# Sliding Modes in Electrical Drives and Motion Control

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**Abstract:** In this paper application of Sliding Mode Control (SMC) to electrical drives and motion control systems is discussed. It is shown that in these applications simplicity in implementation makes concepts of SMC a very attractive design alternative. Application in electrical drives control is discussed for supply via different topologies of the supply converters. Motion control is discussed for single degree of freedom motion control systems as an extension of the control of mechanical coordinates in electrical drives. Extension to multi-body systems is discussed very briefly.

*Keywords:* Sliding Modes, Electric Drives, Motion Control, Power Converters, Motion Control,

## 1. INTRODUCTION

Sliding mode motion results in a system performance that includes disturbance rejection and robustness to parameter variation [Utkin, 1992]**.** It has been presented abundantly in technical literature, both from a theoretical and implementation perspective [Utkin, 1992], [Utkin et al, 1999]. Hence, in this paper, background details on Sliding Mode Control are kept to a minimum.

Our scope is limited on presentation of results of SMC application to electrical drives and motion control systems. The direct application of discontinuous control to motion systems with force as control input led to claims of the chattering phenomena in VSS with sliding modes and the scepticism on the applicability of the discontinuous control based SMC in such systems [Asada et al, 1986]. The solution of the chattering problem attracted many researchers and numerous design methods were proposed to reduce its effects [Young, et al. 1999].

Electrical drives consist of a machine and power converter acting as an interconnecting device between electrical machine and the electrical power source while electrical machine is attached to some mechanical energy source or sink. Thus, electrical drives, in general, act as interconnection devices between electrical and mechanical energy sources. The electrical drive role as interconnection element rises a question of bilateral energy flow and impedance matching for both electrical and mechanical terminals. This requires controlling voltage or current at electrical interconnection terminals and force/torques or velocity at mechanical interconnection terminals.

SMC application in electrical drives seems natural due to discontinuous nature of the outputs of the motor supply converter. Drives control includes control of mechanical motion, control of the electromagnetic processes in the electrical drives resulting in desired changes of the electromagnetic torque/ force [Sabanovic et al., 1981]. The last but not the least is switching pattern selection for the supply converters – known as PWM pattern generation [Sabanovic, 1993]. SMC is characterized by simplicity in control design and robustness against unknown or time-varying plant parameters [Utkin et al, 1993]. However, irrespective of the control algorithm, estimation of the system state (rotor and stator flux) and the mechanical coordinates in the presence of the variable parameters are required [Yan et al, 2002]. Recent advanced developments in SMC observers allow the implementation of a “sensorless” electrical drives control [Rao et al, 2009].

Paper is organized as follows. In section 2 structures and mathematical models are discussed. In section 3 selection of switching pattern for switching converter supplying induction machine is discussed. It is shown that solutions derived using SMC methods can be applied to both DC-to-AC and the AC-to-DC converters. In Section 4 induction machine observer design is discussed briefly. In section 5 issues related to single degree of freedom and multi-body mechanical systems control are discussed.

## 2. STRUCTURES AND MATHEMATICAL MODELS

Motion control systems are related to control of mechanical systems configuration – thus dealing with changes of mechanical coordinates (position and/or velocity) due to the changes of forces/torques acting on the system input(s). The electrical drive as actuator in motion control system needs to generate required force/torque to realize desired motion and adjust electrical impedance on electrical terminals. Both requirements need to be realized by controlling power converter which is attached between electrical machine and the electrical power source. In some cases, due to topological properties of the converters, concurrent realization of both requirements on electrical and mechanical interconnection terminals may not be realizable.

### *2.1 Power Converters and Electrical Machines*

In order to avoid discussion of many different cases we will consider induction machine (IM) as an example. Due to dynamical complexity this example may show a way of controlling other machines as well. In field oriented frame of references ($d,q$), dynamics of the electromagnetic energy conversion, with voltages as inputs and electromagnetic torque (force) as output, for IM can be written as:

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\lambda & \omega \\ -\omega & \lambda \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \gamma\eta\phi_d \\ -\gamma\omega\phi_d \end{bmatrix} + \frac{1}{\sigma L_s}\begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{1}$$

$$\gamma = L_m/\sigma L_s L_r\,;\eta = R_r/L_r\,;\sigma = 1 - L_m^2/L_s L_r$$

$$\frac{d}{dt}\begin{bmatrix} \phi_d \\ \rho \end{bmatrix} = \begin{bmatrix} -\eta\phi_d \\ \omega \end{bmatrix} + \eta L_m \begin{bmatrix} 1 & \\ & 1/\phi_d \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{2}$$

$$\tau = (3L_m/2L_r)\phi_d i_q \tag{3}$$

Here $\mathbf{i}_{dq}^T = [i_d \quad i_q]$ is stator current vector, $\mathbf{u}_{dq}^T = [u_d \quad u_q]$ stands for the stator voltage vector, $\phi_{dq}^T = [\phi_d \quad 0]$ is the rotor flux vector, $\rho$ stands for the position of rotor flux vector in stationary frame of references $(\alpha,\beta)$; $\theta,\omega$ stand for rotor position and velocity governed by $\dot{\theta} = \omega;\ J\dot{\omega} = \tau - \tau_L;$, $\tau_L$ is the external load, $L_m$, $L_s$ and $L_r$ are mutual, stator and rotor inductances, respectively. The dynamics of the electromagnetic torque and the rotor flux can be expressed in another way specifying directly dynamics of the rotor flux modulus and the electromagnetic torque [Sabanovic et al, 1981]

