FOUR PAYMENT MODELS FOR THE MULTI-MODE RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM WITH DISCOUNTED CASH FLOWS

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ABSTRACT

In this paper, the multi-mode resource constrained project scheduling problem with discounted cash flows is considered. The objective is the maximization of the net present value of all cash flows. Time value of money is taken into consideration, and cash in- and out-flows are associated with activities and/or events. The resources can be of renewable, nonrenewable, and doubly constrained resource types. Four payment models are considered: Lump sum payment at the terminal event, payments at prespecified event nodes, payments at prespecified time points and progress payments. For finding solutions to problems proposed, a genetic algorithm (GA) approach is employed, which uses a special crossover operator that can exploit the multi-component nature of the problem. The models are investigated at the hand of an example problem. Sensitivity analyses are performed over the mark up and the discount rate. A set of 93 problems from literature are solved under the four different payment models and resource type combinations with the GA approach employed resulting in satisfactory computation times. The GA approach is compared with a domain specific heuristic for the lump sum payment case with renewable resources and is shown to outperform it.

1. INTRODUCTION

The resource constrained project scheduling problem (RCPSP) is the problem of scheduling activities under resource and precedence restrictions with the objective of minimizing the makespan. RCPSP is proven to be NP-hard (Blazewicz *et al*., 1983). In RCPSP, resources are classified as renewable, nonrenewable and doubly constrained (Slowinski, 1980). Renewable resources are constrained on a period basis only. Nonrenewable resources are constrained on a project basis. There are other resources which are available in limited quantities every time period and in addition, their total availability throughout the project is also constrained. Such resources are called doubly constrained resources. Cash, for example, can become a doubly constrained resource.

A more realistic formulation of RCPSP allows for the crashing of an activity's duration by assigning it extra units of resources at additional cost. Whenever there are a finite number of such alternatives, these alternatives are represented by time-resource consumption pairs called the modes of the activity. These modes represent a time-resource trade-off problem for the activity. Another type of trade-off considered here is the resource-resource trade-off, where a resource is substituted for another resource while keeping the duration of the activity constant. The presence of such trade-offs increases the problem complexity leading to the multi-mode RCPSP.

Financial aspect of the problem comes into picture through cash in- and out-flows associated with the activities and/or events. Time value of money is taken into consideration through discounting the cash flows. The most commonly employed financial objective is the maximization of the net present value (NPV) of the cash flows where cash inflows are treated as positive and cash outflows as negative. These problems are called resource constrained project scheduling problems with discounted cash flows (RCPSPDCF).

Recent surveys on RCPSP and RCPSPDCF are provided by Özdamar and Ulusoy (1995), Herroelen *et al*. (1997), Herroelen *et al*. (1998) and Kolisch and Padman (1998). As can be deduced from these surveys, minimization of makespan and maximization of NPV are the two most commonly emphasized objectives in the field. Models incorporating NPV criteria usually assume a project due date and/or a large positive cash flow at the end of the project, since otherwise activities involving negative cash flows would be delayed indefinitely to maximize NPV (Elmaghraby and Herroelen, 1990). Under these assumptions, the two criteria of makespan minimization and NPV maximization support each other. This mutual support between the two criteria has also been demonstrated by Smith-Daniels and Aquilano (1987) and later by Ulusoy and Özdamar (1995).

An initial version of the discounted cash flow problem in project scheduling is introduced by Russell (1970) with no resource constraints involved. Grinold (1972) extended the model by Russell by introducing a project deadline. In the unconstrained model, cash flows occur at the events and the objective is to maximize the NPV of all transactions by scheduling the events. Exact solution procedures for the resource constrained version are given among others by Doersch and Patterson (1977), Icmeli and Erenguc (1996), and Baroum and Patterson (1999). Heuristic procedures are presented among others by Russell (1986), Smith-Daniels and Aquilano (1987), Padman and Smith-Daniels (1993). The problem is further extended by Dayanand and Padman (1993, 1997) to determine simultaneously the amount, location and timing of the payments by the client so as to maximize the contractor's NPV. Several models for different project environments are introduced. Dayanand and Padman (1994) suggest for this problem a two stage search heuristic, where a simulated annealing phase is followed by a rescheduling phase. The payment scheduling from the point of view of the client is also treated by Dayanand and Padman (1998). Dayanand and Padman (1999) elaborate further on the models from both the contractor's and the client's view and stress the need for models reflecting a joint view. Ulusoy and Cebelli (2000) extend the payment scheduling model by bringing together the contractor and the client in a joint model. They search for an equitable solution where an equitable solution is defined as one where both the contractor and the client deviate from their respective ideal solutions by an equal percentage. The ideal solution for the contractor would be to receive the whole payment at the start of the project. For the client, on the other hand, it would be a single payment at the termination of the project.

In real life situations there are at least two parties involved in the project: the client, i.e., the owner of the project and the contractor who undertakes the execution of the project. The legal basis of the execution of a project is provided by a contract organizing aspects of the interactions between the stakeholders. There are a large number of contract types with considerable amount of detail involved. A treatise of different contract types is given by

Herroelen *et al*. (1997). For the purposes of this paper, we are interested in the basic payment structures specified in the contracts.

Four types of payment scheduling models are of particular interest in practice: Lump-sum payment, payment at event occurences, payment at equal time intervals, and progress payment.

Lump-sum payment (LSP) is one of the more commonly used payment structures in the literature. Here, the whole payment is paid by the client to the contractor upon successful termination of the project.

In the payments at event occurences (PEO) model, payments are made at predetermined set of event nodes. The problem is to determine the amount and timing of these payments.

In the equal time intervals (ETI) model, the client makes H payments for the project. The first (H-1) of these payments are scheduled at equal time intervals over the duration of the project, and the final payment is scheduled on project completion.

In the progress payment (PP) model, the contractor receives the project payments from the client at regular time intervals until the project is completed. For example, the contractor might receive at the end of each month a payment for the work accomplished during that month multiplied by a profit rate agreed upon by both the client and the contractor. The difference between the ETI and PP models is that in the latter case the number of payments is not known in advance. A mixed-integer formulation of the progress payment problem with the objective of maximizing the NPV of the cash flows for the contractor is presented by Kazaz and Sepil (1996). The formulation does not include any resource constraints. A solution procedure based on Benders decomposition technique is proposed. Later, Sepil and Ortac (1997) proposed three different heuristics to the resource constrained extension of the problem and tested those extensively. The progress payment model corresponds to what Dayanand and Padman (1998) refer to as the periodic payment model. Dayanand and Padman provide mixed integer linear programming formulations for the so-called basic client, equal time intervals and periodic payment models. They provide insights about the characteristics of optimal payment schedules obtained with each model.

The detailed definition of the multi-mode RCPSPDCF under consideration is given in Section 2. The genetic algorithm (GA) approach is presented in Section 3. In Section 4, four different payment models are investigated at the hand of an example problem. Section 5 discusses results of numerical tests and Section 6 concludes.

