Conflict Resolution: Role of Strategic Communication, Reputation and Audience Costs*

Selçuk Özyurt†
Sabancı University

June 10, 2011

Abstract

This paper investigates roles of strategic communication, reputation and audience costs in crises bargaining, which is modeled as a continuous-time war of attrition game between two players (e.g., the leaders of two states). Initially, states send (costly) messages that signal their preferences concerning negotiated versus military settlements. Once the dispute is carried to the public, at each moment a state can choose to back down, attack or escalate the crisis further. If a state backs down, its leader suffers audience costs that increase as the public confrontation proceeds. Furthermore, each state is suspected to be a commitment (an irrational) type who will never back down, which allows players to build reputation on obstinacy. Equilibrium analysis shows that higher sensitivity to audience costs is not always an advantage. A state that can generate higher audience costs (such as democracies) is in unfavorable position whenever the cost of attacking or the uncertainty regarding the opponent's irrationality is high. Escalation would make war an optimal outcome even for rational players, but war is not an inevitable outcome. The model also provides a rich set of empirically testable hypotheses.

*This research was supported by the Marie Curie International Reintegration Grant (# 256486) within the European Community Framework Programme.

†Faculty of Arts and Social Sciences, Sabancı University, 34956, Istanbul, Turkey. (e-mail: ozyurt@sabanciuniv.edu)

1. Introduction

Conflicts are unavoidable parts of economic, business and political negotiations, and are frequently characterized as "wars of nerves". In business, for example, this form arises when two (or more) firms compete with each other, each one losing money but hoping that the competitors will eventually give up and exit the industry. A remarkable example is the competition between British Satellite Broadcasting (BSB) and Sky Television who fought for control of the satellite broadcasting business in the U.K. in the late 1980s. Through October 1990, the two firms accumulated losses more than £1 billion. The conflict resolved in November 1990, when two competing firms announced that they would merge into a single firm, BSkyB, with control split evenly. Clearly, examples are not limited to business. Political conflicts or wage negotiations between firms and unions frequently turns into a "wars of attrition" game.[1]

How crises unfold and why rational actors end up with inferior outcomes such as delay in agreement, war, strike or unprofitable acquisitions or mergers are important questions begging for explanation. Fearon (1994) provides a novel approach to crisis bargaining and models an international crisis as a political "war of attrition" game with three defining features. First, at each moment a state can choose to attack, yield to opponent, or escalate the crisis further. Second, if a state backs down, its leaders suffer audience costs that increase as the crisis escalates.[2] This formalization is motivated by an empirical claim that crises are public events carried out in front of domestic political audiences, and that the leaders who make a public threat suffer this cost when they fail to carry through on it.[3] The third attribute is (two-sided) uncertainty regarding the states' cost of attacking. Subject to these specifications, a crisis always has a unique horizon –a level of escalation after which neither side will yield because the cost of backing down is higher than attacking, making war inevitable.

In this paper, I build a model upon the dialects and the setup used in Fearon (1994) to investigate an empirical observation that sides that are engaged in a dispute often times send (costly) messages or threats to each other and that these actions affect how the crises develop (see, for example, Guisinger and Smith 2002, Leventoglu and Tarar

[1] The war of attrition has been used to explain the evolution of natural oligopolies (Bulow and Klemperer, 1997), patent races (Lee and Wilde, 1980), labor strikes (Kennan and Wilson, 1989), litigation and biological competition (Smith, 1974, Smith and Parker, 1976, and Riley, 1980), R & D tournaments (Taylor, 1995), and firms exiting from a declining industry (Ghemawat and Nalebuff,1985, and Fudenberg and Tirole, 1986).

[2] These costs might arise, for example, from the actions of domestic audiences concerned with whether the leadership is successful or unsuccessful at foreign policy.

[3] For further discussions about the micro-foundations of audience costs, see Smith (1998), Guisinger and Smith (2002), Schultz (2001) and Slantchev (2006). For emprical evidence that audience cost exist, see Eyerman and Hart (1996), Gelpi and Griesdorf (2001), Partell and Palmer (1999), Prins (2003) and Tomz (2007).

2005, Sartori 2002, and Smith 1998).[4] I add two new features to the crises bargaining game. First, each state can send one of two messages – *Faint* (f) or *Strong* (s), which determines the growth rate of the states' audience costs. Second, I assume that states can perfectly anticipate each other's cost of war, but introduce another source of uncertainty, allowing players to build reputation on obstinacy. Both states have some small, positive probability of being a commitment (an irrational) type who will never back down.

The formalization has two major benefits. First, it offers a rich set of comparative statics results that provide insights into the dynamics of disputes. One of the most important result, which is also known as *democratic peace* in political science literature, implies that war is less likely if two states' ability of generating audience costs converge or if the states' sensitivity to audience costs increases.[5] Moreover, the model predicts that if the cost of attacking (or war) increases, then states become less aggressive, and it is less likely that the conflict ends with war but more likely that escalation lasts longer.

Second, it helps investigating the roles of strategic communication and reputation that is missing in existing formal models of crisis bargaining.[6] Audience costs are an important factor enabling states to learn about an opponent's willingness to use force in a dispute. However, the results in this paper suggests, unlike the conventional wisdom, that the ability of generating higher audience costs is not always better. Fearon (1994), for example, shows that a state's *ex-ante* payoff increases with its ability to generate audience costs and concludes that this would explain why leaders want to be able to create significant audience costs in international contests. However, audience costs is not enough to explain an additional empirical observation that leaders sometimes make very strong public threats or commitments in international crises, and at other times make much more limited ones or none at all (Leventoglu and Tarar, 2009). The model I provide makes it evident indirectly that states' previously-established reputations (possibly, built in similar crises in the past) would explain this phenomenon. Equilibrium analysis shows that for almost all parameter values, states with high reputation of being "*passive-commitment*" type most likely send the message that generates lower audience costs.[7]

In the model, there are two "threats" that states can utilize to make their commitments credible; possibility of war and obstinate types. A state that is more sensitive to audience costs, say *state 1*, will benefit the possibility of a costly end after some escalation

[4]The setup I use in this paper can be used to explain similar problems in wage negotiations and business conflicts with appropriate modifications of the jargon and the payoff structure.

[5]Although the idea of *democratic peace* has circulated since Immanuel Kant, it was not scientifically evaluated until the 1960s. It relies on one of the most throughly tested observation in international politics that democracies (for some appropriate definition of democracy), rarely, or even never, go to war with one another. For further discussion, see, for example, Lipson (2003) and Schultz (1999).

[6]See for example, Fearon (1997), Kurizaki (2007), Leventoglu and Tarar(2005) and Sartori (2002).

[7]State i's reputation of being "*passive-commitment*" type is its (prior) probability that the obstinate state i is of passive type who sends the weak message.

(i.e., war).[8] On the other hand, the state that is less sensitive to audience cost, i.e. *state 2*, will derive advantage from using the second threat because it can build its reputation faster.[9] Therefore, the effectiveness of these two threats determines the advantageous state. If the cost of war or the uncertainty regarding the states' irrationality is high (so that the danger of war following, possibly, a lengthy escalation is a risky option), the state that is less sensitive to audience costs will be in an advantageous position.

Furthermore, I show that the possibility of strategic communication would benefit only the first state. Namely, a state that can generate higher audience costs communicates about its true preferences concerning negotiated versus military settlements more credibly. However, I also show that communication has no benefit to the states if they are not in the same dyad, i.e., if two states' sensitivity to audience costs are distinctly dissimilar. I also find that the state which is more sensitive to audience costs is (almost) always more *aggressive*. That is, state 1 sends the strong message with a higher probability.

Section 2 explains the model. Equilibrium strategies of the crises bargaining game for the case with no communication between states are characterized in Section 3. Section 4 examines the case with strategic communication. Section 5 presents the comparative statics results. Finally, Section 6 makes some closing remarks.

2. The Model

Two states (or leaders)- 1 and 2- are in dispute over a prize worth $v > 0$. The crises occurs in continuous time. At the beginning, time $t = 0$, state leaders simultaneously choose a message, and then the dispute and the messages become public. "Messages" can be sent only once and be thought of either as political promises or as taking actions such as mobilizing or preparing troops.[10] There are two messages states can pick- *Faint* (f) or *Strong* (s). Moreover, at all times $t \geq 0$ before the crisis ends, each state can choose to escalate, yield, or attack. The crises ends when one or both states attack or yield. Escalating the conflict can be interpreted as simply waiting or as taking actions to support the messages sent at time 0.

Payoffs are given as follows. If either state attacks before the other concedes, the dispute ends with war and both states receive the (net expected) payoff of $-w < 0$. If state i sends the signal $m_i \in M = \{f, s\}$ at time 0 and concedes at time t before the other

[8] Since state 1 can generate higher audience cost, it would be the first player to attack.

[9] Similar to Fearon (1994), I find that regardless of the initial conditions, the state that is less sensitive to audience costs is always more likely to back down in disputes that become public contests.

[10] As Fearon (1997) expreses, the messages are *tying-hand* (not *sunk-cost*) signals, which work by creating (audience) costs that the leadership would accumulate and suffer in case of a failure in the management of the crises.

has yielded or attacked, then its opponent j receives the prize while i suffers audience cost equal to $c_i^{m_i}(t)$, a continuous and strictly increasing function of the amount of escalation with $c_i^{m_i}(0) = 0$. In case of simultaneous concessions, the prize is divided equally.[11] For analytical simplicity, I consider the linear case $c_i^{m_i}(t) = tc_i^{m_i}$ for each message m_i, where the audience-cost coefficient $c_i^{m_i} > 0$ is indicating how rapidly escalation creates audience costs for state i who sends message m_i.

States and messages are not symmetric in their ability of generating audience costs. More specifically, for each state i and message $m \in M$, we have $c_i^f = \rho c_i^s$ and $c_2^m = \theta c_1^m$, where $\rho, \theta \in (0, 1)$. The parameter ρ captures the messages' relative power of generating audience cost while θ measures the states' relative ability. Given that both states send the same message, state 1 always generates higher audience cost since $\theta < 1$. In this sense, I call state 1 *democratic state*. To minimize the number of the variables, I normalize the cost coefficient c_1^s to 1 and so, the audience-cost coefficients of the states are as given by the following table.

Messages	State 1	State 2
Strong (s)	$c_1^s = 1$	$c_2^s = \theta$
Weak (f)	$c_1^f = \rho$	$c_2^f = \theta\rho$

Figure 1 schematically depicts the structure of the game, with payoffs indicated in the case that state 1 concedes or attacks first at time 0 and $t_1 > 0$. Call this *Crisis Game*, where all parameters are common knowledge, G.

Finally, the crises game G is complicated by the possibility of "reputations for obstinacy" – both states have some small, positive probability of being a commitment type who will never back down. More formally, an obstinate (or commitment) type state i is identified by the message $m_i \in M$ and implementing a simple strategy: It always sends the message m_i at time 0, never yields to its opponent, and is willing to attack to acquire the prize. However, commitment types are not completely "irrational" or "myopic", but understand the equilibrium, and start the war (immediately) once they are convinced that their opponent is also committed.[12] The initial probability that a state is obstinate

[11] If one state chooses to attack at time t and the other chooses to yield or attack at the same time, both states receive $-w$. However, if both yield at time t, then state i receives $\frac{v}{2} - tc_i^{m_i}$. Finally, if states escalate the conflict forever, both get some payoff strictly less than $-w$. These particular assumptions are not crucial because simultaneous concessions or attacks, or escalation with infinite horizon occurs with probability 0 in equilibrium. An equilibrium property that there is no escalation forever also follows from the assumptions on commitment types.

[12] Commitment types are used in the bargaining literature first by Myerson (1991) (r-insisting types) and studied extensively later by Abreu and Gul (2000), Kambe (1999), Compte and Jehiel (2002), Atakan and Ekmekci (2009) and Ozyurt (2011). Abreu and Sethi (2003) supports the existence of commitment types from evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such "irrational" types is not compatible with evolutionary stability in a bargaining environment.

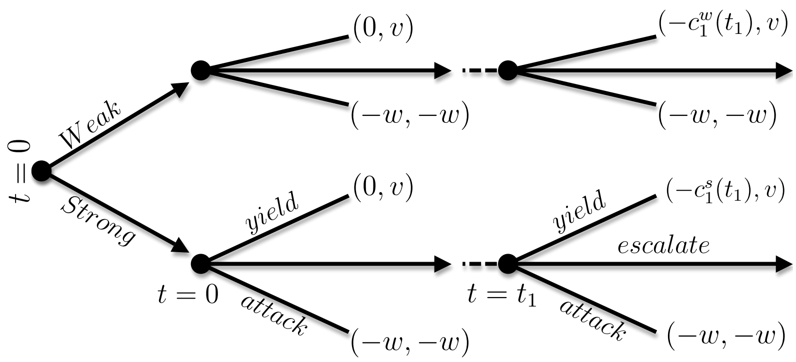

Figure 1: A Schematic representation of the Crises Game for state 1

is denoted by $z \in (0,1)$. The initial prior, z, is common for both states. Let $z_i \in (0,1)$ denote the conditional probability that state i is type $f \in M$, who sends the weak message, given that state i is obstinate.[13] Each state knows its own type but does not know the opponent's true type. The probability z can be interpreted as the uncertainty about the states' commitment to the prize in the current crises, whereas z_i can be thought as the commitment type state i's *previously-established* reputation on dealing with crisis situations.[14]

Strategies

State i's strategy has two parts. The first part $\mu_i \in [0,1]$ denotes the probability of sending the strong message. Given μ_i, denote by $z_i(m)$ the posterior probability that state i is the commitment type conditional on the message $m \in M$ is sent by state i. Therefore, the Bayes' rule implies that

$$z_i(s) = \frac{z(1-z_i)}{z(1-z_i)+(1-z)\mu_i} \qquad \text{and} \qquad z_i(f) = \frac{zz_i}{zz_i+(1-z)(1-\mu_i)}$$

The second part of state i's strategy $\sigma_i = (\mathscr{F}_i, \mathscr{Q}_i)$ has two parts. Let $\mathbb{F}$ be the set of all right-continuous distribution functions defined over $[0,\infty]$.[15] Then, $\mathscr{F}_i : M^2 \to \mathbb{F}$ maps each message profile, $\bar{m} = (m_1, m_2) \in M^2$ to a right-continuous distribution function $F_i^{\bar{m}}(t) : [0,\infty] \to [0,1]$ denoting the probability that state i will yield to state j by time t inclusive. Similarly, $\mathscr{Q}_i : M^2 \to \mathbb{F}$ maps each message profile, $\bar{m}$ to a right-continuous

[13] That is, z_i (or equivalently $1-z_i$) is the probability that the obstinate state i is of *passive* (or equivalently, *aggressive*) type.