$$\begin{gathered} \frac{d^2\phi_d}{dt^2} = f_\phi + \frac{\eta L_m}{\sigma L_s}u_d \\ f_\phi = -\eta^2(L_m i_d - \phi_d) + \eta L_m(\gamma\eta\phi_d + \omega i_q - \lambda i_d) \end{gathered} \tag{4}$$

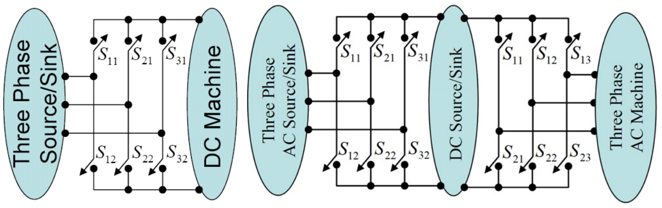
$$\begin{gathered} \frac{d\tau}{dt} = f_\tau + \frac{3L_m}{2L_r}\frac{1}{\sigma L_s}\phi_d u_q \\ f_\tau = \frac{3L_m}{2L_r}(-\eta\phi_d + \eta L_m i_d)i_q - \phi_d(\gamma\omega\phi_d - \omega i_d - \lambda i_q) \end{gathered} \tag{5}$$

Model (4), (5) may be used for direct design of the control voltage to enforce flux and torque tracking. Vector $\mathbf{u}_{dq}$ is defined as

$$\underbrace{\begin{bmatrix} u_d \\ u_q \end{bmatrix}}_{u_{dq}} = \underbrace{\begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix}}_{T_{\alpha\beta}^{dq}}\underbrace{\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}}_{u_{\alpha\beta}} \tag{6}$$

where $\mathbf{u}_{\alpha\beta}^T = [u_\alpha \quad u_\beta]$ stand for supply voltage in $(\alpha,\beta)$ frame of references.

The control properties are greatly influenced by the topology of supply converters. Fig. 1(a) shows interconnection of a DC machine to three-phase source. In Fig. 1(b) an AC machines supplied from three phase source with a DC source as interconnecting element is depicted. Structure in Fig. 1(b) can be analyzed as a serial connection of a AC to DC (Fig. 1(a)) and a DC to AC sources/sinks. Topological structures of the AC to DC and DC to AC converters are the same, thus the same control algorithm can be applied to both [Sabanovic et al, 1993]. To avoid complexities, we will assume that DC voltage $V_0$ is maintained by input AC to DC converter. The instantaneous state of switches determines a converter configuration vector $S_i(s_{kj})$, $i = 1,...,m$, $k,j = 1,2,3$ with $s_{kj} = 1$ if switch $S_{kj}$ is ON and $s_{kj} = 0$ if switch $S_{kj}$ is OFF.




(a) (b)

Fig.1 Structure of switching converters for interconnection of DC or AC machines to a three phase source

Converter output voltages $u_1,u_2,u_3$ can take value $V_0$ or $0$. The load phase voltage $\mathbf{u}_{\alpha\beta}$ for delta connection, and converter output voltages $u_i = V_0$ if $s_{1j} = 1$, $u_i = 0$ if $s_{2j} = 1$, is given by (see Fig. 2)

$$\underbrace{\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}}_{u_{\alpha\beta}} = \underbrace{\frac{1}{3}\begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}}_{T_{abc}^{\alpha\beta}}\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}}_{T_{123}^{abc}}\underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{u_{123}} \tag{7}$$

Insertion (7) into (6) allows the transformation $(1,2,3) \to (d,q)$ to be expressed as

$$\mathbf{u}_{dq}(S_i) = \mathbf{T}_{\alpha\beta}^{dq}(\rho)\mathbf{T}_{abc}^{\alpha\beta}\mathbf{T}_{123}^{abc}\mathbf{u}_{123}(S_i) = \mathbf{T}_{123}^{dq}(\rho)\mathbf{u}_{123}(S_i) \tag{8}$$

$$\begin{aligned} & S_i = [s_{11}, s_{12}, s_{13}, s_{21}, s_{22}, s_{23}] \\ & i = 1,..,8 \\ & S_1 = [1,0,0,0,1,1], S_2 = [1,1,0,0,0,1], \\ & S_3 = [0,1,0,1,0,1], S_4 = [0,1,1,1,0,0] \\ & S_5 = [0,0,1,1,1,0], S_6 = [1,0,1,0,1,0], \\ & S_7 = [1,1,1,0,0,0], S_8 = [0,0,0,1,1,1] \end{aligned}$$