2. PROBLEM DEFINITION

The definition of the multi-mode RCPSPDCF investigated in this paper is as follows: A single project is given with a set of N activities, a set of renewable resources, and a set of nonrenewable resources. Zero-lag finish-to-start precedence constraints are imposed on the sequencing of activities. Activity preemption is not allowed. There are resource constraints on the availability of renewable as well as nonrenewable resources in each time period. The nonrenewable resources can also be constrained from above on their total usage throughout the project. Thus the problem is a doubly constrained one. The resource usage over an activity is taken to be uniform. Associated with each activity j, there is a set of modes (M_j) representing time-resource and/or resource-resource trade-offs. Associated with each resource, there is a unit consumption cost. Cash outflows (CF_j^-) occur at the start of each activity j. Payments (P_k) are received at payment points $k \in K$, where K is the set of payment points. The activities are to be scheduled such that the makespan of the project (C_{max}) does not exceed a given due date (DD). The objective is the maximization of NPV of all the cash flows which is given by the following relationship:

$$NPV = \sum_{j} CF_j^- \,(1+r)^{-ST_j} + \sum_{k \in K} P_k \,(1+r)^{-T_k} \qquad (1)$$

where r is the discount rate, ST_j is the start time of activity j, and T_k is the occurrence time for the payment point k. CF_j^- are computed as follows: First, the total costs of resource usage per unit time are computed by multiplying the unit cost of each resource by the number of units required by activity j and summing the products over all resource types involved. Then, the sum is multiplied by the duration of activity j to obtain the CF_j^-.

The budget is assumed here to be predetermined. The budgeted amount is transferred to the contractor by the client in compliance with a payment program agreed upon by the client and the contractor. In the PEO, ETI and the PP cases, payment to be paid at each payment point consists of the costs incurred plus a fixed mark up (β). In practice, β is selected in such a way that the sum of the payments at the payment points is less than the budget. The difference is the retainage held by the client and is paid to the contractor at the completion of the project.

Following the classification scheme proposed by Herroelen *et al*. (1999) the problems considered in this paper can be classified for the case with renewable resources only as follows: (LSP:$1,1$ / cpm, δ_n,mu, c_j / npv); (ETI & PP: $1,1$ / cpm, δ_n,mu, per / npv); (PEO:$1,1$ / cpm, δ_n,mu, $sched$ / npv). For problems with both renewable and nonrenewable/doubly constrained resources, the classifications become the following: (LSP: $m,1T$ / cpm,δ_n,mu, c_j / npv); (ETI & PP: $m,1T$ / cpm, δ_n,mu, per / npv); (PEO:$m,1T$ / cpm,δ_n,mu,$sched$ / npv).

3. A GENETIC ALGORITHM APPROACH

GAs have been originally developed by John Holland (1975) as artificial adaptive systems simulating natural evolution and have proven themselves as powerful search algorithms. They have been employed to attack many difficult problems from a variety of fields, especially combinatorial optimization problems. The main premise of genetic algorithms is that they are robust and thus are successfully applicable to a range of problems (Goldberg, 1989).

Various GA approaches are proposed in the literature to RCPSP with the objective of minimizing the makespan. Lee and Kim (1996) attack the single-mode RCPSP with renewable resources through the use of tabu search, simulated annealing, and GA. Their extensive numerical analysis showed that these search heuristics work better than three existing heuristics: the minimum slack method, the iterative algorithm of Li and Willis (1992), and the SEARCH method of Khattab and Choobineh (1991). Mori and Tseng (1997) employ a direct chromosome representation in which each gene corresponds to an activity and includes the mode assignment, the scheduling order and start-finish times of the corresponding activity. Only renewable resources are considered. The GA approach is compared to a stochastic scheduling method by Drexl and Grünewald (1993) and is shown to provide superior solutions. A GA approach to the multi-mode RCPSP with both renewable and nonrenewable resources is provided by Özdamar (1999). The proposed GA is a hybrid GA which incorporates problem specific scheduling knowledge. It employs an indirect chromosome encoding with two dimensions: activity modes and ordered set of scheduling rules. Results reported are encouraging in that near optimal solutions are obtained within a reasonable amount of computational time. Another GA approach to the multi-mode RCPSP with both renewable and nonrenewable resources is proposed by Hartmann (1997). The encoding is based on a precedence feasible set of activities and their mode assignments. A local search extension is employed to improve the solutions found by the basic GA. Extensive experiments are conducted with several different variants of the GA and results are compared with three other heuristics from literature. The proposed GA outperforms the other algorithms with regard to a lower average deviation from the optimal makespan. Hartmann (1998) presents a permutation based GA for the single-mode RCPSP with renewable resources and compares it with both priority value based and priority rule based GAs to find it superior to both.

A GA approach to a multi-mode RCPSP with renewable, nonrenewable, and doubly constrained resources and a financial objective function is provided by Ulusoy and Cebelli (2000). A double-loop GA approach is proposed where the outer loop represents the client and the inner loop the contractor. For each distribution of the budget over the event nodes proposed by the client (the outer loop), the contractor (the inner loop) searches for the schedule of activities maximizing his/her NPV.

The GA approach employed here is based on the inner loop of the double loop approach proposed by Ulusoy and Cebelli (2000). The chromosomes for the initial population are generated randomly by a solution generator. To this end, an eligible set of activities is kept by the generator which is initially formed by the activities with no predecessors. At each step of the generation, one of the activities from the eligible set is chosen and put into the next free position on the chromosome. One of the possible modes of that activity is selected randomly to complete the gene. The eligible set is updated by deleting the selected activity and by adding precedence feasible activities. The process iterates until the chromosome is completed. In the presence of nonrenewable and/or doubly constrained resources, if the chromosome is infeasible with respect to the overall resource limit, then it is discarded. Otherwise, it is added to the initial population. As many chromosomes as the population size are generated to constitute the initial population. The initial population becomes the current generation.

A new generation is created from the current generation by reproduction and by a combination of crossover, reproduction, and mutation. All selections for reproduction and crossover from the current population are with replacement. A predetermined portion of the most fit members (the elite individuals) of each population is reproduced into the next generation (elitist selection). Crossover is applied with probability p_c. If crossover is to be applied, then two chromosomes are selected as parents based on their relative fitness (fitness-based selection) from the current population and a single offspring is formed. Otherwise, fitness-based selection is applied to the current population to select a chromosome to be reproduced as the offspring. The offspring thus generated is subjected first to reposition mutation with probability $pm_{repostn}$ and then to bit mutation with probability pm_{bit}. For the case of nonrenewable and doubly constrained resources, feasibility check for the overall limit is applied to the offspring. If the offspring turns out to be infeasible, then it is discarded. Otherwise, it joins the new population of the next generation. The offspring generation process continues until as many individuals are added to the new population as the population size. A replication of the GA procedure is obtained once a predetermined number of generations is created. The GA procedure is replicated until a predetermined number of replications is reached.

3.1. Representation and Evaluation of Chromosomes

3.1.1. Representation

The problem as defined in the previous section is an example of what is called multi-component combinatorial optimization problems with sequencing and selection components (MCCOP_SS). MCCOP_SS constitute a class of combinatorial optimization problems which have a sequencing component and one or more selection components (Sivrikaya-Şerifoğlu, 1997). They cover a wide class of problems including problems from a variety of fields such as scheduling, circuit and network design, and vehicle routing. In a typical scheduling problem, for example, jobs need to be sequenced and for each job one or more of the available resources need to be selected. In RCPSPDCF activities are to be sequenced. The selection problem associated with each activity is the selection of one of its possible modes. The extra difficulty here is that there are precedence relations constraining the sequencing of activities.

The chromosome representation incorporates both components of the problem, namely that of activity sequencing and mode selection. There are *N* genes on the chromosome, one for each activity. A gene incorporates both the activity number it is associated with and the mode selected for that activity. This representation applies to both RCPSP and RCPSPDCF. Only a change in the objective function is needed to switch from one to the other.