[14] Equivalently, $1-z_i$ would be interpreted as the commitment type state i's already established reputation of being aggressive.

[15] Let $[0,\infty] = [0,\infty) \cup \{\infty\}$.

distribution function $Q_i^{\bar{m}}(t) : [0, \infty] \to [0, 1]$, which denotes the probability that state i attacks by time t (inclusive).

Note that, for any $\bar{m}$, the distribution functions $(F_i^{\bar{m}}, Q_i^{\bar{m}})$ are state j's belief about state i's play during the escalation, and that $F_i^{\bar{m}}(t) + Q_i^{\bar{m}}(t) \leq 1$ for all $t \geq 0$. Therefore, $F_i^{\bar{m}}(t)$ is state i's strategy from the point of view of state j, and thus in equilibrium, it never reaches 1 since state j believes that state i is the commitment type m_i with probability $z_i(m_i)$. That is, in equilibrium, we must have $\lim_{t\to\infty} F_i^{\bar{m}}(t) \leq 1 - z_i(m_i)$.

Given $F_j^{\bar{m}}$ and $Q_j^{\bar{m}}$, state i's expected payoff (in the subgame following the message realization $\bar{m}$) of conceding at time t is

$$\begin{aligned} U_i(t, F_j^{\bar{m}}, Q_j^{\bar{m}}) := & \int_0^t v \, dF_j^{\bar{m}}(y) - \int_0^t w \, dQ_j^{\bar{m}}(y) + \left[1 - F_j^{\bar{m}}(t) - Q_j^{\bar{m}}(t)\right] \left[-tc_i^{m_i}\right] \\ & + \left(\frac{v}{2} - tc_i^{m_i}\right) \left[F_j^{\bar{m}}(t) - F_j^{\bar{m}}(t^-)\right] - w \left[Q_j^{\bar{m}}(t) - Q_j^{\bar{m}}(t^-)\right] \end{aligned} \tag{1}$$

with $F_j^{\bar{m}}(t^-) = \lim_{y\uparrow t} F_j^{\bar{m}}(y)$ and $Q_j^{\bar{m}}(t^-) = \lim_{y\uparrow t} Q_j^{\bar{m}}(y)$.[16]

3. Absence of Strategic Communication

In this section, I assume that each state has a unique message to send, and that their audience costs are $c_1(t) = c_1 t$ and $c_2(t) = c_2 t$ where $c_1 > c_2$.[17] Since each state can send only one message, strategy of state i is just the pair of distribution functions $\sigma_i = (F_i, Q_i)$ defined in the previous section. The special cases I investigate in this section both convey the flavor of the analysis and are the basic building blocks for the multiple message case studied subsequently in the next section.

Complete Rationality

Suppose for now that both states are known to be rational, i.e., $z = 0$. Rational state i does not escalate the dispute beyond the time t_i where its audience costs is equal to the cost of war, i.e. $t_i c_i = w$. Since state 1 can generate higher audience costs (as $c_1 > c_2$), it would be the first player to attack (as $t_1 < t_2$). However, state 2 can anticipate that delaying the concession has no benefit, and thus in equilibrium, concedes at time 0. Hence, in equilibrium, the conflict resolves before it escalates, and payoffs of state 1 and 2 are v and $-w$, respectively.

[16]Note that $U_i(t, F_j^{\bar{m}}, Q_j^{\bar{m}})$ is evaluated at time 0.

[17]Thus, there is no communication between the states.

Now, I resume the case where $z > 0$. Since each state has a unique message to send, the posterior belief regarding the states' commitment type is also z. In equilibrium, each state concedes by choosing randomly the timing of backing down with a decreasing hazard rate. Escalation of the conflict stops at some finite (deterministic) time t^*, a function of primitives, with certainty. No state attacks before time t^*, and rational state 1 attacks with some positive probability at time t^* whenever the cost of war is low.

In equilibrium, state 2's instantaneous acceptance rate (i.e. the hazard rate) is higher. That is, state 2 can build its reputation much faster. This is a standard result in war of attrition games: Since state 1 generates higher audience cost, it can be indifferent between conceding now and delaying concession a bit if the gain from delay is sufficiently higher. State 1's benefit from delay is determined by the likelihood of the second state's concession during the period of delay, implying that in equilibrium, state 2 must yield at a rate faster than that of state 1.

As there is an uncertainty on states' commitment type, there are two devices that rational players can utilize to make their commitment credible. The first one is the cost of war and the ability of generating audience cost, and the second one is the possibility of mimicking the commitment type. If the cost of war is low (i.e., $z \leq \frac{v}{v+w}$), then only the first one is the credible commitment device. As we see in the complete rationality case, this tool gives the advantage to state 1 because it can generate higher audience cost. However, if the cost of war is high, so that $z > \frac{v}{v+w}$, then it is risky to use this option. Thus, the only credible commitment device is the possibility of imitating the commitment type. In this case, state 2 has the full advantage. In equilibrium, state 1 anticipates that state 2 can build its reputation faster, and thus makes positive concession at time 0, in order to keep up with the pace of the second state's reputation building. Thus, the second state's *ex-ante* equilibrium payoff is strictly positive while the first state's is 0.

Proposition 1. *The unique sequential equilibrium of the crises game G is characterized by the following conditions: For $i = 1, 2$,*

1. $F_i(t) = 1 - \frac{va_i}{v+tc_j}$ *for all* $t \leq t^*$*,*

2. $a_i \in [0, 1]$ *and* $[1 - a_1][1 - a_2] = 0$*,*

3. t^* *solves* $F_2(t^*) = 1 - z$ *and* $F_1(t^*) \leq 1 - z$*, and*

4. $Q_i(t) = 0$ *for all* $t < t^*$ *and* $Q_i(t) = 1 - F_i(t^*)$ *for all* $t \geq t^*$*.*

I defer the proofs to Appendix. It is well-known (see, for instance Abreu and Gul, 2000) that the unique sequential equilibrium of the continuous-time war of attrition game is characterized by the following three properties. (i) at all times $t > 0$ player i concedes at a constant hazard (instantaneous acceptance) rate that makes the opponent indifferent between escalating and backing down, (ii) at most one player backs down with a positive probability at time 0, and (iii) there exists a finite time t^* at which each player's posterior probability of being the commitment type reaches 1 simultaneously and escalation stops.

The first property in our case is replaced with a decreasing hazard rate $\frac{dF_i/dt}{1-F_i(t)} = c_j/(v + tc_j)$. Since the audience cost (of state j) increases with time, the instantaneous acceptance rate (of state i) must be bigger at earlier times of the escalation to make the opponent indifferent between yielding and escalating at all times.

Property (ii) – only one state can back down at time 0– implies the second condition in Proposition 1. Furthermore, (iii) is true when the cost of war is very large. However, when w is small, it is possible that rational state 1 is indifferent between attacking and yielding at the time t^*. In this case, rational state 1 will attack even if it is convinced that the other state is the commitment type. Therefore, in equilibrium, state 2's posterior probability of being the commitment type has to reach 1 at time t^*, but this does not have to hold for state 1 since state 1 can generate its audience cost faster. Therefore, t^* must solve $F_i(t^*) = 1 - z$ for at least one of the states $i \in \{1, 2\}$. This requirement pins down the identity of the player who needs to back down at time 0 as well as the probability of such a concession and hence establishes the uniqueness of equilibrium. Finally, the forth condition of Proposition 1 implies that war is strictly inferior to backing down at all times $t < t^*$.

Lemma 1. *The crises game G ends at time $t^* = \min\{t_1^*, t_2^*\}$ where for $i, j \in \{1, 2\}$ and $i \neq j$, $t_i^* = \min\{\frac{w}{c_i}, \tau_i\}$ and $\tau_i = \frac{v(1-z)}{zc_j}$.*

Proof. Rational state 1 will not escalate the dispute once it is convinced that state 2 is the commitment type. Given that state 2 does not back down with positive probability at time 0, state 1 will be convinced regarding state 2's commitment at the time τ_2 solving $F_2(\tau_2) = 1 - \frac{v}{v+\tau_2 c_1} = 1 - z$, implying $\tau_2 = \frac{v(1-z)}{zc_1}$. Also, state 1 will not back down after time t satisfying $tc_1 = w$. Therefore, in equilibrium, state 1 must stop escalation before time $\frac{w}{c_1}$ and τ_2. On the other hand, state 1 has to stop escalation once attacking becomes optimal for state 2. Thus, in equilibrium, the escalation must stop before time $t^* = \min\{\frac{w}{c_1}, \frac{w}{c_2}, \tau_1, \tau_2\}$. However, since only one state can back down with positive probability at time 0, the escalation will continue with some positive probability until time t^* and stop at this time with certainty. □

The following two lemma solve the equilibrium value of t^* as a function of the primitives, and find the equilibrium strategies $F_1(t)$ and $F_2(t)$.

Lemma 2. *In equilibrium, if $t_i^* > t_j^*$ where $i, j \in \{1, 2\}$ and $i \neq j$, then we have*

$$F_j(t) = 1 - \frac{v}{v + tc_i} \quad \text{and} \quad F_i(t) = 1 - \frac{va_i}{v + tc_j}$$

where

$$a_i = \begin{cases} z + \frac{c_j z(1-z)}{c_i z}, & \text{if } \frac{w}{c_j} > \frac{v(1-z)}{zc_i} \\ z + \frac{zw}{v}, & \text{otherwise,} \end{cases}$$

Proof. Since $t_i^* > t_j^*$, we have $F_j(0) = 0$ implying $F_j(t) = 1 - \frac{v}{v+tc_i}$. Moreover, since $F_i(t_j^*) = 1 - \frac{va_i}{v+t_j^* c_i} = 1 - z$ we have $a_i = z + \frac{c_j t_j^* z}{v}$ where $t_j^* = \min\left\{\frac{w}{c_j}, \frac{v(1-z)}{zc_i}\right\}$. □

Lemma 3. *In equilibrium, if $t_1^* = t_2^*$, then we have $F_i(t) = 1 - \frac{v}{v+tc_j}$ for $i, j \in \{1, 2\}$ and $j \neq i$.*

Proof. Since $t_1^* = t_2^*$, we have $F_1(0) = F_2(0) = 0$ implying $F_i(t) = 1 - \frac{v}{v+tc_j}$ for $i = 1, 2$. □

Equilibrium Payoffs

State i is called ***strong*** if $t_i^* < t_j^*$ and ***weak*** otherwise. Equivalently, state i is strong if and only if state j backs down with positive probability at time 0, i.e $F_j(0) > 0$, and according to Proposition 1, at most one state can be strong in equilibrium.[18] Since each state is indifferent between backing down and escalating until the time t^*, the equilibrium expected payoff of rational state i is the same at all times $t \in [0, t^*]$, and is equal to what state i can achieve at time 0. Hence, state i's equilibrium payoff in the crises game (evaluated at time 0) is

$$\begin{aligned} U_i &= vF_j(0) + [-c_i(0)](1 - F_j(0)) \\ &= vF_j(0) \end{aligned} \tag{2}$$

Note that, equilibrium payoff of the weak state is always 0, whereas it is (strictly) positive for the strong state. Next proposition determines the identity of the strong state in equilibrium.

Proposition 2. *In equilibrium, the democratic state (state 1) is strong if and only if $z \leq \frac{v}{v+w}$. For all other values of z, state 2 is strong in equilibrium.*

Proof. Since $c_1 > c_2$, $\frac{w}{c_1} < \frac{w}{c_2}$ and $\tau_1 = \frac{v(1-z)}{zc_2} > \tau_2 = \frac{v(1-z)}{zc_2}$. Hence, state 1 is strong whenever $\frac{w}{c_1} \leq \frac{v(1-z)}{zc_1}$. The last inequality yields the desired inequality. □

[18] In equilibrium, both states are weak if $t_i^* = t_j^*$ holds.

4. Presence of Strategic Communication

In this section, I resume the case where each state can send one of two messages in the set $M = \{f, s\}$. What is new in multi-message case is that the democratic state (state 1) can be strong in equilibrium even when the cost of war is high. However, this is possible only if z_1 – the prior belief that state 1 is the commitment type f, who sends faint message – is sufficiently high, i.e., $z_1 \geq \frac{\rho(1-\theta)}{\theta(1-\rho)}$.

It is clear from the last inequality that, in equilibrium, state 1 will never be strong for high values of w if, for example, the asymmetry between the messages' ability of generating audience cost (ρ) is more than or equal to the asymmetry between the states' power of generating audience cost (θ). This observation is conveying an important hint regarding the dynamics of the equilibrium strategies. As $\rho \geq \theta$ holds, state 1 will be generating its audience cost faster than state 2 for all messages, and this is the advantage of state 2 making it strong player when the cost of war is high. However, when $\rho < \theta$, state 2 loses its advantage once state 1 chooses the weak and 2 sends the strong message.