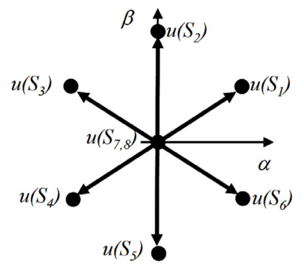



Fig.2 Converter configuration vectors $S_i, i = 1,2,...,8$ and corresponding voltages $\mathbf{u}(S_i)$ in $(\alpha,\beta)$ frame of references

Complete dynamical description of a motion control system includes mechanical motion $\dot{\theta} = \omega;\ J\dot{\omega} = \tau - \tau_L;$, the electromechanical energy conversion (1) - (3) or (4) - (5) and the operation of the supply converter (8). It is complex, high order nonlinear dynamics that can be, from a control point of view, treated in many different ways. In the multi-body motion control systems the drives are treated as s source of the torque/force and the dynamics of mechanical motion is more complex.

### *2.2 Motion Control Systems*

Configuration space dynamics of multi-body, rigid, fully actuated $n-$dof system can be expressed as

$$\mathbf{A}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau} \tag{9}$$

Here $\mathbf{q}\in\Re^{n\times 1}$ denotes the configuration vector; $\mathbf{A}(\mathbf{q})\in\Re^{n\times n}$ stands for positive definite kinetic energy matrix with bounded strictly positive elements; $\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})\in\Re^{n\times 1}$ stands for vector Coriolis forces, viscous friction and centripetal forces and is bounded; $\mathbf{g}(\mathbf{q})\in\Re^{n\times 1}$ stands for vector of gravity terms bounded; $\boldsymbol{\tau}\in\Re^{nx1}$ stands for vector of generalized joint forces bounded (In further text we will sometimes refer to $\boldsymbol{\tau}$ as the control vector or input force vector). Motion control assumes force $\boldsymbol{\tau}\in\Re^{nx1}$ as control input. As shown in analysis of electrical drives torque is result of electromagnetic interaction and can be controlled by changing electromagnetic state of electrical machine.

## 3. ELECTROMAGNETIC FORCE/TORQUE CONTROL

### *3.1 Selection of Reference Current*

Rotor flux and component of the stator current orthogonal to rotor flux $i_q$ need to be controlled in order to uniquely determine electromagnetic force/torque. Rotor flux magnitude dynamics (2) is first order systems with current $i_d$ a scalar control input and reference tracking can be achieved if sliding mode is enforced in

$$S_d^{\phi}=\left\{\phi_d,i_d:\sigma_d^{\phi}=\phi_d^{ref}-\phi_d=0\right\} \tag{10}$$

Current reference

$$i_d^{ref}=i_d^{eq}-K_d\left|\sigma_d^{\phi}\right|^{2\xi-1}sign\left(\sigma_d^{\phi}\right),\ \tfrac{1}{2}<\xi\le 1 \tag{11}$$

with $K_d>0$ enforces sliding mode motion in manifold $\sigma_d^{\phi}=0$; $i_d^{eq}$ stands for equivalent current calculated from $\dot{\sigma}_d^{\phi}=0$. This can be veryfied from the time derivative $\dot{V}=-k2^{\xi}V^{\xi}$, $k>0$, $\frac{1}{2}\le\xi\le 1$ of Lyapunov function $V=\left(\sigma_d^{\phi}\right)^2/2$. For $\phi_d=\phi_d^{ref}\neq 0$ and $\tau^{ref}$ the $i_q$ can be determined as $i_q^{ref}=2\tau^{ref}L_r/\left(3L_m\phi_d^{ref}\right)$. This way desired current vector is determined such that desired magnitude of rotor flux and desired electromagnetic force/torque are generated. Alternatively, if for example velocity of the motor needs to be controlled to track its smooth reference $\omega^{ref}$, then from $V=\left(\sigma_q^{\omega}\right)^2/2$, $\sigma_q^{\omega}=\omega-\omega^{ref}$ the current

$$i_q^{ref}=i_q^{eq}-K_q\left|\sigma_q^{\omega}\right|^{2\xi-1}sign\left(\sigma_q^{\omega}\right),\ \tfrac{1}{2}<\xi\le 1 \tag{12}$$

enforces velocity tracking. $i_q^{eq}$ stands for equivalent current calculated from $\dot{\sigma}_q^{\omega}=0$. Both components of the reference current are continuous.

### *3.2 Selection of Converter Configuration Vector*

Temporal changes of the converter configuration vector should be selected such that actual current $\mathbf{i}_{dq}$ tracks selected reference $\mathbf{i}_{dq}^{ref}$ or that sliding mode is enforced in manifod $S_{dq}^{i}=\left\{i_d,i_q:\sigma_d^{i}=i_d^{ref}-i_d=0\ \&\ \sigma_q^{i}=i_q^{ref}-i_q=0\right\}$. Another way is to start from dynamics (4), (5) and enforce sliding mode in manifold $S_{dq}^{\phi,\tau}$ defined as an intersection of the manifolds $\sigma_d^{\phi}=\dot{e}_{\phi}+C_{\phi}e_{\phi}=0$, $e_{\phi}=\phi_d^{ref}-\phi_d$ and $\sigma_q^{\tau}=\tau^{ref}-\tau=0$. In both cases design procedure is the same. Selection of the components of input voltage as