Consider a problem with 6 activities. A chromosome representing a potential solution for this problem might be

2	1	4	3	5	6
1	1	2	2	2	1

There are six genes on this chromosome, one for each of the six activities. Each gene encodes a pair of numbers, the first one being the number of an activity and the second one being the selected mode for that activity. This chromosome would sequence the activities in the order 2, 1, 4, 3, 5, 6, which has to be a feasible sequence in terms of the precedence relations. The activities 2, 1, and 6 are to be accomplished in their first modes; activity 4, 3, and 5 in their second modes.

3.1.2. Evaluation of chromosomes

To schedule the activities in the sequence on the chromosomes, a forward scheduling scheme is preferred due to the failures of the backward scheduling scheme as discussed in length by Li and Willis (1992). A simple scheduler schedules the activities in the given order on the chromosome by keeping track of the resource availabilities. If, for a given chromosome, the schedule results in the violation of the deadline, then that chromosome is discarded and a new chromosome is generated.

Once the activities are scheduled, NPV of the project can be found from the resulting cash flow diagram and the equation (1). Fitness of the chromosome is then taken to be equal to the NPV value.

It should be noted that the scheduler is not a simple nondelay scheduler. A nondelay scheduler would schedule activities at the earliest possible start time which is allowed by the precedence and resource constraints. In the scheduler proposed here, activities are scheduled such that a further restriction holds; namely, that no activity starts earlier than any one of the activities residing at earlier loci on the chromosome. Thus, start times never decrease with increasing position on the chromosome. If that condition is not met, then this implies that an activity starts later than one of its followers on the chromosome, which practically means that a swap of the two corresponding activities has occurred on the chromosome. This is not desirable since swaps of activities on the chromosome are expected to occur as a result of the genetic search process only.

3.2. Operators

3.2.1. Crossover operator

The heart of a genetic algorithm is its crossover operator. The crossover operator employed here is MCUOX (multi-component uniform order-based crossover operator), a general and powerful operator developed to be used in GA applications to instances of MCCOP_SS (Sivrikaya-Serifoglu, 1998). So far, MCUOX has been successfully employed in two different MCCOP_SS instances: (i) the simultaneous scheduling of machines and automated guided vehicles (Ulusoy *et al*., 1997); (ii) parallel machine scheduling with earliness and tardiness penalties (Sivrikaya-Şerifoğlu and Ulusoy, 1999).

In the context of RCPSP, MCUOX can be rephrased as follows: Starting from the first activities on the parents, iteratively, one of the parents is chosen randomly and its next unconsidered activity becomes the next activity on the child. If the mode selected for that activity is the same on both parents, then that mode is also selected for the activity on the child; if not, one of the mode selections of the parents is chosen randomly.

One of the most important advantages of MCUOX is that it does not violate the precedence constraints. This property proves to be very useful as there are precedence constraints on the sequencing of activities; and the fact that the application of MCUOX will not violate them eliminates the need for repair functions as a complement to the crossover operator in generating feasible offspring.

3.2.1. Mutation operators

There are two mutation operators, one for each dimension of the search: The reposition mutation operator is employed for activities and the bit mutation operator for mode selections. Reposition mutation operator chooses a position on the chromosome randomly and transfers the contents of the corresponding gene into another randomly chosen locus. Bit mutation chooses one of the possible mode assignments associated with the activity on the gene where bit mutation is to apply, and reassigns it to the activity in question. This may result in the same mode assignment and means in practical terms that the effective bit mutation rate is lower than specified.

The application of the reposition mutation may cause precedence violations. A simple repair function complements the reposition mutation operator in generating feasible mutants.

Here the reposition mutation operator is chosen as opposed to the more popular swap mutation, which operates by swapping the positions of two randomly selected genes, for the following reason: Swap mutation operator repositions two activities which means that it can give rise to two violations of the precedence relations and hence to two repairs. As repair means reorganization of the material on the chromosome, its use should be avoided whenever possible.

4. RCPSPDCF PROBLEM WITH FOUR DIFFERENT PAYMENT MODELS

4.1. An Example Problem

An example problem with 14 activities and three resources has been chosen to illustrate the effects of the different payment models considered in this study: LSP, PEO, ETI, and PP. The activity-on-arc representation of the project network is shown in Figure 1 with the arc numbers denoting the activity numbers. Activites 1 and 16 are defined as dummy activities with duration zero and no resource requirements. Their modes are set at mode 1. The activity mode data with durations and resource requirements is presented in Table 1. The limits per period for each resource are as follows: for renewable resource 1 (RR1), the limit is 4; for renewable resource 2 (RR2), it is 3; and for nonrenewable/doubly constrained resource (NRR/DCR), the per period limit is 5 and the overall limit is 600. The deadline is 284; and the discount rate r is taken to be 0.005 per period.

Figure 1 about here

Table 1 about here

The budget is calculated here as being equal to the sum of the activities' costs discounted to the terminal event of the project and multiplied by a factor of 1.3 to secure a positive NPV for the contractor and thus a profit. The project schedule on which all these calculations are based is obtained by solving for the minimum makespan. Hence, the contractor's profit margin in this case is greater than 0.30. For this example problem, the budget is calculated to be 10671.7.

In the LSP type contracts, the client retains all the funds till the successful termination of the project and thus minimizes the risk of premature termination of the project. The contractor has to support all the operations through his/her own funds. In both the PEO and ETI cases, the retainage is much smaller than in the LSP case but still there is some positive retainage reducing the risk of the client against premature termination of the project. For the example problem, when calculating the amount of payments at each payment point in different payment models, the percentage of retainage is tried to be kept at equal levels.

In the LSP case, the whole budget is paid to the contractor at the terminal event node (node 11) as a single payment.

For the PEO case, there are eight payment event nodes {3, 5, 6, 7, 8, 9, 10, 11}. Each event node has a cost equal to the sum of costs of activities whose arcs are incident into that event node. Each payment amount is assumed to be equal to β (β=1.1) times the summation of the costs of the events which occurred after the previous payment time.

For the ETI case, the payments are taken to be made at equal time intervals except for the last payment, which is made at the terminal event node. The length of the last interval is set free since although a deadline is given, the makespan cannot be known in advance. For determining the equal time interval of payments, the minimum makespan is found first by applying the GA procedure proposed but this time with the objective of minimizing the makespan. The equal time interval of payments is obtained by dividing the minimum makespan by (H-1), where H is the number of payments. The final payment, i.e., the H^{th} payment is made at the end of the project. If the contractor finishes the project at that minimum makespan, than s/he will receive (H-1) payments where the last payment is added to the $(H-1)^{th}$ payment. To compute the payments at the payment points, the expenses incurred by the contractor up to the time of the current payment since the last payment are summed up and multiplied by β. The expenses for the activities which are only partially completed at the payment point, are assumed to be linearly proportional to activity duration. The last payment is calculated as the difference between the budget and the sum of all payments upto that point. In the example problem, the minimum makespan is found to be 141 and the number of payments H is selected to be four. Thus, the payment times are determined to be 47, 94, and 141 with the last payment scheduled to the termination of the project.

In the PP case, payments are made every 30 days until the end of the project. Payment amounts are computed in the same way as in the ETI model. Note that in PEO, ETI and PP models, since each payment is a fixed proportion β (β=1.1) of the expense incurred by the contractor up to the time of payment, the contractor makes a 10% profit with each payment when no discount rate is taken into account.

4.2. Solutions

The chromosomes representing the best solutions obtained for all payment model cases are given in Figure 2. The corresponding starting times, costs, C_{max} values, and NPVs are reported in Table 2.