Thus, for large values of z_1 and θ, rational state 1 can exploit the disadvantage of state 2 and use both the threat of war and of uncertainty regarding its rationality to have a positive expected payoff in equilibrium whenever the cost of war is high. Here is how it will work: In equilibrium, both states send each message with a positive probability. However, as z_1 takes higher values, state 1 will pick the strong message with a lower probability, and so the posterior probability that state 1 is a commitment type conditional on sending the strong message will be higher. Therefore, state 1 can push its initial reputation – posterior belief – ahead of state 2's at time 0 by sending the strong message. With this, state 1 can eliminate the second state's advantage – building reputation faster – when the cost of war is high. On the other hand, picking the faint message with a high probability might put state 1 in a disadvantageous position in terms of the initial reputations (depending on the value of θ and z_2). However, in equilibrium, state 1 can eliminate this as well by threatening state 2 by war. That is, state 1 plays a strategy that dictates it to attack with a positive probability after some escalation. Following a strategy that threatens the opponent by war is risky as the cost of war is high. However, in equilibrium, rational state 1 can decrease this risk by using this threat only when state 2 sends the weak message, in which case escalation lasts much longer.

The next result is nothing but a restatement of the Proposition 1 for the multiple message case.

Proposition 3. *Given $\mu_1, \mu_2 \in [0,1]$ and $\bar{m} \in M^2$, sequential equilibrium strategies, $(F_i^{\bar{m}}, Q_i^{\bar{m}})$ for each state $i \in \{1,2\}$, of the crises game are unique and characterized by the following conditions:*

1. $F_i^{\bar{m}}(t) = 1 - \frac{va_i(\bar{m})}{v+tc_j^{m_j}}$ *for all* $t \leq t^*_{\bar{m}}$,

2. $a_i(\bar{m}) \in [0,1]$ *and* $[1-a_1(\bar{m})][1-a_2(\bar{m})] = 0$,

3. $t^*_{\bar{m}}$ *solves* $F_2^{\bar{m}}(t^*_{\bar{m}}) = 1 - z_2(m_2)$ *and* $F_1^{\bar{m}}(t^*_{\bar{m}}) \leq 1 - z_1(m_1)$, *and*

4. $Q_i^{\bar{m}}(t) = 0$ *for all* $t < t^*_{\bar{m}}$ *and* $Q_i^{\bar{m}}(t) = 1 - F_i^{\bar{m}}(t^*_{\bar{m}})$ *for all* $t \geq t^*_{\bar{m}}$.

Remark 1. In equilibrium, both states send each message with a positive probability. That is, $\mu_i^* \in (0,1)$ for $i = 1,2$.

Proposition 3 is stating that in equilibrium, for any realized message profile $\bar{m}$, if one state is strong (and thus has a positive expected payoff in the subgame following the message realization $\bar{m}$), then the other state is weak (and has 0 expected payoff in that subgame). The next result proves a stronger version of this claim. In equilibrium, if state i is strong at some realized message profile $\bar{m}$, then state i is strong for all possible realizations of the messages.

Proposition 4. *There exists no equilibrium of the crises game G where both states are strong. That is, there is no equilibrium in which both states' expected payoff in the game G is positive.*

Therefore, in equilibrium, when state i is strong, its expected payoff in the crises game G is positive and state $j(\neq i)$ is weak with the equilibrium payoff of 0. Let (μ^*, σ^*), where $\mu^* = (\mu_1^*, \mu_2^*)$ and $\sigma^* = (\sigma_1^*, \sigma_2^*)$, be an equilibrium strategy profile of the crises game G. Proposition 3 is characterizing σ^* (for both players). The following results characterizes the first part of the equilibrium strategy profile, μ_i^*, whenever state i is strong. For weak states, the first part is fully characterized in the appendix.

Proposition 5. *If (μ^*, σ^*) is an equilibrium strategy profile and $z \leq \frac{v}{v+w}$, then state 1 is strong, state 2 is weak, $\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$ whenever $\theta > \rho$, and $1 - \frac{zz_1w\theta}{v\rho(1-z)} > \mu_1^* > \frac{\theta z(1-z_1)w}{v(1-z)}$ whenever $\theta \leq \rho$.*

Parallel to Proposition 2, state 1 – who can generate higher audience cost – is the strong state in equilibrium whenever the cost benefit ratio of the conflict ($\frac{w}{v}$) or the prior probability of commitment types (z) is small. However, unlike Proposition 2, the second state's advantage does not hold for all other values of z, and this is formally stated in the following two results.

Proposition 6. *If* (μ^*, σ^*) *is an equilibrium strategy profile,* $z > \frac{v}{v+w}$ *and* $z_1 \leq \frac{\rho(1-\theta)}{\theta(1-\rho)}$*, then state 2 is strong, state 1 is weak and* $\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$.

Proposition 7. *If* (μ^*, σ^*) *is an equilibrium strategy profile,* $z > \frac{v}{v+w}$, $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$ *and* $\theta \geq \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$*, then state 1 is strong, state 2 is weak and* μ_1^* *is equal either to* $\frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$ *or to* $1 - \frac{zz_1w}{v(1-z)}$.

For the set of parameter values that are covered by Propositions 5 through 7, there is a single type of equilibrium; in all equilibria, the same state is the strong player. However, the next result shows that for all other parameter values, two types of equilibria coexist; state 1 is the strong player in one type and 2 is strong in the other.

Proposition 8. *If* (μ^*, σ^*) *is an equilibrium strategy profile,* $z > \frac{v}{v+w}$, $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$ *and* $\theta < \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$*, then*

(*i*) *if state 1 is strong, then* μ_1^* *is equal either to* $\frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$ *or to* $1 - \frac{zz_1w}{v(1-z)}$.

(*ii*) *if state 2 is strong, then* $\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$.

5. Comparative Statics

In this section, I present some comparative statics results and defer the proofs of the claims to Appendix. In equilibrium, rational state 1 attacks with a positive probability only if it is strong. However, state 2 never attacks. The probability that state 1 will attack in an equilibrium depends on the messages sent by the states at time 0. More specifically, we have $Q_1^{\bar{m}}(t) = \frac{v}{v+tc_2^{\bar{m}}}$. As it is clear from this formulae and the following table, "probability of war" decreases as θ increases to 1 or as the states' sensitivity for audience costs increases.[19]

$\bar{m}$ (*Messages*)	t^* (*Stopping time of Escalation*)	$Q_1^{\bar{m}}(t^*)$
ss	w	$\frac{v}{v+\theta w}$
sf	w	$\frac{v}{v+\theta\rho w}$
fs	$\frac{w}{\nu}$ (where $\nu = \max\{\rho, \theta\}$)	$\frac{v}{v+\theta\frac{w}{\nu}}$
ff	$\frac{w}{\rho}$	$\frac{v}{v+\theta w}$

Observation 1 (Democratic Peace). *It is less likely that the crises game ends with war as two states' ability of generating audience cost converge, i.e* $\theta \to 1$ *or as the states' sensitivity to audience costs increases.*

[19]Note that, since state 1's audience cost coefficient (c_1^s) is normalized to 1, $c_2^{\bar{m}}$ is either θ or $\rho\theta$ depending on the message state 2 sends. However, one can relax this assumption by setting $c_1^s = c > 1$. Thus, state 2's audience cost coefficients would be $c\theta$ or $c\rho\theta$. As a result, we can conclude that as states' sensitivity to audience costs, i.e. c, increases, the probability of war decreases.

The next table summarizes the results in the previous section and presents the equilibrium strategies μ_1^* and μ_2^* for all possible parameter values.

	STATE 1	Equilibrium Strategy: μ_1^*
(1)	State 1 is strong (and if, $z \leq \frac{v}{v+w}$)	$\mu_1^* \in \left[\frac{\theta z(1-z_1)w}{v(1-z)}, 1-\frac{zz_1 w\theta}{\rho(1-z)v}\right]$ if $\rho \geq \theta$ $\mu_1^* = 1-\frac{zz_1 w}{v(1-z)}$ if $\rho < \theta$
(2)	Strong ($z > \frac{v}{v+w}$ and $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$)	$\mu_1^* = 1 - \frac{zz_1 w}{v(1-z)}$ or $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$
(3)	Weak ($z > \frac{v}{v+w}$)	$\mu_1^* \in \left[\frac{\theta\rho(1-z_1)}{z_2+\rho(1-z_2)}, 1 - \frac{\theta z_1}{z_2+\rho(1-z_2)}\right]$
	STATE 2	Equilibrium Strategy: μ_2^*
(4)	State 2 is strong (and if, $z > \frac{v}{v+w}$)	$\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$
(5)	Weak ($z \leq \frac{v}{v+w}$)	$\mu_2^* \in \left[\frac{z(1-z_2)w}{v(1-z)}, 1 - \frac{zz_2 w}{v(1-z)}\right]$
(6)	Weak ($z > \frac{v}{v+w}$ and $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$)	$\mu_2^* \in \left[\frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}, 1-\frac{z_2}{\theta[z_1+\rho(1-z_1)]}\right]$ or $\mu_2^* \in \left[\frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}, 1-\frac{wzz_2}{v(1-z)}\right]$
(7)	Weak ($z > \frac{v}{v+w}$ and $\mu_1^* = 1 - \frac{zz_1 w}{v(1-z)}$)	$\mu_2^* \in [\underline{\mu}_2, \bar{\mu}_2]$

where

$$\underline{\mu}_2 = \max\left\{\frac{\rho z(1-z_2)w}{\theta v(1-z)}, \frac{(1-z_2)}{\theta(1-z_1)}\left(1 - \frac{zz_1 w}{v(1-z)}\right)\right\}$$

and

$$\bar{\mu}_2 = \min\left\{1 - \frac{zz_2 w}{v(1-z)}, \left(1 - \frac{z_2}{\theta\rho(1-z_1)}\right)\left(1 - \frac{zz_1 w}{v(1-z)}\right)\right\}$$

By using this table, one can easily check that the following two claims are true.

Observation 2. *In equilibrium for the parameter values satisfying $z \leq \frac{v}{v+w}$, state 1 is weakly more aggressive than 2.*[20] *When $z > \frac{v}{v+w}$, the same is true in all equilibria except the one in which state 1 is strong and the escalation ends with war (with a positive probability) only when state 2 sends the faint message.*[21]

The next observation points out that states with low (previously-established) reputation of being aggressive-commitment type most likely send the message that generates lower audience costs.

Observation 3. *In equilibrium for almost all parameter values, and for any state i, if z_i is sufficiently high, then $\mu_i^* < \frac{1}{2}$.*

The next observation summarizes how primitives affect the possibility of war, states' aggressiveness, and the length of horizon (or equivalently the length of escalation) in the crises game.[22] The length of escalation depends on the messages states send. Escalation

[20] More formally, the highest value of μ_1^* in row (1) is bigger than or equal to the highest value of μ_2^* in row (5).

[21] That is, the highest value of μ_2^* in the second range in row (6) is weakly higher than the highest value of μ_1^* in row (2).

[22] By horizon (or escalation), I mean the level of escalation after which neither side will yield, and so the crises game will end either by the concession of the states or by war.

time is longer as the states are less aggressive, and for any given set of parameters, the length of the horizon is the longest when the realized message profile is (ff).

Observation 4. *In equilibrium,*

1. *if the cost of war* (w) *increases, then the states become less aggressive, i.e., they choose the weak message with a higher probability*[23]*, it is less likely that the conflict ends with war and more likely that escalation lasts longer.*

2. *as the value of the prize* (v) *increases, the length of the escalation (weakly) increases*[24]*, states become (weakly) more aggressive*[25] *and it is more likely that the conflict ends with war.*

3. *if two messages' ability of generating audience cost converge, i.e.* $\rho \to 1$*, then states are (weakly) more aggressive and thus the length of escalation is likely to decrease because the realized message profile is likely to be* (ss)*, and*

4. *as* z *increases, states are (weakly) less aggressive.*

The final observation summarizes the relationship between θ and the probability of initial resolution (i.e., the likelihood that the crises game ends at time 0 with no escalation).

Observation 5. *As two states' ability of generating audience cost converge, i.e* $\theta \to 1$*, then*

1. *state* 1 *becomes less aggressive, state* 2 *gets more aggressive, and the length of escalation (weakly) decreases.*

2. *the probability of initial resolution does not change if the cost-benefit ratio of the conflict or* z *is small, i.e.* $z \leq \frac{v}{v+w}$*, and*

3. *given that* $z > \frac{v}{v+w}$*, the probability of initial resolution increases only if state 1 is strong.*

[23]Recall that for a given set of parameters, there may exist multiple equilibrium strategy μ^*. In that case, I consider only the highest values. Also, μ^* does not necessarily, continuously decrease with w because as w increases, the equilibrium strategies might change.

[24]That is, there are some parameter values (for the primitives other than v) in which increase in v does not effect the (equilibrium) escalation time, and for all other parameter values, it increases the length of the horizon.

[25]That is, there are some parameter values (for the primitives other than v) in which increase in v does not effect the value of the highest μ^*, and for all other parameter values, it increases the highest value of μ^*.

6. Concluding Remarks

This paper develops a reputation-based model to highlight the roles of strategic communication and audience costs on conflict resolution. Given the message selections of the states, equilibrium has a unique horizon - a level of escalation after which either both states' reputation simultaneously reaches one, or neither side will yield because the cost of backing down is higher than attacking, making war inevitable.

Equilibrium analysis shows, in contrast to the conventional wisdom, that the ability of generating higher audience costs is not always an advantage. A state that is sensitive to audience costs is in unfavorable position whenever the cost of war or the states' initial reputation of being a commitment type is high. Escalation and increasing audience costs make war an optimal outcome for rational players, but war is not an inevitable outcome because states would convince each other regarding their determinacy before attacking becomes an optimal action.

The model also has rich set of empirically testable hypotheses. For example, a state that has high reputation of being passive-commitment type most likely sends the weak message that generates lower audience costs. A state that is sensitive to audience costs is more aggressive (for almost all parameter values) and can drive benefit from strategic communication only if states are in the same dyad. Dispute ends with war with a lower probability as two states ability of generating audience cost converge or the states sensitivity to audience costs increases. The length of escalation increases but aggressiveness of the states decreases with the cost of war.

Appendix

Proof of Proposition 1. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and is analogous to the proof of Proposition 1 in Abreu and Gul (2000), so I skip the details. Let $\tau^i = \inf\{t \geq 0 | F_i(t) = \lim_{k\to\infty} F_i(k)\}$, where $\inf \emptyset := \infty$. Then, by optimality we have $\tau^1 = \tau^2 (:= t^*)$: A rational state will not delay yielding once it knows that its opponent will never yield. Moreover, optimality implies that $t^* c_1 \leq w$ because no state will back down if the cost of yielding is more than the cost of war. Hence, we must have $Q_i(t) = 0$ for all $t < t^*$.