$$\begin{aligned}u_d&=u_d^{eq}-k_d sign\left(\sigma_d^{v}\right),k_d>0,v=i,\phi\\ u_q&=u_q^{eq}-k_q sign\left(\sigma_q^{\chi}\right),k_q>0,\chi=i,\tau\end{aligned} \tag{13}$$

enforces component-vice sliding mode in selected manifold. The components of the equivalent voltages are determined from $\dot{\sigma}_d^{i}=0$ and $\dot{\sigma}_q^{i}=0$, or $\dot{\sigma}_d^{\phi}=0$ and $\dot{\sigma}_q^{\tau}=0$. For example in the current control loops the $d-$component and $q-$component of the equivalent control can be expressed as

$$\begin{aligned}u_d^{eq}&=\sigma L_s\left(\tfrac{di_d^{ref}}{dt}-\gamma\eta\phi_d-\omega i_q+\lambda i_d\right)\\ u_q^{eq}&=\sigma L_s\left(\tfrac{di_q^{ref}}{dt}+\gamma\omega\phi_d+\omega i_d+\lambda i_q\right)\end{aligned} \tag{14}$$

Both $u_d^{eq}$ and $u_q^{eq}$ are continuous.

### *3.2 Selection of Converter Switching Pattern*

Further design will be easy if voltages (13) can be realized exactly by converters, but unfortunately that is not the case. As shown in Fig. 2 converter configuration vector can have seven distinct values – thus only seven values in (13) can be exactly reproduced. This requires finding another way of realizing control (13). For Lyapunov function $V(\sigma)>0$, any converter configuration vector $S_k, \mathrm{k}=1,...,8$ that guaranty $\dot{V}(S_k)<0$, satisfies Lyapunov stability requirements. Conceptually, selection of one of the eight configurations $S_k, k=1,2,...,8$ requires mapping vector $\mathbf{u}_{dq}^{T}=\begin{bmatrix}u_d & u_q\end{bmatrix}$ given by (13) to vector $\mathbf{u}_{123}(S_k), k=1,...,8$. Indeed, many different solutions can be found in literature [Sabanovic et al, 1981], [Sabanovic, 1993], [Utkin et al, 1999]. Below a few, most interesting algorithms will be explained in more details.

*Algorithm 1* is based on (8) and the projection of (*d,q*) reference frame to (*a,b,c*) reference frame using generalized inverse

$$\mathbf{u}_{123}=\left[\mathbf{T}_{123}^{dq}\right]^{+}\mathbf{u}_{dq}=\tfrac{3}{2}\left[\mathbf{T}_{123}^{dq}\right]^{T}\mathbf{u}_{dq} \tag{15}$$

This algorithm was first published in [Sabanovic et al, 1981] and is a base for the space vector PWM algorithms. Based on this idea many so-called transition tables [Jezernik et al, 2010], [Chen et al, 1997] are derived. This algorithm implicitly enforces zero average of the sum of converter outputs $u_1,u_2,u_3$ thus gives larger switching frequency compared with all other solutions.

*Algorithm 2:* is based on usage of the existing degree of freedom in (8) and add additional requirement to control specification in the form

$$\begin{bmatrix} \dot{\boldsymbol{\sigma}}_{dq} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{123}^{dq} \\ \mathbf{b}^T \end{bmatrix} \mathbf{u}_{123} - \begin{bmatrix} \mathbf{u}_{dq}^{eq} \\ u_{\varphi}^{eq} \end{bmatrix} \tag{16}$$

$$\mathbf{b}^T = [b_1 \quad b_2 \quad b_3]$$

Selecting $\mathbf{b}^T$ such that augmented matrix has full rank allows calculation of $\mathbf{u}_{123}(S_i)$ such that sliding mode is enforced in manifold $\boldsymbol{\sigma}_{dq} = 0 \,\&\, \varphi = 0$. In [Sabanovic et al, 1981] and [Utkin et al, 1999] $\mathbf{b}^T$ is selected to ensure balanced voltages with $u_{\varphi}^{eq} = 0$. Extensions to, for example, switching optimization, enforcement of desired average voltage of neutral point etc., can be easily implemented by selecting $u_{\varphi}^{eq}$ and $\mathbf{b}^T$.

*Algorithm 2:* is selected with requirements to give minimum rate of change of the current control error [Sabanovic et al, 1993]. Temporal changes in converter configuration control should be selected from

$$S_i \Leftarrow \begin{cases} \min \left\| \mathbf{u}_{dq}^{eq} - \mathbf{u}(S_j) \right\| \\ sign\left\{ \left[ u_d^{eq} - u_d(S_j) \right] \sigma_d^i = -1 \right\}, j = 1,...,8 \\ sign\left\{ \left[ u_q^{eq} - u_q(S_j) \right] \sigma_q^i = -1 \right\} \end{cases} \tag{17}$$

This algorithm gives good results in steady state operations (when change of the current is limited to a current ripple) but is not performing very well when large transients in the current are needed (as for sudden change in load).