Figure 2 about here

Table 2 about here

The budget value of 10671.7 is paid as a lump sum to the contractor by the client at the terminal event node 11 in the LSP case. The project duration is 150 and the resulting NPV for the contractor is 1409.20.

In the PEO case, the budget is distributed over the eight payment nodes as shown in Table 3. The occurrence times for the prespecified event nodes are also reported in Table 3. The project makespan is 151. The resulting NPV for the contractor is 2325.80.

In the ETI case, the budget is distributed over four payment points as shown in Table 4. The resulting payment amounts are also given in Table 4. The project makespan is found to be 157. The resulting NPV for the contractor is 2111.80.

For the PP case, the payment distribution is given in Table 5. The project makespan is 157 and the resulting NPV for the contractor is 2345.03.

Table 3 about here

Table 4 about here

Table 5 about here

The cumulative cash flow profiles for all four cases are given in Figure 3a-3d. As would be expected, the negative peaks are at their highest and NPV is the lowest for the LSP case. The contractor has a higher NPV in the PEO and PP cases than in the ETI case. Note that the profiles are always in the negative cash domain in the LSP case by definition, and would be expected to be in the positive cash domain only for relatively short periods of time for the remaining cases, if ever. This results in a heavy financial burden on the contractor. Recall that the payment structure here does not impose a predetermined initial advance payment.

Figure 3a-3d about here

From the results reported above on NPVs for different payment models, it appears that an increasing number of payments improves the resulting NPVs.

4.3. A comparison of the schedules

In the following, the differences in the final chromosomes obtained (see Figure 2) are investigated. Some of these changes are minor and simply a result of the random nature of GA without any actual influence on NPV. The changes in the sequences of the activities 8, 9, and 10 in all four cases are of this nature. As can be seen from Table 2, resource and precedence constraints allow these activities to start at the same time and be executed in parallel; therefore any changes in their ordering on the chromosome is immaterial with respect to NPV. Some other expectations regarding chromosome ordering are due to precedence constraints which dictate that activity 2 has to start first. And, as can also be followed from Table 1, resource constraints dictate that activities 3, 4, 6, 7, 12 and 15 have to be executed by themselves; all of these activities require 4 units of RR1, which is the upmost quantity available, and some of them require the limit quantities for RR2 and NRR also.

Comparison of the LSP and PEO schedules: Two differences can be detected. Activity 6 is deferred in the schedule for the PEO problem. Thus, the payment at event node 5 which has a greater amount (almost two times of the payment at event node 6) can be received earlier than the one at event node 6 without changing the occurrence time of other payment events. Hence this gives rise to a greater NPV. The change in the mode assignment of activity 12 may seem unreasonable at first. However, the change of the mode from 1 to 3 results in a small increase in the NPV. The decrease in cost resulting from this mode change of activity 12 is slightly larger than the decrease in the NPV of the payment deferred due to the mode change of activity 12.

Comparison of the PEO and ETI schedules: The first difference to be interpreted is the exchange of positions of activities 3 and 4. In the PEO solution, activity 3 precedes activity 4 which is due to the fact that there is a payment at event node 3 and the contractor wants to get that payment as early as s/he can. In the ETI solution, activity 4 precedes activity 3. The first payment occurs later than the completion of these activities, and hence the exchange of these activities does not affect the first payment amount. The delay of activity 3 beyond activity 4 results in the delay of the greater cost and thus decreases the total discounted cost. The most considerable change in the chromosomes is the position of activity 7. Since the cost of activity 7 is large, and the payments are made according to the costs incurred, it is scheduled earlier in the ETI solution, to increase the second payment amount. However, in the PEO problem, as the cost of activity 7 is included in the payment at event node 8, and it is not the only activity that affects the payment at event node 8, the earlier scheduling of activity 7 could not change that payment time, rather its cost will be incurred early and hence decrease NPV. The change in the mode assignments of activities 13 and 14 and the location of activity 13 allows for the parallel scheduling of these activities. Since the third payment in the ETI problem is after the starting time of these activities, and since the cost of the activities is changed due to the changes in the mode assignments, the third payment and the last payment amounts are also changed. Furthermore, makespan is increased due to the second and third mode assignments. All these changes results in an increase in NPV. In the PEO problem, there is no need to make these changes, because the payment at event node 10 only includes the cost of activity 14, small enough when compared to the third payment amount in the ETI problem.

Comparison of the LSP and ETI schedules: The position of activity 4 on the solution chromosome occurs later in the LSP problem than in the ETI problem, and higher costs associated with activities 3 and 6 seem to have occurred earlier in the LSP problem. However, the duration of the activities also affect NPV in addition to these costs and hence this order gives a higher NPV in the LSP problem. Since the position of activity 7 does not affect the single payment at the end in the LSP problem, and its cost is one of the largest costs in the project, its start time is postponed as much as possible without changing the makespan. However, in the ETI problem activity 7 affects the payment, and its earlier scheduling results in early receipt of a greater payment amount. Since there is a single payment in the LSP case, which is approximately two times of the payment at the end of the project in the ETI problem, the first modes which have shorter durations are more preferable in order to receive the budget earlier. So, activities 12, 14, and 13 are in their first modes in the LSP problem while, in the ETI problem, activity 14 is in its second, and the other two activities are in their third modes.

Comparison of the ETI and PP schedules: The ETI and PP problems are very similar, they differ in the length of the payment periods: In the ETI problem, the (three) equidistant payments are made every 47 days; whereas in the PP problem, , the (a priori unknown number of) equidistant payments are made every 30 days. Therefore, any changes in the respective schedules are related to this difference. The differences in the ordering of the activities 8, 9, 10 are immaterial since they are executed in parallel. So are the differences in the ordering of activities 13 and 14: These activities are executed in their third and second modes respectively which makes it possible to start them at the same time and be executed in parallel. The meaningful differences are in the ordering of activities 3 and 4 and in the positioning of activity 5. In the ETI case, the first payment at time point 47 includes costs of both of the activities 3 and 4 anyway (sum of durations of activities 2, 3 and 4 is 37 while the payment time is 47); so by executing activity 4 earlier, the more costly activity 3 is deferred, which has a positive effect on the NPV. In the PP case, the first payment is at time point 30 and a choice is made in favor of the more costly activity 3 to increase the payment amount. By scheduling activity 5 before activities 6 and 7 in the PP case, its cost is fully included in the second payment and the most costly activity 7 is deferred to the third payment.

Considering all four payment models together, the following observations are made. In the LSP case, the makespan is tried to be minimized so as to receive the only payment as soon as possible. In the case of PEO, the payment event nodes with larger payments due are tried to be scheduled earlier so as to increase their contribution to NPV. But this does not necessarily lead to smaller makespan values. Since the payment times are not decision variables in the case of ETI and PP, the procedure tries to increase the amount of earlier payments by proper scheduling of the activities relative to the payment points.

4.4. Some Sensitivity Analyses

Some sensitivity analyses are conducted on two parameters: The mark up β and the discount rate r. The experiments are conducted on the example problem as the base case using the PP model with doubly constrained resources. To investigate the effect of changes in β, the results obtained using β=1.2 and β=1.0 are compared to those obtained with β =1.1 of the base case. It is observed that as β increases, NPV increases. Part of this increase is due to the increase in the mark up. Part of the increase comes from the changes in the schedule. When β=1.2, the schedule changes compared to the base case in such a way that activities with higher unit cost are executed earlier so as to receive the corresponding payment with a relatively larger mark up earlier. When β=1.0, on the other hand, the schedule again changes compared to the base case. Activities with higher unit cost are deferred to later periods so as to reduce their negative impact on NPV.