Lemma A.1. *If state i's strategy F_i is constant on some interval $[t_1, t_2] \subseteq [0, t^*)$, then state j's strategy F_j (where $j \neq i$) is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. *For any state i, F_i does not have a mass point over $(0, t^*]$.*

Lemma A.3. $F_1(0)F_2(0) = 0$.

Therefore, according to Lemma A.1 and A.2, F_i is strictly increasing and continuous over $[0, t^*]$. Therefore, the utility function of state i given in Equation (1) is also continuous on $[0, t^*]$. Then, it follows that $D^i := \{t | U_i(t, F_j, Q_j) = \max_{s\in[0,t^*]} U_i(s, F_j, Q_j)\}$ is dense in $[0, t^*]$. Hence, $U_i(t, F_j, Q_j)$ is constant for all $t \in [0, t^*]$. Consequently, $D^i = [0, t^*]$. Therefore, $U_i(t, F_j, Q_j)$ is differentiable as a function of t. Similarly, F_i is differentiable because the utility function is differentiable on $[0, t^*]$. Differentiating the utility function and applying the Leibnitz's rule, we get $F_i(t) = 1 - \frac{va_i}{v+tc_j}$ where $a_i = 1 - F_i(0)$. By Lemma A.3, we know that $F_1(0)F_1(0) = 0$ implying condition (ii). Optimality implies that $F_2(t^*) = 1 - z$ whereas $F_1(t^*) \leq 1 - z$ because it might be the case that $c_1 t^* = w$. Therefore, $Q_2(t) = 1 - z$ and $Q_1(t) = 1 - F_1(t^*)$ for all $t \geq t^*$.

I do not provide the proofs of following three Lemma since they are the same as the proofs of Lemma 1-3 presented in Section 3.

Lemma A.4. *Given μ^* and the realized messages, $(m_1, m_2) = \bar{m}$, the crises game G ends at time $t^*_{\bar{m}} = \min\{t^1_{\bar{m}}, t^2_{\bar{m}}\}$ where for $i, j \in \{1, 2\}$ and $i \neq j$ we have $t^i_{\bar{m}} = \min\left\{\frac{w}{c_i^{m_i}}, \frac{v(1-z_i(m_i))}{z_i(m_i)c_j^{m_j}}\right\}$.*

Lemma A.5. *Given μ^* and the realized messages, $\bar{m} = (m_1, m_2)$, if $t^i_{\bar{m}} > t^j_{\bar{m}}$ where $i, j \in \{1, 2\}$ and $i \neq j$, then in equilibrium, we have*

$$F_j^{\bar{m}}(t) = 1 - \frac{v}{v + tc_i^{m_i}} \quad \text{and} \quad F_i^{\bar{m}}(t) = 1 - \frac{va_i(\bar{m})}{v + tc_j^{m_j}}$$

where

$$a_i(\bar{m}) = \begin{cases} z_i(m_i) + \frac{c_j^{m_j} z_i(m_i)(1-z_j(m_j))}{c_i^{m_i} z_j(m_j)}, & \text{if } \frac{w}{c_j^{m_j}} > \frac{v(1-z_j(m_j))}{z_j(m_j)c_i^{m_i}} \\ z_i(m_i) + \frac{z_i(m_i)w}{v}, & \text{otherwise,} \end{cases}$$

Lemma A.6. *Given μ^* and the realized messages, $\bar{m} = (m_1, m_2)$, if in equilibrium we have $t^1_{\bar{m}} = t^2_{\bar{m}}$, then $F_i^{\bar{m}}(t) = 1 - \frac{v}{v+tc_j^{m_j}}$ for $i, j \in \{1, 2\}$ and $j \neq i$.*

Given μ^* and realized message profile $\bar{m}$, state i is called ***strong*** if $t^i_{\bar{m}} < t^j_{\bar{m}}$ and ***weak*** otherwise. The equilibrium payoff of state i (evaluated at time 0) is $U_i(\mu^*, \bar{m}) = vF_j^{\bar{m}}(0) + [-c_i^{m_i}(0)](1 - F_j^{\bar{m}}(0)) = vF_j^{\bar{m}}(0)$. Thus, the equilibrium payoff of the weak state (in the subgame following $\bar{m}$) is 0, whereas it is positive for the strong state. Given μ^*, realized messages $\bar{m} = (m_1, m_2)$ and that state i does not back down with positive probability at time 0, state j will be convinced regarding state i's commitment at time $\tau_i^{\bar{m}}$, which solves $F_i^{\bar{m}}(\tau_i^{\bar{m}}) = 1 - z_i(m_i)$. Thus, we have

Messages:	State 1	State 2	$\tau_1^{\bar{m}}$ & $\tau_2^{\bar{m}}$
	Strong	Strong	$\tau_1^{ss} = \frac{v(1-z_1(s))}{z_1(s)\theta}$ & $\tau_2^{ss} = \frac{v(1-z_2(s))}{z_2(s)}$
	Strong	Faint	$\tau_1^{sf} = \frac{v(1-z_1(s))}{z_1(s)\theta\rho}$ & $\tau_2^{sf} = \frac{v(1-z_2(f))}{z_2(f)}$
	Faint	Strong	$\tau_1^{fs} = \frac{v(1-z_1(f))}{z_1(f)\theta}$ & $\tau_2^{fs} = \frac{v(1-z_2(s))}{z_2(s)\rho}$
	Faint	Faint	$\tau_1^{ff} = \frac{v(1-z_1(f))}{z_1(f)\theta\rho}$ & $\tau_2^{ff} = \frac{v(1-z_2(f))}{z_2(f)\rho}$

***Proof of Proposition 4*.** Suppose for a contradiction that there is such an equilibrium (μ^*, σ^*) where both states' ex-ante payoffs are positive. If a state's expected payoff is positive when it sends the strong message, then its expected payoff must also be positive if it sends the weak message, and vice-versa. Therefore, there are two possible cases we need to consider.

Case 4.1. *State 2 is strong when the realized messages are (ss) and (ff) whereas state 1 is strong when (sf) and (fs)*: In this case we must have (1) $w > \tau_1^{ss} > \tau_2^{ss}$, (2) $w > \tau_2^{sf} > \tau_1^{sf}$, (3) $\frac{w}{\rho} > \tau_2^{fs} > \tau_1^{fs}$ if $\theta \leq \rho$ or (otherwise $\frac{w}{\theta} > \tau_2^{fs} > \tau_1^{fs}$), and finally (4) $\frac{w}{\rho}, \tau_1^{ff} > \tau_2^{ff}$. However, these four conditions cannot hold simultaneously because inequalities (1) and (2) imply that $\mu_2^* < \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$ whereas (3) and (4) imply $\mu_2^* > \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$.

Case 4.2. *State 2 is strong when the realized messages are (sf) and (fs) whereas state 1 is strong when (ss) and (ff)*: In this case we must have (1) $w > \tau_2^{ss} > \tau_1^{ss}$, (2) $w, \tau_1^{sf} > \tau_1^{sf}$, (3) $\frac{w}{\rho}, \tau_1^{fs} > \tau_2^{fs}$ if $\theta \leq \rho$ or (otherwise $\frac{w}{\theta}, \tau_1^{fs} > \tau_2^{fs}$), and finally (4) $\frac{w}{\rho} > \tau_2^{ff} > \tau_1^{ff}$. Similar arguments in Case 4.1 suffice to show that these four conditions will not hold simultaneously, which finishes the proof.

– – – – –

The strategy profile (μ^*, σ^*) is called **strict equilibrium** of the crises game G if it is an equilibrium profile, and state i's equilibrium payoff in all subgames (following any realized message profile) is positive given that state i is the strong state under this strategy. For the rest of the analysis, I restrict my attention only to strict equilibria whenever they exist. Strict equilibrium does not exists only in the subgame following that state 1 and 2 send the weak and strong messages, respectively, and $\theta > \rho$.

In equilibrium, σ^* is unique but μ^* is not. In some equilibria (μ^*, σ^*) where state i is strong, state i's equilibrium payoff in some subgames can be 0, though its expected payoff in the crises game is positive. Restricting attention only to strict equilibria (whenever they exist) will eliminate such equilibrium strategy profiles, and this restriction does not affect the main essence of the remaining analysis and results. However, it substantially simplifies the proofs by reducing the number of possible cases we need to consider.

Lemma A.7. *Let (μ^*, σ^*) be an equilibrium profile where state 2 is strong. Then the followings are true:* (1) $\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$, (2) $\theta < \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$, *and* (3) $z_2 > \frac{v(1-z)}{zw(1-\rho)} - \frac{\rho}{1-\rho}$.

Proof. If state 2 is strong, $(A1)$ and one of (Bi)'s (given below) must hold.

$(A1)$	$\tau_2^{ss} < w, \tau_1^{ss}$ and $\tau_2^{sf} < w, \tau_1^{sf}$	$(B1)$	$\tau_2^{fs} < \frac{w}{\theta}, \tau_1^{fs}$ and $\tau_2^{ff} < \frac{w}{\rho}, \tau_1^{ff}$
	(if $\theta > \rho$)	$(B2)$	$\frac{w}{\theta} \leq \tau_2^{fs}, \tau_1^{fs}$ and $\tau_2^{ff} < \tau_1^{ff}, \frac{w}{\rho}$
	(if $\theta \leq \rho$)	$(B2)'$	$\tau_2^{fm} < \tau_1^{fm}$ for each $m \in M$ and $\frac{w}{\rho} = \tau_2^{fm}$ for at least one m.

First note that, if $\theta > \rho$, there is no equilibrium where both $(A1)$ and $(B2)$ hold: To prove this, suppose for a contradiction there is an equilibrium (μ^*, σ^*) such that $(A1)$ and $(B2)$ hold. Let $u_2(m_2)$ denote the equilibrium expected payoff of state 2 under the strategy (μ^*, σ^*), which is evaluated before messages are revealed, when it picks the message m_2. Recall that Proposition 3 characterizes σ^*. Let $y = z(1 - z_1) + (1 - z)\mu_1^*$ denotes the probability that state 1's realized message will be strong. Then, $u_2(m_2) = v[xF_1^{s,m_1}(0) + (1 - x)F_1^{f,m_1}(0)]$ where $F_1^{m_1,m_2}(0) = 1 - a_1(m_1, m_2) = z_1(m_1) + \frac{(1-z_2(m_2))z_1(m_1)c_2^{m_2}}{z_2(m_2)c_1^{m_1}}$. Therefore, $u_2(s) = v[y(1 - a_1(s, s)) + (1 - y)(1 - a_1(f, s))]$ and $u_2(f) = v[y(1 - a_1(s, f)) + (1 - y)(1 - a_1(f, f))]$.

Thus, we have $u_2(s) = v\left[y\left(1 - z_1(s) - \frac{(1-z_2(s))z_1(s)\theta}{z_2(s)}\right) + (1 - y)\left(1 - z_1(f) - \frac{z_1(f)w}{v}\right)\right]$ and $u_2(f) = v\left[y\left(1 - z_1(s) - \frac{(1-z_2(f))z_1(s)\theta\rho}{z_2(f)}\right) + (1 - y)\left(1 - z_1(f) - \frac{(1-z_2(f))z_1(f)\theta\rho}{\rho z_2(f)}\right)\right]$. In equilibrium state 2 must be indifferent between messages, i.e. $u_2(s) = u_2(f)$, implying that $(*)$ $(1 - z_1)[\theta\rho\tau_2^{sf} - \theta\tau_2^{ss}] = z_1[w - \theta\tau_2^{sf}]$. Solving $(*)$ yields $\mu_2^* = \frac{A(1-z_2)}{z_2+A(1-z_2)}$ where $A := \frac{\tau_2^{ss}}{\tau_2^{sf}} = \frac{\theta[z_1+\rho(1-z_1)]}{\theta(1-z_1)+\frac{z_1 w}{\tau_2^{ss}}}$. Also, since $\tau_2^{sf} < w$, $(*)$ implies $\tau_2^{ss} < \rho\tau_2^{sf}$, i.e. $A < \rho$. However, $A < \rho$ yields $\tau_2^{ss} < \frac{w\rho}{\theta}$. However, since $\tau_2^{fs} = \frac{\tau_2^{ss}}{\rho}$, we must have $\tau_2^{fs} < \frac{w}{\theta}$ contradicting with $(B2)$.

On the other hand, when $\theta \leq \rho$, there is no equilibrium where both $(A1)$ and $(B2)'$ hold: Suppose for a contradiction that there is such an equilibrium. The condition $\frac{w}{\rho} = \tau_2^{fs}$ implies that $\mu_2^* = \frac{z(1-z_2)w}{v(1-z)}$. However, this equality contradicts with condition $(A1)$ because $\tau_2^{ss} = w$. Similar arguments for $m = f$ yields the desired contradiction.

Therefore, in equilibrium where state 2 is strong, only $(A1)$ and $(B1)$ will hold. We have $u_2(s) = v\left[y\left(1 - z_1(s) - \frac{(1-z_2(s))z_1(s)\theta}{z_2(s)}\right) + (1 - y)\left(1 - z_1(f) - \frac{(1-z_2(s))z_1(f)\theta}{\rho z_2(s)}\right)\right]$ and $u_2(f) = v\left[y\left(1 - z_1(s) - \frac{(1-z_2(f))z_1(s)\theta\rho}{z_2(f)}\right) + (1 - y)\left(1 - z_1(f) - \frac{(1-z_2(f))z_1(f)\theta\rho}{\rho z_2(f)}\right)\right]$ where $y = z(1 - z_1) + (1 - z)\mu_1^*$. The equilibrium condition $u_2(s) = u_2(f)$ implies that $\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$.