*Algorithm 3:* In algorithms (15) - (17) switching pattern does not depend on the actual amplitude of the error. An interesting improvement of algorithm (17) is proposed in [Chen et al, 1997]. The idea is simple: value of switching function at the end of the switching interval $T$ is minimized and switching pattern of the converter configuration vector is selected as

$$S_i \Leftarrow \left\{ S_j \left| \min_j \left\| \boldsymbol{\sigma}_{dq}^i(0) + \dot{\boldsymbol{\sigma}}_{dq}^i(S_j)T \right\| \right. \right\}, j = 1,...,8 \tag{18}$$

Further improvement is possible if two control vectors are allowed to be used in one switching interval. Assuming that from (18) vector $S_k$, $k = 1,...,8$ is selected, by applying simple linear approximation the second vector is determined from (19):

$$S_i \Leftarrow \left\{ S_j \left| \min_j \left\| \boldsymbol{\sigma}_{dq}^i(0) + \dot{\boldsymbol{\sigma}}_{dq}^i(S_k)\nu T + \dot{\boldsymbol{\sigma}}_{dq}^i(S_j)(1-\nu)T \right\| \right. \right\}, \tag{19}$$

$$j, k = 1,...,8; 0 \le \nu \le 1$$

Algorithm (19) allows optimization of the switching pattern by selecting optimal value for $0 \le \nu \le 1$. This algorithm shows very good behaviour in steady state and transient conditions.

Selection of temporal changes of the converter configuration vector establishes relationship between control of electromagnetic states of electrical machines and the operation of switching converter.

## 4. SMC BASED OBSERVERS FOR ELECTRIC DRIVES

Estimation of state and/or parameters in motion control plays an important role in the system realization. As shown above, information on the flux is needed for the electrical machines control. Here we will show application of the sliding mode observer design methods taking an IM machine as an example. The dynamics of IM (1), (2) in $(\alpha, \beta)$ frame of references is taken as a starting point

$$\frac{d}{dt}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \gamma \begin{bmatrix} \eta & \omega \\ -\omega & \eta \end{bmatrix} \begin{bmatrix} \phi_\alpha \\ \phi_\beta \end{bmatrix} - \lambda \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{1}{\sigma L_s} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \tag{20}$$

$$\frac{d}{dt}\begin{bmatrix} \phi_\alpha \\ \phi_\beta \end{bmatrix} = \begin{bmatrix} -\eta & -\omega \\ \omega & -\eta \end{bmatrix} \begin{bmatrix} \phi_\alpha \\ \phi_\beta \end{bmatrix} + \eta L_m \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{21}$$

where $\phi_{\alpha\beta}^T = [\phi_\alpha \quad \phi_\beta]$, $\mathbf{i}_{\alpha\beta}^T = [i_\alpha \quad i_\beta]$ and $\mathbf{u}_{\alpha\beta}^T = [u_\alpha \quad u_\beta]$ are rotor flux, stator current and stator voltage vectors, respectively. Stator current and voltage are assumed measured. Observer structure can be selected in the same form as (20), (21). One of the first ideas on IM identification with sliding modes [Izosimov, 1983], is based on selection of the rotor time constant $\eta$ and angular velocity $\omega$ as control. It has been shown that if sliding mode is enforced in $\sigma_1 = \phi_{\alpha\beta}^T \mathbf{e}_i$ and $\sigma_2 = \phi_{\alpha\beta} \times \mathbf{e}_i$, $\mathbf{e}_i^T = \mathbf{i}_{\alpha\beta} - \hat{\mathbf{i}}_{\alpha\beta}$ the estimated flux converges to real one. Improved version if this observer is with adaptation of the rotor time constant presented in [Utkin et al., 1999] , [Rao et al, 2009]. In [Sahin et al., 1995] stator current observer is selected as

$$\frac{d}{dt}\begin{bmatrix} \hat{i}_\alpha \\ \hat{i}_\beta \end{bmatrix} = \begin{bmatrix} E_\alpha \\ E_\beta \end{bmatrix} + \frac{1}{\sigma L_s} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} + \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \tag{22}$$

where $V_\alpha$ and $V_\beta$ are components of the observer control vector and $\mathbf{E}_{\alpha\beta}^T = [E_\alpha \quad E_\beta]$ is vector that includes known components of stator current dynamics. If sliding mode is enforced in manifold

$$S_{\alpha\beta}^i = \left\{ i_\alpha, i_\beta : \sigma_{\alpha i} = \hat{i}_\alpha - i_\alpha = 0 \,\&\, \sigma_{\beta i} = \hat{i}_\beta - i_\beta = 0 \right\} \tag{23}$$

Equivalent control can be expressed as:

$$\begin{aligned} V_{\alpha eq} &= \gamma\eta\phi_\alpha + \gamma\omega\phi_\beta - \lambda i_\alpha - E_\alpha \\ V_{\beta eq} &= \gamma\eta\phi_\beta - \gamma\omega\phi_\alpha - \lambda i_\beta - E_\beta \end{aligned} \tag{24}$$