The results of experiments with the discount rate *r* is given in Table 6 below. As *r* is increased, the PP model starts to behave more like the LSP model in trying to decrease the makespan and hence to realize the receipt of the last payment earlier. The changes in the solution chromosomes are initially along these lines. When *r* is increased to 0.006 and further to 0.007, the durations of activities 12, 13 and 14 and subsequently the duration of activity 6 are crashed. Durations of activities 8 and 10 are not changed for any increase in r as they are executed in parallel with activity 9, which has only a single mode of execution with a duration of 20 time periods. When r=0.006, activity 13 with a higher cost per unit duration, is deferred. When r=0.007, the most costly activity 7 is deferred also as much as possible, just like in the LSP model. When r=0.008, activity 4 is leapfrogged to defer the execution of activities 3 and 5, which have higher costs per unit duration compared to activity 4. The chromosome thus obtained does not change thereafter for any further increases in *r*.

Table 6 about here

When the same experiment is conducted for the LSP problem, no change occurs in the chromosome. This indicates that costs have already been deferred as much as possible without increasing the makespan.

5. COMPUTATIONAL RESULTS

5.1. Set of Test Problems

A set of 93 problems from the literature (Ulusoy and Özdamar, 1989) is used as the test bed. The distributions of the test problems according to the number of activities and the number of renewable resources are given in Table 7a and 7b respectively. The number of event nodes varies between 8 and 30. The number of modes per activity takes on values between 1 and 3 with the average number of modes per activity being 1.47.

Table 7a and 7b about here

Sixtyfour of the problems represent a 2^4 full factorial design, with four replicates for each effect. These test problems consist, on the average, of 20 activities and three resource types. The four network/resource characteristics which represent the factors in the design are complexity ratio (CPX), aspect ratio (ASP), resource utilization factor (UF), and dominant obstruction value (DOV). The first two are network characteristics whereas the last two represent the resource characteristics of the project. CPX is the ratio of the number of activities to the number of events. ASP is the ratio of the number of critical activities to the number of non-critical ones. UF is the ratio of a resource type's total resource requirements over critical path length to its total availability. UF of a project is taken to be the maximum ratio over all resource types. DOV measures the overflow of per period resource requirements over the resource limits in the earliest start schedule. These factors are considered at two levels in the factorial design. The low and high level limits for the characteristics are (0.58, 1.55), (0, 1.20), (2.75, 1) and (0.3, 3) for UF, DOV, CPX and ASP, respectively. CPX is negatively correlated with project duration extension and therefore, the low and high levels for CPX are reversed. Each treatment combination has four replicates thus constituting the 64 problems of the design. The remaining 29 problems are free of design and involve up to 53 activities. These problems are a subset of 47 problems details of which are given in Ulusoy and Özdamar (1989).

Two frequently employed resource characteristics for project networks are the resource factor and the resource strength (Kolisch, 1995). Resource factor is a measure of the average number of resources requested per activity. Resource strength expresses the relationship between the resource demand and the resource availability. The range of resource factor values over all 93 problems for the first renewable resource is (0.44 - 1), for the second and third renewable resources (0.28 – 1), and for the fourth one, the range is (0.28 – 0.48). The corresponding ranges for the resource strengths are (0 - 1.25), (0 – 1.57), (0 – 1.29) and (0 – 0.40) for the first four renewable resources. The resource factor and the resource strength for the fifth renewable resource are 0.571and 0, respectively.

The above problems are extended to include nonrenewable and doubly constrained resources. At most two nonrenewable resources are added to the problems. Here, 16 out of 93 problems have two nonrenewable resources. It is assumed that each mode of each activity uses at least 1 unit of nonrenewable resources; that is the resource factor of nonrenewable resources for each problem is taken as 1. The data of nonrenewable resource usages and their unit costs are generated such that the nonrenewable resources' discounted total cost is approximately equal to the discounted total cost of renewable resources obtained as a result of the makespan minimization problem. For each problem, the limit of nonrenewable resources is computed in the algorithm by taking a resource strength of 0.60. For the case of doubly constrained resources, all nonrenewable resources become doubly constrained with the addition of a period limit to each nonrenewable resource. The overall limit remains the same for the doubly constrained resources as the overall limit for the corresponding nonrenewable resources. The period limit is set equal to the maximum doubly constrained resource usage among all modes of all activities, so as not to make some modes redundant. For some problems this limit turns out to be small enough to make the overall limit redundant and hence, the corresponding doubly constrained resource behaves like a renewable resource. For some problems, this limit turns out to be high and hence, the corresponding doubly constrained resource behaves like a nonrenewable resource.

5.2. Parameter Finetuning

As the GAs are able to provide good solutions to search problems in reasonable times they are employed as meta-level algorithms to finetune the parameters of other GAs (Grefenstette, 1986). In line with this approach, a meta-level GA has been designed and employed here for the optimization of the parameters of the GA for the LSP problem. First, the case with renewable resources only is treated. Seven parameters are subjected to the finetuning process via the meta-GA. These parameters and the range of values investigated are listed in Table 8 below in the order in which they appear on the chromosomes. One chromosome of the meta-GA might be for example [50, 160, 8, 0.20, 0.30, 0.00, 3] describing a GA with a population size of 50 and number of generations of 160 to be replicated 8 times. The probability of crossover, reposition mutation and bit mutation are 0.20, 0.30 and 0.00 respectively. The number of elitist n_E is 3, so at each generation the best three chromosomes are transferred in tact into the next generation. From Table 8, it follows that the meta-GA is to search through a space of more than 2^{28} possible parameter sets.

Table 8 about here

To evaluate a chromosome in the meta-GA, the GA with the parameters specified in the chromosome is run as many times as specified by the nRep parameter. The payoff value is then taken to be the average of best solution (NPV) values of all these replications. Fitness of the chromosome *X*, *F(X)*, equals the payoff value, i.e., $F(X) = \bar{Z}(X)$, where $\bar{Z}(X)$ is the average of the best solution values obtained using the parameters in X to run the GA for nRep replications.

The problem with the maximal sum of number of activities and number of resources is chosen from the test suite to be used in the fine-tuning study. This problem has 53 activities and 3 resources. The discount rate is taken to be 0.005. The parameters of the meta-GA are as follows: population size is 51, probability of crossover is 0.60, probability of bit mutation is 0.125, and number elitist n_E is 1.

The meta-GA is run on this problem for 60 generations. The resulting best chromosome has yielded the following GA parameters: popsize=140, nGen=240, nRep=8, pc=0.65, pm_{bit} =0.05, $pm_{repostn}$=0.50, and G=19/140=0.14.

For the case of renewable and nonrenewable resources, a similar meta-GA has been applied. The same parameters as above are employed for the meta-GA. In the application for these cases, not the best one but the best five individuals with different chromosome structures are chosen as elite individuals to be reproduced into the next generation.

The resulting best chromosome has yielded the following GA parameters: popsize=140, nGen=250, nRep=22, pc=0.90, pm_{bit} =0.02, $pm_{repostn}$=0.05, and G=11/140=0.08.

These same GA parameters are used for the doubly constrained resources case.

5.3. Computational Performance

The computational times measured for different types of payment models and resource types are reported in Table 9. For the experiments, a project with 27 activities, 13 nodes, 3 renewable resources, 1 nonrenewable resource is selected and it is solved repetitively 50 times. When testing the procedure for the doubly constrained resource case, the nonrenewable resource becomes doubly constrained. The average CPU times measured from the start of the GA up to the generation and evaluation of the 1000th feasible chromosome are shown in Table 9. Recall that infeasible chromosomes are discarded. The experiments are performed on a PC with Pentium II processor and 32 MB RAM using C++ compiler.