Furthermore, $(A1)$ implies $\mu_1^* > \frac{\theta\rho(1-z_1)}{z_2+\rho(1-z_2)}$ and $(B1)$ implies $1 - \mu_1^* > \frac{\theta z_1}{z_2+\rho(1-z_2)}$. The last two inequality yields $\theta < \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$. Also, since $\tau_2^{s,m} < w$ for $m \in M$, $\tau_2^{f,s} < \frac{w}{\theta}$ and $\tau_2^{f,f} < \frac{w}{\rho}$, we have $z_2 > \frac{v(1-z)}{zw(1-\rho)} - \frac{\rho}{1-\rho}$. □

Lemma A.8. *Suppose that one of the following conditions hold: (i) $z \leq \frac{v}{v+w}$, (ii) $\frac{v}{v+w} < z$ and $\theta \leq \rho$, or (iii) $z > \frac{v}{v+w}$ and $z_1 \leq \frac{\rho(1-\theta)}{\theta(1-\rho)}$. Then, there exists no equilibrium strategy profile (μ^*, σ^*) where both states are weak.*

Proof. Suppose for a contradiction that there exists such (μ^*, σ^*) . Therefore, we must have (1) $\tau_1^{ss} = \tau_2^{ss} \leq w$, (2) $\tau_1^{sf} = \tau_2^{sf} \leq w$, (3) $\tau_1^{fs} = \tau_2^{fs} \leq \frac{w}{\rho}$ and (4) $\tau_1^{ff} = \tau_2^{ff} \leq \frac{w}{\rho}$. By (1) and (2) we have $\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$. Similarly, by (1) and (3) we have $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$.

However, with these μ_1^* and μ_2^*, (1)-(3) hold if and only if (5) $\theta = \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$. However, the equality in (5) cannot be true if $z_1 \leq z_2$ because by assumption $\theta < 1$. On the other hand, when $z_1 > z_2$ we have $\frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)} > \rho$ contradicting with (ii), i.e. $\theta \leq \rho$.

Furthermore (5) contradicts with the condition (iii), more specifically $z_1 \leq \frac{\rho(1-\theta)}{\theta(1-\rho)}$ because otherwise we should have $\theta > \frac{\theta[\rho+z_2(1-\rho)]}{\rho}$, implying that $0 > z_2(1-\rho)$.

Finally, with the μ_1^* at hand, (2) holds if and only if $\frac{v(1-z)}{\theta z w} - \frac{\rho}{(1-\rho)} \leq z_1$. Since we have $z_1 < 1$ the last inequality implies that $\frac{v}{v+\theta w} < z$ contradicting with the condition (i). Three contradiction for each three possible cases end the proof. □

Proof of Proposition 5. Assume that (μ^*, σ^*) is an equilibrium strategy profile and $z \leq \frac{v}{v+w}$. Suppose for a contradiction that state 1 is weak. Then state 2 must be strong by lemma A.5.

Since state 2 is strong, by Lemma A.7 we have $(*)$ $z_2 > \frac{v(1-z)}{zw(1-\rho)} - \frac{\rho}{1-\rho}$. However, since $z \leq \frac{v}{v+w}$, $(*)$ implies $z_2 > 1$ contradicting with our assumption that $z_2 < 1$. Hence, whenever $z \leq \frac{v}{v+w}$, state 1(state 2) must be strong (weak).

Now I will characterize the equilibrium strategies of state 1.

Case 5.1. *Suppose that $\theta \leq \rho$.*

In this case one of the following (Ai) and (Bj) must hold

$(A1)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B1)$	$\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$ and $\tau_1^{fs} < \frac{w}{\rho}, \tau_2^{fs}$
$(A2)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B2)$	$\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$ and $\frac{w}{\rho} \leq \tau_1^{fs}, \tau_2^{fs}$
$(A3)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B3)$	$\frac{w}{\rho} \leq \tau_1^{fs}, \tau_2^{fs}$ and $\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$
$(A4)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B4)$	$\tau_1^{fs} < \frac{w}{\rho}, \tau_2^{fs}$ and $\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$

Lemma 5.1. *There is no equilibrium where $(A3)$ or $(B3)$ holds.*

Proof. Suppose $(A3)$ holds. Then, $w \leq \tau_1^{ss} < \tau_1^{sf} < w$ which yields a contradiction. Similarly, if $(B3)$ holds, then $\frac{w}{\rho} \leq \tau_1^{fs} < \tau_1^{ff} < \frac{w}{\rho}$, also yielding a contradiction. □

Lemma 5.2. *There is no equilibrium where $(A1)$ and $(B1)$ hold.*

Proof. Suppose for a contradiction that there is such an equilibrium. Then, the equilibrium condition $u_1(s) = u_1(f)$ implies that

$$\left[\rho\left(\frac{1-z_1(f)}{z_1(f)}\right) - \frac{1-z_1(s)}{z_1(s)}\right]\left[\frac{xz_2(s)}{\theta} + (1-x)\frac{z_2(f)}{\theta\rho}\right] = 0$$

The last equality yields the equilibrium value of $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. However, given the value of μ_1^* and condition $(A1)$, i.e. $\tau_1^{sf} < w$, we have $z > \frac{v}{v+\theta w}$ contradicting our assumption.

□

Lemma 5.3. *There is no equilibrium where* $(A1)$ *and* $(B4)$ *[or* $(A4)$ *and* $(B1)$ *] hold.*

Proof. I will show the case where $(A1)$ and $(B4)$ hold. Similar arguments proves the case where $(A4)$ and $(B1)$ hold. In equilibrium, we have

$$u_1(s) = v\left[x\left(1 - z_2(s) - \frac{(1-z_1(s))z_2(s)}{z_1(s)\theta}\right) + (1-x)\left(1 - z_2(f) - \frac{(1-z_1(s))z_2(f)}{z_1(s)\theta\rho}\right)\right]$$

and $u_1(f) = v\left[x\left(1 - z_2(s) - \frac{(1-z_1(f))z_2(s)\rho}{z_1(f)\theta}\right) + (1-x)\left(1 - z_2(f) - \frac{z_2(f)w}{v}\right)\right]$. The equilibrium condition $u_1(s) = u_1(f)$ implies that

$$xz_2(s)\left[\frac{1-z_1(s)}{\theta z_1(s)} - \frac{(1-z_1(f))\rho}{z_1(f)\theta}\right] = (1-x)z_2(f)\left[\frac{w}{v} - \frac{1-z_1(s)}{z_1(s)\theta\rho}\right]$$

or equivalently $(1-z_2)\left[\tau_1^{ss} - \rho\tau_1^{fs}\right] = z_2\left[w - \tau_1^{sf}\right]$. Since $(A1)$ holds, the right hand side of the last equality is positive. Thus, we have $\tau_1^{ss} \geq \rho\tau_1^{fs}$. Rearranging the last equality, we have $(1-z_2)\left[\tau_1^{ss} - \rho\tau_1^{fs}\right] = z_2\left[w - \frac{\tau_1^{ss}}{\rho}\right]$ implying that

$$\frac{\tau_1^{ss}}{\tau_1^{fs}} = \frac{\rho\left[\rho(1-z_2) + \frac{z_2(f)}{\tau_1^{fs}}\right]}{z_2 + \rho(1-z_2)} := A \tag{3}$$

If $A < \rho$, then we contradict with $\tau_1^{ss} \geq \rho\tau_1^{fs}$. Therefore, suppose that $A \geq \rho$. Since $\tau_1^{ss} = \frac{v(1-z_1(s))}{z_1(s)\theta}$ and $\tau_1^{fs} = \frac{v(1-z_1(f))}{z_1(f)\theta}$, Equation (3) implies that $\mu_1^* = \frac{A(1-z_1)}{z_1+A(1-z_1)}$. Also note that since $\tau_1^{sf} < w$, $\mu_1^* < \frac{w\theta\rho z(1-z_1)}{v(1-z)}$. Given the equilibrium value of μ_1^* we have $(*)$ $\frac{A}{z_1+A(1-z_1)} < \frac{w\theta\rho z}{v(1-z)}$. Also, since $\tau_1^{ff} \geq \frac{w}{\rho}$, we have $1 - \mu_1^* \geq \frac{w\theta\rho z z_1}{v(1-z)\rho}$. The last inequality implies that $(**)$ $\frac{\rho}{z_1+A(1-z_1)} \geq \frac{w\theta\rho z}{v(1-z)}$. Inequalities $(*)$ and $(**)$ imply that $A < \rho$ which yields the desired contradiction. □

Lemma 5.4. *There is no equilibrium where* $(A4)$ *and* $(B4)$ *hold.*

Proof. The equality $u_1(s) = u_1(f)$ implies $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$ because

$$u_1(s) = v\left[x\left(1 - z_2(s) - \frac{(1-z_1(s))z_2(s)}{z_1(s)\theta}\right) + (1-x)\left(1 - z_2(f) - \frac{z_2(f)w}{v}\right)\right]$$

and $u_1(f) = v\left[x\left(1 - z_2(s) - \frac{(1-z_1(f))z_2(s)\rho}{z_1(f)\theta}\right) + (1-x)\left(1 - z_2(f) - \frac{z_2(f)w}{v}\right)\right]$. Since $\tau_1^{ss} < w$ we have $z_1 > \frac{v\rho(1-z)}{\theta wz(1-\rho)} - \frac{\rho}{1-\rho}$. Also, since $z_1 < 1$ we must have $z > \frac{v\rho}{v\rho+\theta w}$, which is higher than $\frac{v}{v+w}$ as $\rho \geq \theta$, yielding a contradiction. □

Lemma 5.5. *There is no equilibrium where both* $(A1)$ *and* $(B2)$ *hold.*

Proof. Suppose for a contradiction that $(A1)$ and $(B2)$ hold. The equilibrium condition $u_1(f) = u_1(s)$ implies that $\mu_1^* = \frac{w\theta\rho z(1-z_1)}{v(1-z)[\rho+z_2(1-\rho)]}$. However, $(A1)$ implies that $\tau_1^{sf} < w$ and given the value of μ_1^* we have $z_2 > 1$. □

Lemma 5.6. *There is no equilibrium where both $(A2)$ and $(B1)$ hold.*

Proof. Suppose for a contradiction that $(A2)$ and $(B1)$ hold. The equilibrium condition $u_1(f) = u_1(s)$ implies that $\mu_1^* = 1 - \frac{w\theta z z_1}{v(1-z)[\rho + z_2(1-\rho)]}$. However, $(B1)$ implies that $\tau_1^{ff} < \frac{w}{\rho}$ and given the value of μ_1^* we have $z_2 > 1$. □

Lemma 5.7. *There exists an equilibrium where both $(A2)$ and $(B2)$ hold and μ_1^* satisfies $\frac{\theta z(1-z_1)w}{v(1-z)} < \mu_1^* < 1 - \frac{\theta z z_1 w}{v\rho(1-z)}$ for all values of z_1.*

Proof. Note that in this case state 1's threat of war is binding. Therefore, $(A2)$ and $(B2)$ hold for state 1, i.e. $\tau_1^{ss} \geq w$ and $\tau_1^{fs} \geq \frac{w}{\rho}$, whenever μ_1^* is in the range given above. Note that this open interval is non-empty for all values of z_1. □

Lemma 5.8. *There is no equilibrium where both $(A2)$ and $(B4)$ hold.*

Proof. Suppose for a contradiction that $(A2)$ and $(B4)$ hold. The equilibrium condition $u_1(f) = u_1(s)$ implies that $\mu_1^* = 1 - \frac{w\theta z z_1}{v(1-z)\rho}$. However, given the value of μ_1^* we have $\tau_1^{fs} = \frac{w}{\rho}$ contradicting with $(B4)$. □

Lemma 5.9. *There is no equilibrium where both $(A4)$ and $(B2)$ hold.*

Proof. Suppose for a contradiction that $(A4)$ and $(B2)$ hold. The equilibrium condition $u_1(f) = u_1(s)$ implies that $\mu_1^* = \frac{w\theta z(1-z_1)}{v(1-z)}$. However, given the value of μ_1^* we have $\tau_1^{ss} = w$ contradicting with $(A4)$. □

Case 5.2. *Suppose that $\theta > \rho$.*

In this case one of (Ai) and (Bj) must hold

$(A1)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B1)$	$\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$ and $\tau_1^{fs} < \frac{w}{\theta}, \tau_2^{fs}$
$(A2)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B2)$	$\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$ and $\frac{w}{\theta} = \tau_1^{fs} < \tau_2^{fs}$
$(A3)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B3)$	$\frac{w}{\theta} = \tau_1^{fs} < \tau_2^{fs}$ and $\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$
$(A4)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B4)$	$\tau_1^{fs} < \frac{w}{\theta}, \tau_2^{fs}$ and $\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$

By Lemma 5.2, if $(A1)$ and $(B1)$ hold, then $\mu_1^* = \frac{\rho(1-z_1)}{z_1 + \rho(1-z_1)}$ and by $\tau_1^{sf} < w$ we must have $z > \frac{v}{v+\theta w}$ contradicting our assumption. Similarly, by Lemma 5.3 we do not have equilibrium where $(A1)$ and $(B4)$ [or $(A4)$ and $(B1)$] hold. By Lemma 5.5, $(A1)$ and $(B2)$ cannot hold. By Lemma 5.6, $(A2)$ and $(B1)$ does not hold. Furthermore, $(A2)$ and $(B4)$ does not hold by Lemma 5.8 because with the value of μ_1^*, $\tau_1^{fs} = \frac{w}{\rho}$. But, since $\theta > \rho$, $\tau_1^{fs} = \frac{w}{\rho} > \frac{w}{\theta}$ contradicts with $(B4)$. Remaining cases are considered in the following four exhaustive Lemma.