From (24) two variables can be determined as function of the rest. In sliding mode vector $\mathbf{V}_{\alpha\beta eq}^T = [V_{\alpha eq} \quad V_{\beta eq}]$ can be measured. Selection of vector $\mathbf{E}_{\alpha\beta}^T = [E_\alpha \quad E_\beta]$ offers a range of possibilities in determining flux and/or other variables from (24). In [Sahin et al., 1995] $\mathbf{E}_{\alpha\beta}^T = [-\gamma i_\alpha \quad -\gamma i_\beta]$ is selected and with assumption that $\omega$ and $\eta$ are known rotor flux can be determined from

$$\begin{bmatrix} \hat{\phi}_\alpha \\ \hat{\phi}_\beta \end{bmatrix} = \frac{1}{\gamma} \begin{bmatrix} \eta & \omega \\ -\omega & \eta \end{bmatrix} \begin{bmatrix} V_{\alpha eq} \\ V_{\beta eq} \end{bmatrix} \tag{25}$$

$\mathbf{E}_{\alpha\beta}^{T} = \left[-\left(i_\alpha R_s / \sigma L_s\right) \quad -\left(i_\beta R_s / \sigma L_s\right)\right]$ is selected in [Derdiyok et al., 2001]. Convergence in Lyapunov sense is proven with assumption that $\dot{\omega} = 0, \dot{\eta} = 0$. Moreover it is shown that, by designing additional observer for $\mathbf{L}_{\alpha\beta} = \mathbf{V}_{\alpha\beta eq}$ as

$$\begin{bmatrix} \dot{\hat{L}}_\alpha \\ \dot{\hat{L}}_\beta \end{bmatrix} = \begin{bmatrix} \hat{\eta} & \hat{\omega} \\ -\hat{\omega} & \hat{\eta} \end{bmatrix} \begin{bmatrix} V_{\alpha eq} \\ V_{\beta eq} \end{bmatrix} - \gamma L_m \hat{\eta} \begin{bmatrix} \hat{i}_\alpha \\ \hat{i}_\beta \end{bmatrix} - K \begin{bmatrix} \hat{L}_\alpha \\ \hat{L}_\beta \end{bmatrix} \tag{26}$$

with adaptation law

$$\begin{bmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\omega}} \end{bmatrix} = \begin{bmatrix} L_\alpha + \gamma L_m \hat{i}_\alpha & L_\beta + \gamma L_m \hat{i}_\beta \\ L_\beta & -L_\alpha \end{bmatrix} \begin{bmatrix} \hat{L}_\alpha - V_{\alpha eq} \\ \hat{L}_\beta - V_{\beta eq} \end{bmatrix} \tag{27}$$

concurrent estimation of the rotor flux, velocity and rotor constant is possible. General structure of the electrical machines control in sliding mode is depicted in Fig. 3. The torque/force controller generates desired control voltages (13) and then a PWM algorithm is applied to determine temporal changes of the switches in the supply converter. The selection of the temporal changes of the configuration vectors at the same time realizes the transformation from the field oriented frame of references to the actuator stationary frame of references.

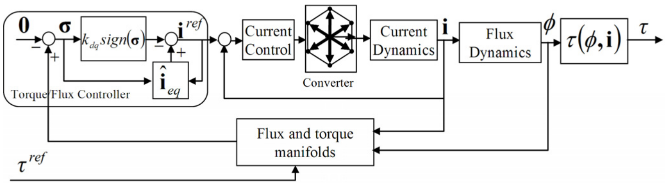



Fig. 3. General structure of the sliding mode actuator control

## 5. SMC IN MOTION CONTROL

### *5.1 Single dof Systems*

Single degree of freedom motion system can be described by

$$\left.\begin{aligned} &\dot{q} = v \\ &a(q)\dot{v} + b(q,v) + g(q) + \tau_{ext} = \tau \end{aligned}\right\} \begin{aligned} &a(q)\ddot{q} + \tau_d = \tau \\ &\tau_d = b(q,v) + g(q) \end{aligned} \tag{28}$$

$q$ and $\dot{q}$ stand for the state variables - position and velocity respectively; $a(q)$ is continuous strictly positive bounded; the forces $b(q,\dot{q})$, $g(q)$ are assumed bounded as well as the exogenous $\tau_{ext}$ and the input force $\tau$. Let closed loop desired motion of system (28) be satisfied if sliding mode is enforced in manifold:

$$S_q = \left\{q, \dot{q} : \sigma_q = c\left(q - q^{ref}\right) + \left(\dot{q} - \dot{q}^{ref}\right) = 0\right\} \tag{29}$$