Table 9 about here

As can bee seen from Table 9, there is a considerable increase in computation times when switching from LSP to PEO, ETI, and PP cases. This increase is caused by the additional operations performed due to the calculation of payment amounts. There is a steady increase in the CPU times when switching from the renewable resources only case to the case with nonrenewable resources and to the one with doubly constrained resources. The increase results from the extra operations performed due to the additional resources, additional feasibility checks, and the need to generate new chromosomes for each infeasible chromosome thrown away due to exceeding the overall limit for the nonrenewable and/or doubly constrained resources. Such infeasible chromosomes are counted for 9 projects which are thought to be good representatives of 93 projects in the problem set. On the average, approximately 5.6 % of the chromosomes generated are infeasible due to overall limit. It is also observed that there is an increase in the infeasibility percentage due to deadline in the PEO case with doubly constrained resources.

5.4. Testing GA against LCBA

The testing of the GA is performed only against the local constraint based analysis (LCBA) of Ulusoy and Özdamar (1995) and only in the context of the LSP problem for the renewable resources case. The reason for the restriction on the solution procedure selected for comparison is that, to the best of the authors' knowledge, there are no other solution procedures reported in the open literature for the multi-mode RCPSPDCF. Furthermore, LCBA is applicable in its present form only to solving the LSP problem for the renewable resources case. Thus, the test bed is limited to LSP problems.

Local constraint based analysis (LCBA) of Ulusoy and Özdamar (1994) is a solution procedure proposed for solving multi-mode RCPSP with the objective of minimizing the makespan. It has also been applied to solve the NPV maximization problem (Ulusoy and Özdamar, 1995). LCBA analyses resource conflicts locally on small segments of the project. At every scheduling time point, through an activity selection process, a set of schedulable activities are identified from which a group of activities are chosen as the highest priority group in the allocation of scarce resources. The delay of activities from this group extends the project duration. The same selection process also identifies another set of activities to be discarded from the set of schedulable activities. After the activities in the highest priority group are all scheduled, activities not discarded from the schedulable set are considered for scheduling. Ulusoy and Özdamar (1995) test the performance of LCBA against a couple of good decision rules and the algorithm of Li and Willis (1992). LCBA and the other decision rules are first used as single pass heuristics and then are embedded into an iterative scheme. The results demonstrate that all decision rules benefit from being embedded in an iterative algorithm; LCBA's performance, in particular, is three-folded by the iterative algorithm. The results of the comparisons show that LCBA performs far better than the other approaches including the algorithm of Li and Willis. Therefore the GA approach will be compared here against LCBA.

The algorithms are run on the test suite, and for each problem both the maximum NPV value and the minimum C_{max} value are recorded for the LSP case with renewable resources. These solutions are compared with the solutions of LCBA with respect to the average percentage of improvement in NPV values and the average percentage of improvement in C_{max} values, where for each problem the efficient solution with the maximum NPV among the iterative runs is chosen as the solution provided by LCBA. The results of these two comparative analyses are subjected to t-tests. Table 10 displays the results.

The results indicate that the GA approach yields an average improvement of 18.37 percent, which is an important improvement in NPVs and an average improvement of 0.97 percent in C_{max} values as compared to LCBA. These results are significant at a significance level of 0.0005, and 0.01 respectively. Here, the results are recorded under the maximization of NPV objective, therefore C_{max} values reported for the GA may not be the best values that it can provide. So, the same GA is run on the same test suite with an objective function of the minimization of makespan. The resulting GA yields an average percent improvement of 5.35 which is significant at a level of 0.0005 (see the last row in Table 10).

Table 10 about here

This fact that the same GA with the same set of parameters performs better than a domain specific heuristic in two different problem environments for a wide range of problems is an illustration of its robustness.

6. SUMMARY

In this paper, the multi-mode RCPSPDCF with the objective of NPV maximization is considered. A GA approach is used to investigate four different payment models for the multi-mode RCPSPDCF with renewable, nonrenewable, and doubly constrained resources. Lump sum payment (LSP), payment at event occurences (PEO), payment at equal time intervals (ETI), and progress payment (PP). It is observed that in the LSP case, the makespan is tried to be minimized so as to receive the only payment as soon as possible. In the case of PEO, the payment event nodes with larger payments due are tried to be scheduled earlier so as to increase their contribution to NPV. But this does not necessarily lead to smaller makespan values. Since the payment times are not decision variables in the case of ETI and PP, the procedure tries to increase the earlier payments by proper scheduling of the activities relative to the payment points.

For the PP model only, sensitivity analysis is performed over the mark up β and the discount rate r. It is observed that as the mark up β is increased, the schedule of the PP model tends to execute activities with relatively higher unit cost earlier and as β is decreased, the schedule tends to defer such activities.

Similar to the conclusion by Bey *et al*. (1981), the cost of capital is shown to have an effect on the time sequencing of activities. Another observation is that as the discount rate r increases in the PP model, it starts to behave more like the LSP model and tries to decrease the makespan.

The GA employing multi-component uniform order-based crossover (MCUOX) is shown to be applicable to both the multi-mode RCPSP and multi-mode RCPSPDCF with a relatively minor change in the definition of the fitness function. For a comparative analysis, the GA is tested against LCBA, one of the better problem specific heuristics in both multi-mode

RCPSP and multi-mode RCPSPDCF for the LSP with the renewable resource case, and is shown to outperform it. Computational tests show that GA can provide good solutions in reasonable CPU times.

Two extensions might be of interest. One would be to investigate the impact of different numbers of payment points on the solutions generated. The other extension would be to include the case where the contractor will borrow funds whenever the cash flow is negative at rates larger than the discount rate.

REFERENCES

Bey, R.B., R.H. Doersch, J.H. Patterson, "The net present value criterion: its impact on project scheduling", *Project Management Quarterly*, 12, pp. 223-233, 1981

Baroum, S.M., J.H. Patterson, "An exact solution procedure for maximizing the net present value of a project", in Weglarz, J. (Ed.), *Handbook on Recent Advances in Project Scheduling*, Kluwer Academic Publishers, 107-134, 1999.

Blazewicz, J., J.K. Lenstra, A.H.G. Rinnooy Kan, "Scheduling subject to resource constraints: classification and complexity", *Discrete Applied Mathematics*, **5**, 11-24, 1983.

Dayanand, N., R. Padman, "Payments in projects: A contractor's model", Working Paper 93-71, The Heinz School, Carnegie Mellon University, Pittsburgh, Pennsylvania, 1993.

Dayanand, N., R. Padman, "A simulated annealing approach for scheduling payments in projects", Working Paper 94-40, The Heinz School, Carnegie Mellon University, Pittsburgh, Pennsylvania, 1994.

Dayanand, N., R. Padman, "On modelling payments in projects", *Journal of the Operational Research Society,* **48**, 906-918, 1997.

Dayanand, N., R. Padman, "Project contracts and payment schedules: The client's problem", Working Paper, The Heinz School, Carnegie Mellon University, Pittsburgh, Pennsylvania, 1998.

Dayanand, N., R. Padman, "On payment schedules in contractor client negotiations in projects: An overview of the problem and research issues", in Weglarz, J. (Ed.), *Handbook on Recent Advances in Project Scheduling*, Kluwer Academic Publishers, 477-508, 1999.

Doersch, R.H., J.H. Patterson, "Scheduling a project to maximize its present value: A zero-one programming approach", *Management Science*, **23**, 882-889, 1977.