Lemma 5.10. *There is no equilibrium where $(A3)$ or $(B3)$ holds.*

Proof. Lemma 5.1 gives the case for $(A3)$. However, if $(B3)$ holds, then $\frac{w}{\theta} = \tau_1^{fs}$ implies $1 - \mu_1 = \frac{wzz_1}{v(1-z)}$. Thus, we have $\tau_1^{ff} = \frac{w}{\theta\rho} > \frac{w}{\rho}$, contradicting with $(B3)$. □

Lemma 5.11. *There exists an equilibrium where both $(A2)$ and $(B2)$ hold and $\mu_1^* = 1 - \frac{wzz_1}{v(1-z)}$.*

Proof. By $(B2)$, i.e. $\tau_1^{fs} = \frac{w}{\theta}$, we have the value of μ_1^* given above. Furthermore, $(A2)$ holds, i.e. $\tau_1^{ss} > w$, whenever $z_1 \leq \frac{v(1-z)}{wz(1-\theta)} - \frac{\theta}{1-\theta}$ which is true for all values of z_1 because the right hand side is always bigger than 1 as $z < \frac{v}{v+w}$. □

Lemma 5.12. *There is no equilibrium where both $(A4)$ and $(B2)$ hold.*

Proof. Suppose for a contradiction that $(A4)$ and $(B2)$ hold. Then by $(B2)$, i.e. $\tau_1^{fs} = \frac{w}{\theta}$, we have the value of μ_1^* given in Lemma 5.9. However, with this value of μ_1^* and $\tau_1^{ss} < w$ (by $(A4)$) we have $z_1 > \frac{v(1-z)}{wz(1-\theta)} - \frac{\theta}{1-\theta}$ which is always bigger than 1 as $z < \frac{v}{v+w}$. □

Lemma 5.13. *There is no equilibrium where both $(A4)$ and $(B4)$ hold.*

Proof. Suppose for a contradiction that $(A4)$ and $(B4)$ hold. Then by Lemma 5.4 we have $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. However, given the value of μ_1^* we have $\tau_1^{fs} < \frac{w}{\theta}$ (by $(B4)$) implying that $z_1 > \frac{v(1-z)}{wz(1-\rho)} - \frac{\rho}{1-\rho}$ which is always bigger than 1 as $z < \frac{v}{v+w}$. □

***Proof of Proposition 6*.** Lemma A.7 gives the equilibrium value of μ_2^*. I will prove the claim (that state 2 must be strong) with a series of Lemma. Suppose for a contradiction that state 2 is weak. By Lemma A.8, there exists no equilibrium strategy profile (μ^*, σ^*) where both states are weak. Thus, state 1 must be strong. The first lemma considers the case where $\rho \geq \theta$. In this case, $z_1 \leq \frac{\rho(1-\theta)}{\theta(1-\rho)}$ is not binding since the right hand side, in this case, is strictly bigger than 1. The second lemma considers the case where $z_1 \leq \frac{\rho(1-\theta)}{\theta(1-\rho)}$ and $\rho < \theta$.

Lemma 6.1. *Suppose that $\frac{v}{v+w} < z$ and $\theta \leq \rho$. There exists no equilibrium strategy profile (μ^*, σ^*) where state 1 is strong and state 2 is weak.*

Proof. Suppose for a contradiction that there exists such (μ^*, σ^*). Then, one of the following (Ai)'s and (Bj)'s must hold.

$(A1)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B1)$	$\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$ and $\tau_1^{fs} < \frac{w}{\rho}, \tau_2^{fs}$
$(A2)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B2)$	$\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$ and $\frac{w}{\rho} \leq \tau_1^{fs}, \tau_2^{fs}$
$(A3)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B3)$	$\frac{w}{\rho} \leq \tau_1^{fs}, \tau_2^{fs}$ and $\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$
$(A4)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B4)$	$\tau_1^{fs} < \frac{w}{\rho}, \tau_2^{fs}$ and $\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$

I will consider all possibilities under following exhaustive cases.

Case 6.1.1. *Either* $(A2)$ *or* $(B2)$ *holds.* : Suppose $(A2)$ holds. Then, $w \leq \tau_2^{ss}$ and $w \leq \tau_2^{sf}$ imply that $\frac{wz(1-z_2)}{v(1-z)} \leq \mu_2^*$ and $\mu_2^* \leq 1 - \frac{wzz_2}{v(1-z)}$. The last two inequality hold when $z \leq \frac{v}{v+w}$ contradicting initial assumption. Same arguments for $(B2)$ yields the desired result.

Case 6.1.2. *Either* $(A3)$ *or* $(B3)$ *holds*: By Lemma 5.1, there is no equilibrium in this case.

Case 6.1.3. *Both* $(A1)$ *and* $(B1)$ *hold*: Lemma 5.2 gives the equilibrium value of μ_1^*. By $\tau_1^{ss} < \tau_2^{ss}$, $\tau_1^{sf} < \tau_2^{sf}$ and μ_1^* we have $\mu_2^* > \frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}$ and $\mu_2^* < 1 - \frac{\rho(z_2)}{\theta[z_1+\rho(1-z_1)]}$. The last two inequality implies that $\theta > \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)} > \rho$ contradicting with the assumption that $\theta \leq \rho$.

Case 6.1.4. *Both* $(A1)$ *and* $(B4)$ *[or* $(A4)$ *and* $(B1)$*] hold*: By Lemma 5.3, there is no equilibrium in this case.

Case 6.1.5. *Both* $(A4)$ *and* $(B4)$ *hold*: The value of μ_1^* is given by Lemma 5.4. Furthermore, since $\tau_1^{ss} < w$ we have (1) $z_1 > \frac{v\rho(1-z)}{\theta wz(1-\rho)} - \frac{\rho}{1-\rho}$. Also, since $\tau_1^{sf} \geq w$, we have (2) $z_1 \leq \frac{v(1-z)}{\theta wz(1-\rho)} - \frac{\rho}{1-\rho}$. Moreover, $\tau_1^{ss} < \tau_2^{ss}$ implies that (3) $\mu_2^* > \frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}$, and $w \leq \tau_2^{sf}$ implies that (4) $\mu_2^* \leq 1 - \frac{wzz_2}{v(1-z)}$.

By (3) and (4), we have $\frac{A}{B} := \frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]} < \frac{v-z(v+wz_2)}{v-zv}$ implying that (5) $z < \frac{(B-A)v}{(B-A)v+Bwz_2}$. Note that $B > A$ because otherwise we would have $\mu_2^* \geq 1$. Now, notice that (1) can be correct in equilibrium if $\frac{v\rho(1-z)}{\theta wz(1-\rho)} - \frac{\rho}{1-\rho} < 1$ implying that (6) $z > \frac{v\rho}{v\rho+\theta w}$. However, (5) and (6) can hold at the same time if (7) $z_2 < \frac{\theta}{\rho} - \frac{1-z_2}{\rho+z_1(1-\rho)}$. Since (7) is true for all z_1 satisfying (2), we can rewrite (7) as $z_2 < \frac{\theta}{\rho} - \frac{(1-z_2)\theta wz}{v(1-z)}$. The last inequality implies that $z_2[\theta\rho wz - v\rho(1-z)] > \theta\rho wz - \theta v(1-z)$ which contradicts that $\theta \leq \rho$.

□

Lemma 6.2. Suppose that $\frac{v}{v+w} < z$, $z_1 \leq \frac{\rho(1-\theta)}{\theta(1-\rho)}$ and $\rho < \theta$. There exists no equilibrium strategy profile (μ^*, σ^*) where state 1 is strong (but state 2 is weak).

Proof. Suppose for a contradiction that there is such (μ^*, σ^*). State 2 is weak (and state 1 is strong) if one of (Ai)'s and one of (Bj)'s hold.

$(A1)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B1)$	$\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$ and $\tau_1^{fs} < \frac{w}{\theta}, \tau_2^{fs}$
$(A2)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B2)$	$\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$ and $\frac{w}{\theta} = \tau_1^{fs} < \tau_2^{fs}$
$(A3)$	$w \leq \tau_1^{ss}, \tau_2^{ss}$ and $\tau_1^{sf} < w, \tau_2^{sf}$	$(B3)$	$\frac{w}{\theta} = \tau_1^{fs} < \tau_2^{fs}$ and $\tau_1^{ff} < \frac{w}{\rho}, \tau_2^{ff}$
$(A4)$	$\tau_1^{ss} < w, \tau_2^{ss}$ and $w \leq \tau_1^{sf}, \tau_2^{sf}$	$(B4)$	$\tau_1^{fs} < \frac{w}{\theta}, \tau_2^{fs}$ and $\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$

First, there exists no equilibrium strategy profile μ^* where $(A2)$, $(A3)$ or $(B3)$ holds. The proof of this claim is the same as the proof of Case 6.1.1 and Lemma 5.1. Also, proof of Lemma 5.4 shows that there is no such equilibrium where $(A1)$ and $(B4)$ [or $(A4)$ or $(B1)$] hold. The remaining four cases are considered in the following four cases.

Case 6.2.1. *Both* $(A1)$ *and* $(B1)$ *hold.*

Proof. Suppose there is such an equilibrium. Then, we must have (1) $\tau_1^{ss} < \tau_2^{ss}, w$, (2) $\tau_1^{sf} < \tau_2^{sf}, w$, (3) $\tau_1^{fs} < \tau_2^{fs}, \frac{w}{\theta}$ and (4) $\tau_1^{ff} < \tau_2^{ff}, \frac{w}{\rho}$. In this case, by Lemma 5.2 we know that $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. Inequalities in (1)-(4) (more specifically $\tau_1^{m_1,m_2} < \tau_2^{m_1,m_2}$ for all $m_1, m_2 \in M$) implies that $\frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]} < \mu_2^* < 1 - \frac{z_2}{\theta[z_1+\rho(1-z_1)]}$ yielding (5) $\theta > \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$. We can re-write (5) so that $z_2 < \frac{\theta[\rho+z_1(1-\rho)]}{1-\rho} - \frac{\rho}{1-\rho}$. However, since $z_2 > 0$, we must have $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$ contradicting our condition on z_1. □

Case 6.2.2. *Both* $(A4)$ *and* $(B4)$ *hold.*

Proof. Similar arguments in the proof of Case 6.1.5 gives us that in equilibrium we must have (1) $z < \frac{(B-A)v}{(B-A)v+Bwz_2}$ where $A = \rho(1-z_2)$ and $B = \theta[z_1 + \rho(1-z_1)]$ where $B > A$ (otherwise $\mu_2^* \geq 1$ contradicting that it is an equilibrium strategy). Since, $z > \frac{v}{v+w}$ holds by assumption, these two inequalities implies that $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$, which leads to the desired contradiction. □

Case 6.2.3. *Both* $(A1)$ *and* $(B2)$ *hold.*

Proof. By $(B2)$ we have $\frac{w\rho z(1-z_2)}{\theta v(1-z)} < \mu_2 < 1 - \frac{wzz_2}{v(1-z)}$ implying $(*)$ $z < \frac{\theta v}{\theta v+\rho w}$. Moreover, since $\tau_1^{fs} = \frac{w}{\theta}$, $\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$. By $(A1)$, we have $\frac{\mu_1^*(1-z_2)}{\theta(1-z_1)} < \mu_2^* < 1 - \frac{z_2\mu_1^*}{\theta\rho(1-z_1)}$ implying that $z_2 < \frac{\theta\rho(1-z_1)}{\mu_1^*(1-\rho)} - \frac{\rho}{1-\rho}$. Since $z_2 > 0$ we have $zz_1w - \theta v(1-z)z_1 > v(1-z)(1-\theta)$. Assuming that $zw > \theta v(1-z)$ we have $z_1 > \frac{v(1-z)(1-\theta)}{zw-\theta v(1-z)}$. Also, since $z_1 < \frac{\rho(1-\theta)}{\theta(1-\rho)}$ we have $z > \frac{\theta v}{\theta v+\rho w}$ contradicting with $(*)$. □

Case 6.2.4. *Both* $(A4)$ *and* $(B2)$ *hold.*

Proof. In this case, $u_1(s) = v\left[x\left(1 - z_2(s) - \frac{(1-z_1(s))z_2(s)}{z_1(s)\theta}\right) + (1-x)\left(1 - z_2(f) - \frac{z_2(f)w}{v}\right)\right]$ and $u_1(f) = v(1-x)\left(1 - z_2(f) - \frac{z_2(f)w}{v}\right)$. The equilibrium condition $u_1(f) = u_1(s)$ implies that $\mu_2^* = \frac{1-z_2}{\theta(1-z_1)}\mu_1^*$. However, condition by $(A4)$ we have $\tau_1^{ss} < \tau_2^{ss}$ implying $\mu_2^* > \frac{1-z_2}{\theta(1-z_1)}\mu_1^*$ which yields the desired contradiction. □

□

Proof of Proposition 7. Suppose that $\frac{v}{v+w} < z$, $\frac{\rho(1-\theta)}{\theta(1-\rho)} < z_1$, $\frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)} \leq \theta$ and that there is an equilibrium where state 2 is strong. For those values of the parameters to be valid, we must have $\rho < \theta$. Lemma A.7 shows that if state 2 is strong, then we must have $\theta < \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$, contradicting with our assumption. Hence, in equilibrium state 2 must be weak. Moreover, by Lemma A.8 both states cannot be weak, implying that state 1 must be strong in equilibrium.

Now I want to characterize the equilibrium strategies of strong state 1. In equilibrium, state 1 is strong whenever one of (Ai)'s and (Bj)'s (in the proof of Proposition 5, Case 5.2) hold. Case 6.1.1 and Lemma 5.1 show that in equilibrium we cannot have $(A2)$, $(A3)$, and $(B3)$. Similarly, Lemma 5.3 shows that there cannot be an equilibrium where $(A1)$ and $(B4)$ [or $(B1)$ and $(A4)$] hold. Furthermore, Case 6.2.4 proves that $(B2)$ and $(A4)$ cannot hold. Therefore, there are only three possibilities. The next three cases consider these possibilities.