Control input

$$\begin{aligned} &\tau = \tau^{eq} - a(q)\frac{\lambda(V)}{\left|\sigma_q\right|} sign\left(\sigma_q\right) \\ &\tau^{eq} = \tau_d - a(q)\left(c\left(\dot{q} - \dot{q}^{ref}\right) - \ddot{q}^{ref}\right) \end{aligned} \tag{30}$$

enforces sliding mode in (29) for appropriate scalar function $\lambda(V) > 0$. This can be verified by deriving derivative of Lyapunov function candidate $V = \sigma_q^2 / 2 > 0$, which for control (25) has form $\dot{V} = \sigma_q \dot{\sigma}_q = -\lambda(V) < 0$. Such a selection allows design of different controller – depending on selection of function $\lambda(V) > 0$. General selection as

$$\dot{V} = -\lambda(V) = -k 2^{\xi} V^{\xi}, \quad k > 0, \ \tfrac{1}{2} \le \xi \le 1 \tag{31}$$

leads to sliding mode. The exponential stability for is guarantied for $k > 0$ & $\xi = 1$. In realization of control (30) equivalent force/torque is required. From control error dynamics $\dot{\sigma}_q = a^{-1}\left(\tau - \tau^{eq}\right)$ and assumption that equivalent force $\hat{\tau}^{eq} = \vartheta$ is changing slowly with respect to mechanical system dynamics, thus it can be modelled by $\dot{\vartheta} = 0$, observer

$$\begin{aligned} &\dot{\hat{z}} = l a^{-1}\left(\tau + l\sigma_q\right) - l a^{-q} \hat{z} \\ &\hat{\vartheta} = \hat{z} - l\sigma_q = \hat{\tau}^{eq} \end{aligned} \tag{32}$$

gives estimated equivalent force which can be used in (30) instead of $\tau^{eq}$.

### *5.2 Multi-body Mechanical Systems – Configuration Space*

Let us look at problem of selection of the control input for multi-body system (9) enforcing sliding mode in manifold

$$\begin{aligned} &S = \left\{\mathbf{q}, \dot{\mathbf{q}} : \boldsymbol{\sigma}\left(\mathbf{q}, \mathbf{q}^{ref}\right) = \mathbf{C}\Delta\mathbf{q} + \Delta\dot{\mathbf{q}} = \mathbf{0}; \mathbf{C} \in \Re^{n\times n}; \mathbf{C} > 0\right\} \\ &\Delta\mathbf{q} = \mathbf{q} - \mathbf{q}^{ref} \end{aligned} \tag{33}$$

Let Lyapunov function candidate be selected as $V = \boldsymbol{\sigma}^T \boldsymbol{\sigma} / 2$. Then control

$$\begin{aligned} &\boldsymbol{\tau} = \boldsymbol{\tau}^{eq} - \mathbf{A}(\mathbf{q})\boldsymbol{\Psi}(\boldsymbol{\sigma}) \\ &\boldsymbol{\tau}^{eq} = \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) - \mathbf{A}(\mathbf{q})\left(\mathbf{C}\Delta\dot{\mathbf{q}} - \ddot{\mathbf{q}}^{ref}\right) \end{aligned} \tag{34}$$

gives $\dot{V} = \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T \boldsymbol{\Psi}(\boldsymbol{\sigma}) < 0$, for $sign(\boldsymbol{\Psi}(\boldsymbol{\sigma})) = sign(\boldsymbol{\sigma})$ where $sign(\mathbf{x})$ stands for component-vice function with elements $sign(x_i)$ being 1 for $x_i > 0$ and $-1$ for $x_i < 0$, $i = 1,2,...,n$. This leads for vide range of the possibilities for selection of the function $\boldsymbol{\Psi}(\boldsymbol{\sigma})$. For example, if $\boldsymbol{\Psi}(\boldsymbol{\sigma})$ is selected discontinuous, finite-time convergence to equilibrium and sliding mode motion can be enforced [Utkin et al, 1999].

### *5.2 Multi-body Mechanical Systems – Operational Space*

Dynamics of task $\mathbf{x} = \mathbf{x}(\mathbf{q}), \mathbf{x} \in \Re^{p\times 1}$ with $\mathbf{q} \in \Re^{n\times 1}, p \le n$ can be expressed as

$$\dot{\mathbf{x}} = \left(\frac{\partial \mathbf{x}(\mathbf{q})}{\partial \mathbf{q}}\right)\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}; \ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} \tag{35}$$

For non-redundant systems $p = n$, projection of operational space force $\mathbf{f}_x$ into configuration space is expressed as $\boldsymbol{\tau} = \mathbf{J}^T \mathbf{f}_x$ [Khatib, 1987]. Inserting (9) and $\boldsymbol{\tau} = \mathbf{J}^T \mathbf{f}_x$ into $\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}}$ task dynamics becomes

$$\boldsymbol{\Lambda}_x \ddot{\mathbf{x}} + \boldsymbol{\mu}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{v}(\mathbf{q}) = \mathbf{f}_x \tag{36}$$

Where $\mathbf{\Lambda}_x = \left(\mathbf{J}\mathbf{A}^{-1}\mathbf{J}^T\right)^{-1}$; $\boldsymbol{\mu}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{\Lambda}_x\left(\mathbf{J}\mathbf{A}^{-1}\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) - \dot{\mathbf{J}}\dot{\mathbf{q}}\right)$ and $\mathbf{v}(\mathbf{q}) = \mathbf{\Lambda}_x\mathbf{J}\mathbf{A}^{-1}\mathbf{g}(\mathbf{q})$. Let task manifold is specified by

$$S_x = \left\{\mathbf{x},\dot{\mathbf{x}} : \boldsymbol{\sigma}_x\left(\mathbf{x},\mathbf{x}^{ref}\right) = \mathbf{C}\Delta\mathbf{x} + \Delta\mathbf{x} = \mathbf{0}; \mathbf{C} \in \Re^{n\times n}; \mathbf{C} > 0\right\} \\ \Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^{ref} \tag{37}$$