Drexl, A., J. Grünewald, "Nonpreemptive multi-mode resource constrained project scheduling", *IIE Transactions*, **25**, 74-81, 1993.

Elmaghraby, S.E., W.S. Herroelen, "The scheduling of activities to maximize the net present value of projects", *European Journal of Operational Research*, **49**, 35-40, 1990.

Goldberg, D.E., *Genetic Algorithms in Search, Optimization & Machine Learning*, Addison-Wesley, 1989.

Grefenstette, J.J., "Optimization of control parameters for genetic algorithms", *IEEE Transactions on Systems, Man, and Cybernetics*, **SMC-16**, 122-128, 1986.

Grinold, R.C., "The payment scheduling problem", *Naval Research Logistics Quarterly*, **9**, 123-136, 1972.

Hartmann, S., "Project scheduling with multiple modes: A genetic algorithm", Technical Report 435, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universitaet Kiel, Kiel, Germany, 1997.

Hartmann, S., "A competitive genetic algorithm for resource-constrained project scheduling", *Naval Research Logistics*, 45, pp. 733-750, 1998.

Herroelen, W.S., B. De Reyck, E.L. Demeulemeester, "Project network models with discounted cash flows: A guided tour through recent developments", *European Journal of Operational Research*, **100**, 97 - 121, 1997

Herroelen, W.S., P. van Dommelen, E.L. Demeulemeester, "Resource-constrained project scheduling: A survey of recent developments", *Computers and OR*, **25**, 279-302, 1998.

Herroelen, W.S., E.L. Demeulemeester, and B. De Reyck, "A classification scheme for project scheduling", in Weglarz, J. (Ed.), *Handbook on Recent Advances in Project Scheduling*, Kluwer Academic Publishers, 1-26, 1999.

Holland, J.H., *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, 1975.

Icmeli, O., S.S. Erenguc, “A branch-and-bound procedure for the resource-constrained project scheduling with discounted cash flows”, *Management Science*, **42**, 1395-1408, 1996.

Kazaz, B., C. Sepil, "Project scheduling with discounted cash flows and progress payments", *Journal of the Operational Research Society*, **47**, 1262-1272, 1996.

Khattab, M., F. Choobineh, "A new approach for project scheduling with a limited resource", *International Journal of Production Research*, **29**, 185-198, 1991.

Kolisch, R., *Project Scheduling under Resource Constraints*, Physica Verlag, Heidelberg, 1995.

Kolisch, R., R. Padman, "An integrated survey of project scheduling", Technical Report 463, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universitaet Kiel, Kiel, Germany, 1998.

Lee, J.-K., Y.-D Kim, "Search heuristics for resource constrained project scheduling", *Journal of the Operational Research Society,* **47**, 678-689, 1996.

Li, K.Y., R.J. Willis, “An iterative scheduling technique for resource-constrained project scheduling”, *European Journal of Operational Research*, **56**, 370-379, 1992.

Mori, M., C.C. Tseng, "A genetic algorithm for multi-mode resource constrained project scheduling problem", *European Journal of Operational Research*, **100**, 134-141, 1997.

Özdamar, L., "A genetic algorithm approach to a general category project scheduling problem", *IEEE Trans. Systems, Man and Cybernetics, Part C: Applications and Reviews*, **29**, 1, pp. 44-59, 1999.

Özdamar, L., G.Ulusoy, “A survey on the resource-constrained project scheduling problem”, *IIE Transactions*, **27**, 574-586, 1995.

Padman, R., D.E. Smith-Daniels, “Early-tardy cost trade-offs in resource constrained projects with cash flows: An optimization guided heuristic approach”, *European Journal of Operational Research*, **64**, 295-311, 1993.

Russell, A.H., "Cash flows in networks", *Management Science*, **16**, 357-373, 1970.

Russell, R.A., “A comparison of heuristics for scheduling projects with cash flows and resource restrictions”, *Management Science*, **32**, 291-300, 1986.

Sepil, C., N. Ortac, "Performance of the heuristic procedures for constrained projects with progress payments", *Journal of the Operational Research Society*, **48**, 1123-1130, 1997.

Sivrikaya-Şerifoğlu, F., *A New Uniform Order-Based Crossover Operator for Genetic Algorithm Applications to Multi-Component Combinatorial Optimization Problems*, Unpublished PhD Dissertation, Boğaziçi University, Istanbul, 1997.

Sivrikaya-Şerifoğlu, F., "A new uniform order-based crossover for concurrent consideration of sequencing and selection problems", Working Paper, Abant İzzet Baysal University, Bolu, Turkey, 1998.

Sivrikaya-Şerifoğlu, F., G. Ulusoy, "Parallel machine scheduling with earliness and tardiness penalties", *Computers and OR*, **26**, 773-787, 1999.

Slowinski, R., Two approaches to problems of resource allocation among project activities - a comparative study, *Journal of the Operational Research Society,* **31**, 711-723, 1980.

Smith-Daniels, D.E., N.J. Aquilano, “Using a late start resource constrained project schedule to improve project net present value”, *Decision Sciences*, **18**, 617-630, 1987.

Ulusoy, G., L. Özdamar, “Heuristic performance and network/resource characteristics in resource-constrained project scheduling”, *Journal of Operational Research Society,* **40**, 1145-1152, 1989.

Ulusoy, G., L. Özdamar, “A constraint-based perspective in resource constrained project scheduling”, *International Journal of Production Research*, **32**, 693-705, 1994.

Ulusoy, G., L. Özdamar, “A heuristic scheduling algorithm for improving the duration and net present value of a project”, *International Journal of Operations & Production Management*, **15**, 89-98, 1995.

Ulusoy, G., S. Cebelli, "An equitable approach to the payment scheduling problem in project management", *European Journal of Operational Research*, **127**, 262-278, 2000.

Ulusoy, G., F. Sivrikaya-Şerifoğlu, Ü. Bilge, A genetic algorithm approach to the simultaneous scheduling of machines and automated guided vehicles", *Computers and OR*, **24**, 335-351, 1997.

Table 1. Activity mode data with durations and resource requirements

Activity	Mode 1				Mode 2				Mode 3			
	Duratn	RR1	RR2	NRR/ DCR	Duratn	RR1	RR2	NRR/ DCR	Duratn	RR1	RR2	NRR/ DCR
2	10	1	1	2								
3	15	4	3	5								
4	12	4	3	3								
5	15	4	3	5	29	1	2	1				
6	20	4	3	5	29	4	1	2				
7	20	4	3	5								
8	10	2	2	5	17	2	1	1				
9	20	1	1	3								
10	12	2	1	4	16	1	1	1				
11	15	0	1	2								
12	2	4	3	3	3	4	2	1	3	4	1	1
13	10	2	3	5	12	1	3	5	19	1	1	2
14	3	4	3	3	5	1	1	1				
15	2	4	2	2								

Table 2. Starting times, costs, C_{max} , and NPVs for the four different payment models

	LSP		PEO		ETI		PP	
Activity	ST	Cost	ST	Cost	ST	Cost	ST	Cost
2	0	160	0	160	0	160	0	160
3	10	675	10	675	22	675	10	675
4	54	444	25	444	10	444	25	444
5	66	675	37	675	86	675	37	675
6	25	551	52	551	37	551	52	551
7	113	900	113	900	66	900	81	900
8	81	221	81	221	101	221	101	221
9	81	400	81	400	101	400	101	400
10	81	192	81	192	101	192	101	192
11	98	225	98	225	118	225	118	225
12	133	74	133	45	133	45	133	45
13	140	430	141	430	136	304	136	304
14	135	111	136	111	136	60	136	60
15	138	52	139	52	155	52	155	52
C_{max}	150		151		157		157	
NPV	1409.20		2325.80		2111.80		2345.03	