Case 7.1. *Suppose that both* $(A1)$ *and* $(B1)$ *hold.*

Proof. In this case, by Lemma 5.2 we know that $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. Inequalities $\tau_1^{m_1,m_2} < \tau_2^{m_1,m_2}$ for all $m_1, m_2 \in M$ implies that $\frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]} < \mu_2^* < 1 - \frac{z_2}{\theta[z_1+\rho(1-z_1)]}$ yielding $\theta > \frac{\rho+z_2(1-\rho)}{\rho+z_1(1-\rho)}$ and thus $z_1 > z_2$. We can rewrite the last inequality as $z_2 < \frac{\theta[\rho+z_1(1-\rho)]}{(1-\rho)} - \frac{\rho}{1-\rho}$, and since $z_2 > 0$ it implies that $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$. □

Case 7.2. *Suppose that both* $(A4)$ *and* $(B4)$ *hold.*

Proof. Lemma 5.4 implies that $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. Since, $\tau_1^{ss} \leq w$, $\tau_1^{sf} \geq w$, $\tau_1^{ff} \geq \frac{w}{\rho}$ and $\tau_1^{fs} \leq \frac{w}{\theta}$, we must have $\frac{v(1-z)}{wz(1-\rho)} - \frac{\rho}{1-\rho} \leq z_1 \leq \frac{v(1-z)}{\theta wz(1-\rho)} - \frac{\rho}{1-\rho}$. By $\tau_1^{ss} < \tau_2^{ss}$ and $\tau_1^{fs} < \tau_2^{fs}$, we have $\mu_2^* > \frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}$. Also, $w \leq \tau_2^{sf}$ and $\frac{w}{\rho} < \tau_2^{ff}$ imply $\mu_2^* \leq 1 - \frac{wzz_2}{v(1-z)}$. Therefore, the last two imply that $z < \frac{(B-A)v}{(B-A)v+Bwz_2}$ where $B = \theta[z_1 + \rho(1-z_1)]$ and $A = \rho(1-z_2)$. Finally, since we have $z > \frac{v}{v+w}$ by assumption, the last two inequality yields that $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$.

Case 7.3. *Suppose that both* $(A1)$ *and* $(B2)$ *hold.*

By Case 6.2.3 and $(B2)$, we have $z < \frac{\theta v}{\theta v+\rho w}$ and $\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$. By $(A1)$, we have $z_1 > \frac{v(1-z)(1-\theta)}{zw-\theta v(1-z)}$ which is higher than $\frac{\rho(1-\theta)}{\theta(1-\rho)}$ as $z < \frac{\theta v}{\theta v+\rho w}$. □

Proof of Proposition 8. The case where state 1 is strong is proved by Proposition 7. The second case is proved by Lemma A.7.

Proof of Observations 2 through 5. The next two tables summarize the equilibrium strategies for the states for all parameter values, and prove observations 2 and 3.

State 1	Equilibrium Strategy: μ_1^*
Strong ($z \leq \frac{v}{v+w}$)	$\mu_1^* \in \left[\frac{\theta z(1-z_1)w}{v(1-z)}, 1-\frac{zz_1w\theta}{\rho(1-z)v}\right]$ if $\rho \geq \theta$ $\mu_1^* = 1-\frac{zz_1w}{v(1-z)}$ if $\rho < \theta$
Strong ($z > \frac{v}{v+w}$ and $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$)	$\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$ or $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$
Weak ($z > \frac{v}{v+w}$)	$\mu_1^* \in \left[\frac{\theta\rho(1-z_1)}{z_2+\rho(1-z_2)}, 1 - \frac{\theta z_1}{z_2+\rho(1-z_2)}\right]$

First two rows are given by Proposition 5. Suppose now that state 1 is weak, i.e. $z > \frac{v}{v+w}$. In this case, we should have $\tau_2^{ss} \leq \tau_1^{ss}, w$, $\tau_2^{sf} \leq \tau_1^{sf}, w$, $\tau_2^{ff} \leq \tau_1^{ff}, \frac{w}{\rho}$ and

$\tau_2^{fs} \leq \tau_1^{fs}, \frac{w}{\nu}$ where $\nu = \min\{\rho, \theta\}$. Since $\tau_1^{ss} < \tau_1^{sf}$ and $\tau_1^{fs} < \tau_1^{ff}$ for all parameter values, two inequalities $\tau_1^{ss} > \tau_2^{ss}$ and $\tau_1^{fs} > \tau_2^{fs}$ imply the range given in the third row.

State 2	Equilibrium Strategy: μ_2^*
Strong ($z > \frac{v}{v+w}$)	$\mu_2^* = \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$
Weak ($z \leq \frac{v}{v+w}$)	$\mu_2^* \in \left[\frac{z(1-z_2)w}{v(1-z)}, 1 - \frac{zz_2w}{v(1-z)}\right]$
Weak ($z > \frac{v}{v+w}$ and $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$)	$\mu_2^* \in \left[\frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}, 1 - \frac{z_2}{\theta[z_1+\rho(1-z_1)]}\right]$ or $\mu_2^* \in \left[\frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}, 1 - \frac{wzz_2}{v(1-z)}\right]$
Weak ($z > \frac{v}{v+w}$ and $\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$)	$\mu_2^* \in [\underline{\mu}_2, \bar{\mu}_2]$

where

$$\underline{\mu}_2 = \max\left\{\frac{\rho z(1-z_2)w}{\theta v(1-z)}, \frac{(1-z_2)}{\theta(1-z_1)}\left(1 - \frac{zz_1w}{v(1-z)}\right)\right\}$$

and

$$\bar{\mu}_2 = \min\left\{1 - \frac{zz_2w}{v(1-z)}, \left(1 - \frac{z_2}{\theta\rho(1-z_1)}\right)\left(1 - \frac{zz_1w}{v(1-z)}\right)\right\}$$

The first row is given by Proposition 6. When $z \leq \frac{v}{v+w}$, state 2 is weak if and only if $w \leq \tau_1^{ss}, \tau_2^{ss}$; $w \leq \tau_1^{sf}, \tau_2^{sf}$; $\frac{w}{\rho} \leq \tau_1^{ff}, \tau_2^{ff}$; and finally $\frac{w}{\rho} \leq \tau_1^{fs}, \tau_2^{fs}$ if $\rho > \theta$ and $\frac{w}{\theta} = \tau_1^{fs} < \tau_2^{fs}$ otherwise. Since $\tau_2^{ss} > w$ implies $\tau_2^{fs} > \frac{w}{\rho}$ and as $\theta > \rho$ we have $\tau_2^{ss} > w$ implying $\tau_2^{fs} > \frac{w}{\rho} > \frac{w}{\theta}$. So, we need to check whether $\tau_2^{ss} \geq w$ and $\tau_2^{sf} \geq w$. The last two inequality yields the range given in the second row.

In order to find the values given in the third and the fourth rows, first note that when $z > \frac{v}{v+w}$ and $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$, state 2 is weak if and only if one of the following holds (by the proof of Propositions 7 and 8): Here for (Ai)'s and (Bi)'s, I refer to proof of Proposition 5, Case 5.2.

1. $(A1)$ and $(B1)$ hold: $\tau_1^{ss} < \tau_2^{ss}$, $\tau_1^{sf} < \tau_2^{sf}$, $\tau_1^{fs} < \tau_2^{fs}$ and $\tau_1^{ff} < \tau_2^{ff}$. In this case strong state 1 chooses $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. These four inequalities imply the first range in the third row.

2. $(A4)$ and $(B4)$ hold: $\tau_1^{ss} < \tau_2^{ss}$, $w < \tau_2^{sf}$, $\tau_1^{fs} < \tau_2^{fs}$ and $\frac{w}{\rho} < \tau_2^{ff}$. In this case strong state 1 chooses $\mu_1^* = \frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$. These four inequalities imply the second range in the third row.

3. $(A1)$ and $(B2)$ hold: $\tau_1^{ss} < \tau_2^{ss}$, $\tau_1^{sf} < \tau_2^{sf}$, $\frac{w}{\theta} < \tau_2^{fs}$ and $\frac{w}{\rho} < \tau_2^{ff}$. In this case strong state 1 chooses $\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$. These four inequalities imply the range in the fourth row.

Now, fix the values of all parameters except w. Let the initial value of the cost of war is w^L where $z \leq \frac{v}{v+w^L}$. As the cost of war increases to w where $w^L < w$, then state 2's strategy (highest value of μ_2^*) decreases (assuming that $z \leq \frac{v}{v+w^L}$ is still satisfied).

Suppose now that the cost of war is increased to w^H where $z > \frac{v}{v+w^H}$. Then, as we increase the cost of war, the highest value of μ_2^* either stays the same or decreases (depending on which equilibrium strategies players picked). Furthermore, as w increases from w^L to w^H, second state's strategy may shift from $\mu_2^* = 1 - \frac{zz_2w}{v(1-z)}$ to some other strategy given in the rows 2-4 in the above table. Note that this value of μ_2^* is higher than the highest value of all possible strategies of the state 2 in rows 2-4. That is, $\mu_2^* = 1 - \frac{zz_2w^L}{v(1-z)} > \frac{\rho(1-z_2)}{z_2+\rho(1-z_2)}$ since $z < \frac{v}{v+w} < \frac{v}{v+w^L(z_2+\rho(1-z_2))}$ and $\mu_2^* = 1 - \frac{zz_2w^L}{v(1-z)} > 1 - \frac{z_2}{\theta[z_1+\rho(1-z_1)]}$ since $z < \frac{v}{v+w} < \frac{v}{v+w^L\theta(z_1+\rho(1-z_1))}$. Hence, we can conclude that as the cost of war increases, state 2 becomes less aggressive. Similar arguments hold for the first state with one exception. As the cost of war increases and $z \leq \frac{v}{v+w}$, highest value of μ_1^* decreases. However, when it takes a value (w^*) where $z = \frac{v}{v+w^*}$, then μ_1^* might change and thus make a jump at that point. Furthermore, the value of that μ_1^* is constant for all values of $w > w^*$.

Length of Escalation

The following table summarizes the length of escalation in all possible equilibria for all possible message realizations:

Equilibrium— Messages	ss	sf	fs	ff
$z \leq \frac{v}{v+w}$ (State 1 strong)	w	w	$\frac{w}{\rho}$ (or $\frac{w}{\theta}$)	$\frac{w}{\rho}$
$z > \frac{v}{v+w}$ (State 2 strong)	(τ_2^{ss}) $\frac{v\rho(1-z)}{z[z_2+\rho(1-z_2)]}$	(τ_2^{sf}) $\frac{v(1-z)}{z[z_2+\rho(1-z_2)]}$	(τ_2^{fs}) $\frac{v(1-z)}{z[z_2+\rho(1-z_2)]}$	(τ_2^{ff}) $\frac{v(1-z)}{z\rho[z_2+\rho(1-z_2)]}$
(State 1 strong and $\mu_1^*=\frac{\rho(1-z_1)}{z_1+\rho(1-z_1)}$*)* $z>\frac{v}{v+w}$, *and* $z_1>\frac{\rho(1-\theta)}{\theta(1-\rho)}$	(τ_1^{ss}) $\frac{v\rho(1-z)}{\theta z[z_1+\rho(1-z_1)]}$	(τ_1^{sf}) *or* w $\frac{v(1-z)}{\theta z[z_1+\rho(1-z_1)]}$ *or* w	(τ_1^{fs}) $\frac{v(1-z)}{\theta z[z_1+\rho(1-z_1)]}$	(τ_1^{ff}) *or* $\frac{w}{\rho}$ $\frac{v(1-z)}{\theta\rho z[z_1+\rho(1-z_1)]}$ *or* $\frac{w}{\rho}$
(State 1 strong and $\mu_1^*=1-\frac{zz_1w}{v(1-z)}$*)* $z>\frac{v}{v+w}$, *and* $z_1>\frac{\rho(1-\theta)}{\theta(1-\rho)}$	(τ_1^{ss}) $\frac{v(1-z)-zz_1w}{\theta z(1-z_1)}$	(τ_1^{sf}) $\frac{v(1-z)-zz_1w}{\theta\rho z(1-z_1)}$	$\frac{w}{\theta}$	$\frac{w}{\rho}$

I will now show that as the cost of war, w, increases, length of the escalation increases. Again, fix the values of all parameters except w. Let the initial value of the cost of war is w^L where $z \leq \frac{v}{v+w^L}$. As the cost of war increases to $w^L < w$, the escalation times (as given by the first row of the above table) increases given that $z \leq \frac{v}{v+w}$. However, when the cost of war increases even further to w^H where $z > \frac{v}{v+w^H}$ then the escalation times (as given by rows 2-4) either stays the same or increases. The following cases show that given the realized messages, the escalation times in row 1 is less than the ones in the rows $2-4$, implying that length of escalation increases as the cost of war increases from w^L to w^H.

Case 1. *Suppose that equilibrium length of escalation is as given in row 2:* We need to show that $\tau_2^{ss} > w^L$, $\tau_2^{sf} > w^L$, $\tau_2^{fs} > \frac{w^L}{\rho}$ or $\frac{w^L}{\theta}$ and $\tau_2^{ff} > \frac{w^L}{\rho}$. Note that $\tau_2^{sf} > w^L$ if and only if $\tau_2^{ff} > \frac{w^L}{\rho}$. The first inequality implies that $(*)$ $\frac{v(1-z)}{zw^L(1-\rho)} - \frac{\rho}{1-\rho} > z_2 > 0$ which is true if and only if $z < \frac{v}{v+\rho w^L}$. Furthermore $\tau_2^{ss} > w^L$ is true if and only if $\tau_2^{fs} > \frac{w^L}{\rho}$ when $\rho > \theta$. The inequality $\tau_2^{ss} > w^L$ implies $(*)$, which is true as argued above. Finally, suppose that $\theta \geq \rho$. Thus, we need to show $\tau_2^{fs} > \frac{w^L}{\theta}$. The last inequality implies that $\frac{v(1-z)\theta}{zw^L(1-\rho)} - \frac{\rho}{1-\rho} > z_2 > 0$ and this is true if and only if $z < \frac{v\theta}{v\theta+\rho w^L}$ and this inequality is true because we have $\frac{v\theta}{v\theta+\rho w^L} > \frac{v}{v+w^L}$.