Then control

$$\mathbf{f}_x = \mathbf{f}_x^{eq} - \mathbf{\Lambda}_x\mathbf{\Psi}_x(\boldsymbol{\sigma}_x) \\ \mathbf{f}_x^{eq} = \boldsymbol{\mu}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{v}(\mathbf{q}) - \mathbf{\Lambda}_x\left(\mathbf{C}\Delta\dot{\mathbf{x}} - \ddot{\mathbf{x}}^{ref}\right) \tag{38}$$

for appropriate $\mathbf{\Psi}_x(\boldsymbol{\sigma}_x)$ enforces sliding mode in manifold (37). The structure of the task space control is shown in Fig. 4.

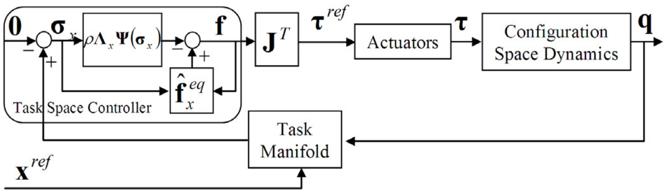



Fig. 4 Task space control of multi-body systems

### *5.3 Multi-body Mechanical Systems – Constrained Systems*

Let constraint manifold is represented by $\phi(\mathbf{q}) = \mathbf{0} \in \Re^{m\times 1}$. Constraint Jacobian $\mathbf{\Phi} = \left(\partial\phi(\mathbf{q})/\partial\mathbf{q}\right)$ is assumed to have full row rank. In the constraint manifold interaction unknown forces $\boldsymbol{\lambda} \in \Re^{m\times 1}$, are balanced by the system motion to obtain

$$\mathbf{A}\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) - \boldsymbol{\tau} = \mathbf{\Phi}^T\boldsymbol{\lambda} \tag{39}$$

Velocity in the constrained direction $\mathbf{\Phi}\dot{\mathbf{q}}$ must be zero. That leads to direct application of the SMC concept – the interaction force can be determined from the requirement that sliding mode is enforced in manifold $\boldsymbol{\sigma}_\phi(\mathbf{q}) = \mathbf{\Phi}\dot{\mathbf{q}} = \mathbf{0}$. From $\dot{\boldsymbol{\sigma}}_\phi(\boldsymbol{\lambda} = \boldsymbol{\lambda}_{eq}) = \mathbf{0}$ follows

$$\boldsymbol{\lambda}_{eq} = -\mathbf{\Phi}^{\#\mathrm{T}}(\boldsymbol{\tau} - \mathbf{b} - \mathbf{g}) - \left(\mathbf{\Phi}\mathbf{A}^{-1}\mathbf{\Phi}^T\right)^{-1}\dot{\mathbf{\Phi}}\dot{\mathbf{q}} \tag{40}$$

Matrix $\mathbf{\Phi}^{\#\mathrm{T}} = \left(\mathbf{\Phi}\mathbf{A}^{-1}\mathbf{\Phi}^T\right)^{-1}\mathbf{\Phi}\mathbf{A}^{-1}$ stands for the transpose of the generalized inverse of constraint Jacobian. By inserting (40) into (39) equations of motion, with constraint consistent initial conditions, become

$$\mathbf{A}\ddot{\mathbf{q}} = \mathbf{\Gamma}^T(\boldsymbol{\tau} - \mathbf{b} - \mathbf{g}) - \mathbf{\Phi}^T\mathbf{\Lambda}_\Phi\dot{\mathbf{\Phi}}\dot{\mathbf{q}}, \quad \phi(\mathbf{q}) = \mathbf{0} \in \Re^{m\times 1} \\ \mathbf{\Gamma}^T = \left(\mathbf{I} - \mathbf{\Phi}^T\left(\mathbf{\Phi}\mathbf{A}^{-1}\mathbf{\Phi}^T\right)^{-1}\mathbf{\Phi}\mathbf{A}^{-1}\right), \quad \mathbf{\Lambda}_\Phi = \left(\mathbf{\Phi}\mathbf{A}^{-1}\mathbf{\Phi}^T\right)^{-1} \tag{41}$$

The unconstrained motion (41) can be enforced by selecting appropriate control input similarly as in task control.

## 6. CONCLUSIONS

Sliding mode design methods and their applications in electrical drives and motion control systems has been discussed. Selection of temporal changes in converter configuration enforcing sliding mode in electric drives current tracking error is shown as an example of switching converter control in the SMC framework. In addition, design of nonlinear SMC based observers for IM machine is shown. Motion control systems are discussed for single dof and multi-body systems. It has been shown that sliding mode control is well suited for control of such systems.

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