned at the hip. Two joints are at the ankle and one at the knee. The numerical values of the various dynamics parameters are taken to match out experimental humanoid robot SURALP (Sabanci University Robotic ReseArCh Laboratory Platform) [14,18] (Fig. 6, Table II). This robot is equipped by joint encoders, an inclinometer at its torso, six-axes force sensors at its ankles and wrists, two CCD cameras at its head. The details of the simulation algorithm can be found in [20].

![Fig. 6 The kinematic arrangement of SURALP](image)

**TABLE II
ROBOT PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Leg Length</td>
<td>280mm</td>
</tr>
<tr>
<td>Lower Leg Length</td>
<td>270mm</td>
</tr>
<tr>
<td>Sole-Ankle Distance</td>
<td>124mm</td>
</tr>
<tr>
<td>Foot Dimensions</td>
<td>240mm x 150mm</td>
</tr>
<tr>
<td>Upper Arm Length</td>
<td>219mm</td>
</tr>
<tr>
<td>Lower Arm Length</td>
<td>255mm</td>
</tr>
<tr>
<td>Robot Weight</td>
<td>114 kg</td>
</tr>
</tbody>
</table>
The walking reference generation on even floor is carried out as in [14]. Parameters used for reference generation are presented in Table III. The control algorithm is a simple one based on independent joint PID position controllers. The joint position references are generated through inverse kinematics from CoM and swing foot references in world frame coordinates. The PID controller gains are obtained via trial and error. The simulation and PID controller cycle time employed is 0.5 milliseconds. The cycle time of the fuzzy adaptation routine is however set to 200 milliseconds. The value of the number of samples, \( N \), in (1) equals to 20. Further parameters of the control system in (1-4) are presented in Table IV. Simulation results obtained with this control system are shown in Figures 7 and 8. The walking surface of the simulation environment contains two parts: An even surface and a plane inclined with 5.6 degrees (10% grade).

**TABLE III**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single support period</td>
<td>0.6 s</td>
</tr>
<tr>
<td>Double support period</td>
<td>0.9 s</td>
</tr>
<tr>
<td>ZMP reference in y direction</td>
<td>8 cm</td>
</tr>
<tr>
<td>ZMP motion under the sole</td>
<td>4 cm</td>
</tr>
<tr>
<td>Step size</td>
<td>10 cm</td>
</tr>
<tr>
<td>Step height</td>
<td>2.5 cm</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>Rule Strength</th>
<th>Numeric Value (Degrees)</th>
<th>Corner</th>
<th>Numeric Value (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \theta_p )</td>
<td>0.5</td>
<td>( \bar{\beta}_{\text{Negative Big}} )</td>
<td>( -5 )</td>
</tr>
<tr>
<td>( \Delta \theta_z )</td>
<td>0</td>
<td>( \bar{\beta}_{\text{Negative Small}} )</td>
<td>( -0.25 )</td>
</tr>
<tr>
<td>( \Delta \theta_N )</td>
<td>( -0.5 )</td>
<td>( \bar{\beta}_{\text{Positive Small}} )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \bar{\beta}_{\text{Positive Big}} )</td>
<td></td>
<td></td>
<td>( 5 )</td>
</tr>
</tbody>
</table>

Snapshots of the robot from the simulation window are shown in Fig. 7. Initially, the simulation starts with the robot in upright posture at a distance of 12 cm to the inclined plane. It can be observed from Fig. 8 that the robot reaches to the inclined plane after approximately 2 seconds is elapsed from the start of the walk. The \( \bar{\beta} \) graph (second plot from the top) shows that the average oscillation of the upper body of the robot is small during this period. After the establishment contact with the inclined plane \( \bar{\beta} \) changes much faster. The fuzzy parameter adaptation system acts according to the fuzzy rule base and finds the appropriate \( \theta \) angle (third plot from the top). This angle is first increased as a response to negative \( \bar{\beta} \) values and converges quickly after the full transition onto
the inclined plane. The lowermost plot in Fig. 8 shows the ankle pitch joint correction angled computed by (4). It can be noted that higher amounts of correction are required initially when the incline plane is met. The need for correction vanishes as the walk progresses on the inclined plane. Simulation are carried out with 15 % grade too and the robot was successful in keeping its balance and climb onto the incline plane though the feet slid back occasionally in the single support phases. Simulation studies are carried out by deactivating the fuzzy adaptation and complementary ankle joint angle correction too. It is observed that the robot fails to climb onto the inclined plane with 10 % slope and falls in the absence of either controller.

V. CONCLUSIONS

Bipedal walk on uneven surfaces is an important research area. Inclined planes are typically encountered in our living environment. This case is studied in this paper. A fuzzy logic control system system for the body pitch orientation and a complementary ankle pitch joint angle correction controller are proposed. The model of a 12-DOF biped walker is used in the simulations which demonstrate the performance of the adaptation algorithm under various slope conditions. Simulation results indicate that the control system is successful in obtaining a stable walk in the transition from a horizontal plane onto an inclined one with a grade up to 15 %. Our future research is motivated into the implementation of the algorithm on our humanoid robot SURALP.

REFERENCES