Case 2. *Suppose that equilibrium length of escalation is as given in row 3:* Rows 3 and 4 can hold if and only if $\theta > \rho$. Also, for all parameter values $\tau_1^{ss} < \tau_1^{sf}$. Therefore, we need to check if $\tau_1^{ss} > w^L$ holds. Given μ_1^*, the last inequality holds if and only if $\frac{v(1-z)\rho}{\theta zw^L(1-\rho)} - \frac{\rho}{1-\rho} > z_1 > 0$ and it is true because $z < \frac{v}{v+\theta w^L}$. Moreover since $\tau_1^{ff} > \tau_1^{fs}$ we need to show $\tau_1^{fs} > w^L$ given μ_1^*. The last inequality implies that $\frac{v(1-z)}{\theta zw^L(1-\rho)} - \frac{\rho}{1-\rho} > z_1 > 0$ and this is also true because $z < \frac{v}{v+\theta\rho w^L}$.

Case 3. *Suppose that equilibrium length of escalation is as given in row 4:* In this case $\mu_1^* > 0$ if and only if $z < \frac{v}{v+w^H z_1}$ and since we pick w^L in a way that $z < \frac{v}{v+w^L}$ we must have $(*)$ $z_1 < \frac{w^L}{w^H}$. On the other hand, $\tau_1^{ss} > w^L$ holds if and only if $z < \frac{v}{v+z_1w^H+\theta w^L(1-z_1)}$. We need to show that the last fraction is greater than $\frac{v}{v+w^L}$. Suppose for a contradiction that it is not true. That is, $w^L \leq z_1w^H+\theta w^L(1-z_1)$ implying that $w^L(1-\theta-\theta z_1) \leq z_1w^H$. Since $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$, we must have $\frac{w^L}{w^H}\frac{(1-\theta)}{(1-\rho)} \leq z_1$ contradicting with $(*)$.

Hence, for all possible cases, as w increases, escalation length also increases.

Initial Concessions

Next three cases analyze the highest values of $F_i^{\bar{m}}(0)$'s (initial concessions of the states) in equilibrium.

Case 1. *State* 1 *is Strong and* $z \leq \frac{v}{v+w}$*:* State 1 is strong for all realized messages. Therefore, $F_1^{\bar{m}}(0) = 0$ and $F_2^{\bar{m}}(0) > 0$. By proof of Proposition 5, in equilibrium, only $(A2)$ and $(B2)$ hold (as given in the proof of Proposition 5). Thus, the highest values of $F_2^{\bar{m}}(0)$'s are

$F_2^{ss}(0) = F_2^{ws}(0) =$	$1 - z_2(s)\left[1+\frac{w}{v}\right]$	$z_2(s) = \frac{z(1-z_2)}{z(1-z_2)+(1-z)\mu_2^*}$ and $\mu_2^* = 1 - \frac{zz_2w}{v(1-z)}$
$F_2^{sw}(0) = F_2^{ww}(0) =$	$1 - z_2(f)\left[1+\frac{w}{v}\right]$	$z_2(s) = \frac{zz_2}{zz_2+(1-z)(1-\mu_2^*)}$ and $\mu_2^* = \frac{z(1-z_2)w}{v(1-z)}$

Case 2. *State* 1 *is Strong and* $z > \frac{v}{v+w}$*:* According to the proof of Propositions 7 and 8, there are only three possible cases: In equilibrium either $(A1)$ and $(B1)$, or $(A4)$ and $(B4)$ or $(A1)$ and $(B2)$ hold ((Ai)'s and (Bi)'s are given in the proof of Proposition 5). Thus, the highest values of $F_2^{\bar{m}}(0)$'s when $(A1)$ and $(B1)$ hold are

$F_2^{ss}(0) =$	$1 - z_2(s)\left[1+\frac{(1-z)\rho}{z\theta[z_1+\rho(1-z_1)]}\right]$	$z_2(s) = \frac{z(1-z_2)}{z(1-z_2)+(1-z)\mu_2^*}$ and $\mu_2^* = 1 - \frac{z_2}{\theta[z_1+\rho(1-z_1)]}$
$F_2^{ws}(0) =$	$1 - z_2(s)\left[1+\frac{(1-z)z_1\rho}{\theta(1-z_1)[z_1+\rho(1-z_1)]}\right]$	$z_2(s)$ and μ_2^* are the same as above
$F_2^{sw}(0) =$	$1 - z_2(f)\left[1+\frac{(1-z)}{z\theta[z_1+\rho(1-z_1)]}\right]$	$z_2(f) = \frac{zz_2}{zz_2+(1-z)(1-\mu_2^*)}$ and $\mu_2^* = \frac{\rho(1-z_2)}{\theta[z_1+\rho(1-z_1)]}$
$F_2^{ww}(0) =$	$1 - z_2(f)\left[1+\frac{z_1(1-z)}{\theta(1-z_1)[z_1+\rho(1-z_1)]}\right]$	$z_2(f)$ and μ_2^* are the same as above

If $(A4)$ and $(B4)$ hold, then the only difference is $F_2^{ww}(0) = 1 - z_2(f)\left[1 + \frac{w}{v}\right]$ where $z_2(f)$ and μ_2^* are as given in the third row.

On the other hand, if $(A1)$ and $(B2)$ hold, then

$F_2^{ss}(0) =$	$1 - z_2(s)\left[1 + \frac{(1-z_1(s))}{z_1(s)\theta}\right]$	$\mu_2^* = \min\left\{1 - \frac{zz_2w}{v(1-z)}, \left(1 - \frac{z_2}{\theta\rho(1-z_1)}\right)\left(1 - \frac{zz_1w}{v(1-z)}\right)\right\}$
$F_2^{ws}(0) =$	$1 - z_2(s)\left[1 + \frac{w}{v+w}\right]$	$z_2(s) = \frac{z(1-z_2)}{z(1-z_2)+(1-z)\mu_2^*}$ μ_2^* is given above
$F_2^{sw}(0) =$	$1 - z_2(f)\left[1 + \frac{(1-z_1(s))}{z_1(s)\theta\rho}\right]$	$z_2(f) = \frac{zz_2}{zz_2+(1-z)(1-\mu_2^*)}$ and μ_2^* is given below
$F_2^{ww}(0) =$	$1 - z_2(f)\left[1 + \frac{v}{v+w}\right]$	$\mu_2^* = \max\left\{\frac{\rho z(1-z_2)w}{\theta v(1-z)}, \frac{(1-z_2)}{\theta(1-z_1)}\left(1 - \frac{zz_1w}{v(1-z)}\right)\right\}$

where $\frac{1-z_1(s)}{z_1(s)} = \frac{(1-z)\mu_1^*}{z(1-z_1)}$ and $\mu_1^* = 1 - \frac{zz_1w}{v(1-z)}$.

Case 3. *State 2 is Strong and* $z > \frac{v}{v+w}$*:* State 2 is strong for all realized messages. Therefore, $F_1^{\bar{m}}(0) > 0$ and $F_2^{\bar{m}}(0) = 0$. Thus, the highest values of $F_1^{\bar{m}}(0)$'s are

$F_1^{ss}(0) =$	$1 - z_1(s)\left[1 + \frac{(1-z)\rho\theta}{z[z_2+\rho(1-z_2)]}\right]$	$z_1(s) = \frac{z(1-z_1)}{z(1-z_1)+(1-z)\mu_1^*}$ and $\mu_1^* = 1 - \frac{\theta z_1}{z_2+\rho(1-z_2)}$
$F_1^{sw}(0) =$	$1 - z_1(s)\left[1 + \frac{(1-z)z_2\theta\rho}{z[z_2+\rho(1-z_2)]}\right]$	$z_1(s)$ and μ_1^* are the same as above
$F_1^{ws}(0) =$	$1 - z_1(f)\left[1 + \frac{(1-z)\theta}{z[z_2+\rho(1-z_2)]}\right]$	$z_1(f) = \frac{zz_1}{zz_1+(1-z)(1-\mu_1^*)}$ and $\mu_1^* = \frac{\theta\rho(1-z_1)}{z_2+\rho(1-z_2)]}$
$F_1^{ww}(0) =$	$1 - z_1(f)\left[1 + \frac{(1-z)\theta z_2}{z[z_2+\rho(1-z_2)]}\right]$	$z_1(f)$ and μ_1^* are the same as above

When ρ, z and w change, there is an ambiguous effect on the value of initial concessions. When the cost of war is small, i.e. $z \leq \frac{v}{v+w}$, change in θ does not change the probability of resolution, $F_2^{\bar{m}}(0)$. However, if $z > \frac{v}{v+w}$, then as θ increases, the probability of resolution $F_1^{\bar{m}}(0)$ decreases for all possible values of primitives and realized messages whenever state 1 is weak. However, the probability of resolution increases (weakly) if state 1 is strong (which happens when θ is large enough so that $z_1 > \frac{\rho(1-\theta)}{\theta(1-\rho)}$).

References

[1] Abreu, D., and F. Gul, (2000):"Bargaining and Reputation," *Econometrica*, 68, 85-117.

[2] Abreu, D., and R. Sethi, (2003): "Evolutionary Stability in a Reputational Model of Bargaining," *Games and Economic Behavior*, 44, 195-216.

[3] Atakan, A.E., and M. Ekmekci, (2009): "Bargaining and Reputation in Search Markets," mimeo, Northwestern University.

[4] Bulow, J., and P. Klemperer, (1999): "The Generalized War of Attrition," *The American Economic Review*, 89, 175-189.

[5] Compte, O., and P. Jehiel, (2002):"On the Role of Outside Options in Bargaining with Obstinate Parties," *Econometrica*, 70, 1477-1517.

[6] Eyerman, J., and R. A. Hart Jr., (1996): "An Empirical Test of the Audience Cost Proposition," *Journal of Conflict Resolution*, 40(4), 597-616.

[7] Farrell, J., and R. Gibbons, (1989): "Cheap Talk Can Matter in Bargaining," *Journal of Economic Theory*, 48, 221-37.

[8] Fearon, D. J., (1994): "Domestic Political Audiences and the Escalation of International Disputes," *American Political Science Review*, 88, (September), 577-592.

[9] Fearon, D. J., (1997): "Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs," *Journal of Conflict Resolution*, 41 (February), 68-90.

[10] Fudenberg, D., and J. Tirole, (1986): "A Theory of Exit in Duopoly," *Econometrica*, 54(4), 943-960.

[11] Gelpi, C. F., and M. Griesdorf, (2001): "Winners or Losers: Democracies in International Crises, 1918-1994," *American Political Science Review*, 95(3), 633-47.

[12] Ghemawat, P., and B. Nalebuff, (1985): "Exit," *RAND Journal of Economics*, 16, 184-194.

[13] Guisinger, A., and A. Smith, (2002): "Honest Threats," *Journal of Conflict Resolution*, 46(2), 175-200.

[14] Hendricks, K., A. Weiss, and R. Wilson (1988): "The War of Attrition in Continuous-Time with Complete Information," *International Economic Review*, 29, 663-680.

[15] Kambe, S. (1999): "Bargaining with Imperfect Commitment," *Games and Economic Behavior*, 28, 217-237.

[16] Kennan, J., and R. Wilson, (1989): "Strategic Bargaining Models and Interpretation of Strike Data," *Journal of Applied Econometrics*, 4, S87-S130.

[17] Kurizaki, S., (2007): "Efficient Secrecy: Public versus Private Threats in Crisis Diplomacy," *American Political Science Review*, 101(3), 543-58.

[18] Lee, T. and L. Wilde, (1980): "Market Structure and Innovation: A Reformulation," *Quarterly Journal of Economics*, 94, 429-436.

[19] Leventoglu, B., and A. Tarar, (2005): "Prenegotiation Public Commitment in Domestic and International Bargaining," *American Political Science Review*, 99(3), 419- 433.

[20] Leventoglu, B., and A. Tarar, (2009): "Public Commitment in Crisis Bargaining," *International Studies Quarterly*, 53(3), 817839.

[21] Lipson, C., (2003): "Reliable Partners: How Democracies Have Made a Separate Peace," Princeton: Princeton University Press.

[22] Maynard S., J., (1974): "The Theory of Games and the Evolution of Animal Conflict," *Journal of Theoretical Biology*, 47, 209-xix.

[23] Maynard S., J., and G. A. Parker, (1976): "The Logic of Asymmetric Contests," *Animal Behaviour*, 24, 159-175.

[24] Myerson, R. (1991): *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press.

[25] Ozyurt, S., (2011): *Aearching a Bargain: Power of Strategic Commitment.* Mimeo, Sabancı University.

[26] Partell, P. J., and G. Palmer, (1999), "Audience Costs and Interstate Crises: An Empirical Assessment of Fearons Model of Dispute Outcomes," *International Studies Quarterly*, 43(2), 389-405.

[27] Powell, R., (2002): "Bargaining Theory and International Conflict," *Annual Review of Political Science*, 5 (June), 1-30.

[28] Powell, R., (2004): "Bargaining and Learning While Fighting," *American Journal of Political Science*, 48(2), 344-361.

[29] Prins, B. C., (2003): "Institutional Instability and the Credibility of Audience Costs: Political Participation and Interstate Crisis Bargaining, 1816-1992," *Journal of Peace Research* 40(1), 67-84.

[30] Riley, J., G., and W. F. Samuelson, (1980): "Strong Evolutionary Equilibrium and The War of Attrition," *Journal of Theoretical Biology*, 82, 383-400.

[31] Sartori, A. E., (2002): "The Might of the Pen: A Reputational Theory of Communication in International Disputes," *International Organization*, 56 (February), 121-149.

[32] Schelling, T. (1960): *The Strategy of Conflict*. Harvard University Pres.

[33] Schultz, K. A., (1999): "Do Democratic Institutions Constrain or Inform?" *International Organization*, 52, 233-66.

[34] Schultz, K. A., (2001): "Looking for Audience Costs," *Journal of Conflict Resolution*, 45(February), 32-60.

[35] Slantchev, B., (2006): "Politicians, the Media, and Domestic Audience Costs," *International Studies Quarterly*, 50, 445-77.

[36] Smith, A., (1998): "International Crises and Domestic Politics," *American Political Science Review*, 92 (September), 623-638.

[37] Taylor, C., (1995): "Digging for Golden Carrots: An Analysis of Research Tournements," *American Economic Review*, 85(4), 872-890.