Table 3. Payment distribution for the PEO case

Payment Event Node	3	5	6	7	8	9	10	11
Occ. Time	25	52	81	98	133	136	139	151
Payment amount	918.6	1230.9	606.1	454.3	1237.5	49.5	122.1	6052.8

Table 4. Payment distribution for the ETI case

Payment Time	47	94	141	157
Payment amount	1407.5	1596.1	1999.5	5668.6

Table 5. Payment distribution for the PP case

Payment Time	30	60	90	120	150	157
Payment amount	1122	1194.6	884.4	1449.8	598.4	5422.52

Table 6. Results of the sensitivity analysis on the discount rate

r	NPV	C_{max}	Chromosome
0.005	2345.03	157	$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 9 & 10 & 8 & 11 & 12 & 13 & 14 & 15 & 16 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$
0.006	2585.50	150	$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 10 & 9 & 8 & 11 & 12 & 14 & 15 & 13 & 16 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 3 & 2 & 1 & 1 \end{bmatrix}$
0.007	2854.85	141	$\begin{bmatrix} 1 & 2 & 3 & 5 & 6 & 4 & 9 & 10 & 8 & 11 & 7 & 12 & 14 & 15 & 13 & 16 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
0.008	3093.56	141	$\begin{bmatrix} 1 & 2 & 4 & 3 & 5 & 6 & 8 & 9 & 10 & 11 & 7 & 12 & 14 & 15 & 13 & 16 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
0.010	3483.35	141	$\begin{bmatrix} 1 & 2 & 4 & 3 & 5 & 6 & 8 & 9 & 10 & 11 & 7 & 12 & 14 & 15 & 13 & 16 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
0.015	4100.60	141	$\begin{bmatrix} 1 & 2 & 4 & 3 & 5 & 6 & 8 & 9 & 10 & 11 & 7 & 12 & 14 & 15 & 13 & 16 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

Table 7a. Distribution of test problems according to the number of activities

Number of activities	Number of problems
15-19	20
20-24	51
25-29	16
30-39	4
41	1
53	1

Table 7b. Distribution of test problems according to the number of renewable resources

Number of renewable resources	Number of problems
1	29
2	47
3	14
4	2
5	1

Table 8. Parameters and their range of values fine-tuned in the meta-GA

Parameter	Identifier	Values
population size	popsize	20,30,40,..,150
number of generations	nGen	10,20,30,..,250
number of replications	nRep	5,8,10,12,15,18,20,22,25,28,30
probability of crossover	pc	0, .10, .20, .30, .40, .45, .50, .55, .60, ..,1
probability of bit mutation	pm_{bit}	0, .05, .10, .15, .20, .., .50, .60, .70, ..,1
probability of reposition mutation	$pm_{repostn}$	0, .05, .10, .15, .20, .30, .40, ..,1
number of elitist	n_E	1,..,20

Table 9. Computation times for the generation and evaluation of 1000 feasible chromosomes (in CPU seconds)

	LSP	PEO	ETI	PP
R Resources	0.203	0.234	0.223	0.215
R & NR Res.	0.219	0.239	0.239	0.235
R & DC Res.	0.235	0.254	0.260	0.255

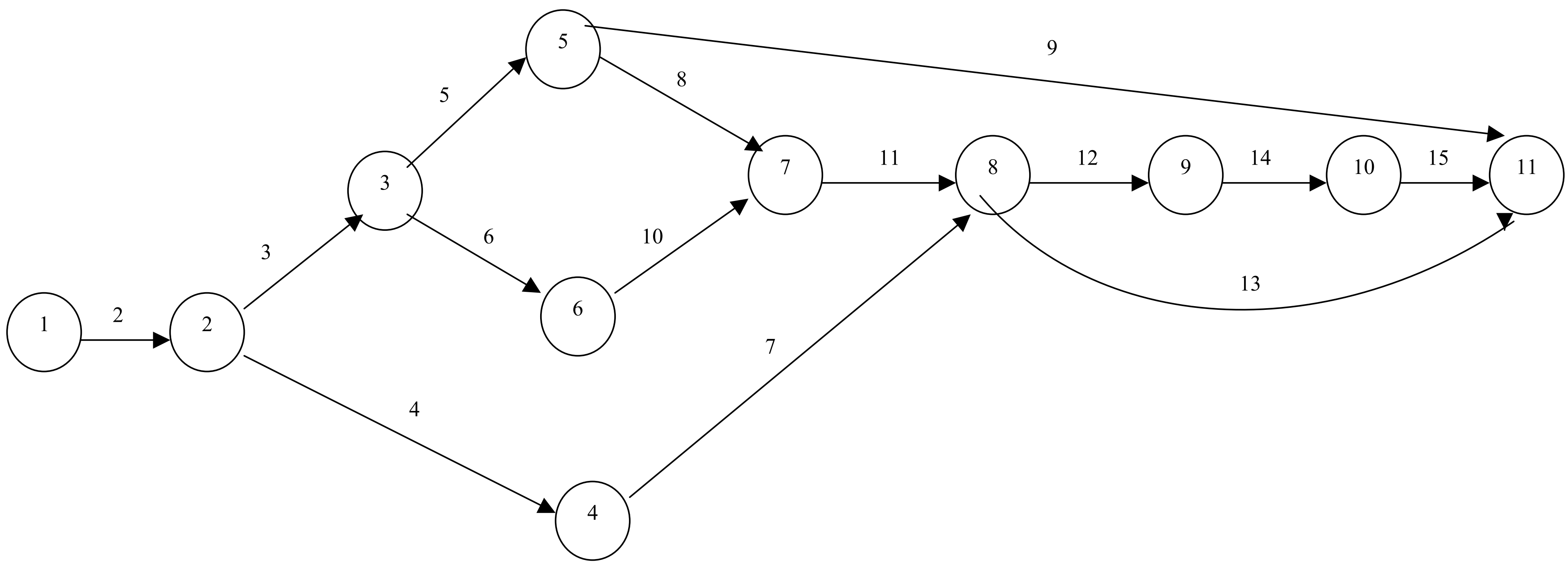
Table 10. Results of the comparisons between GA and LCBA

	Avg impr. (%)	t-test
Comparison of NPV values *	18.37	6.17
Comparison of C_{max} values *	0.97	2.37
Comparison of C_{max} values $	5.35	7.51

*Objective is NPV maximization.

$ Objective is makespan minimization.

Figure 1. The activity-on-arc representation of the project network of the example problem



(a)

1	2	3	6	4	5	8	9	10	11	7	12	14	15	13	16
1	1	1	2	1	1	2	1	2	1	1	1	1	1	1	1

(b)

1	2	3	4	5	6	8	10	9	11	7	12	14	15	13	16
1	1	1	1	1	2	2	2	1	1	1	3	1	1	1	1

(c)

1	2	4	3	6	7	5	10	8	9	11	12	14	13	15	16
1	1	1	1	2	1	1	2	2	1	1	3	2	3	1	1

(d)

1	2	3	4	5	6	7	10	8	9	11	12	13	14	15	16
1	1	1	1	1	2	1	2	2	1	1	3	3	2	1	1

Figure 2. The solution chromosomes for: (a) LSP, (b) PEO, (c) ETI and (d) PP cases

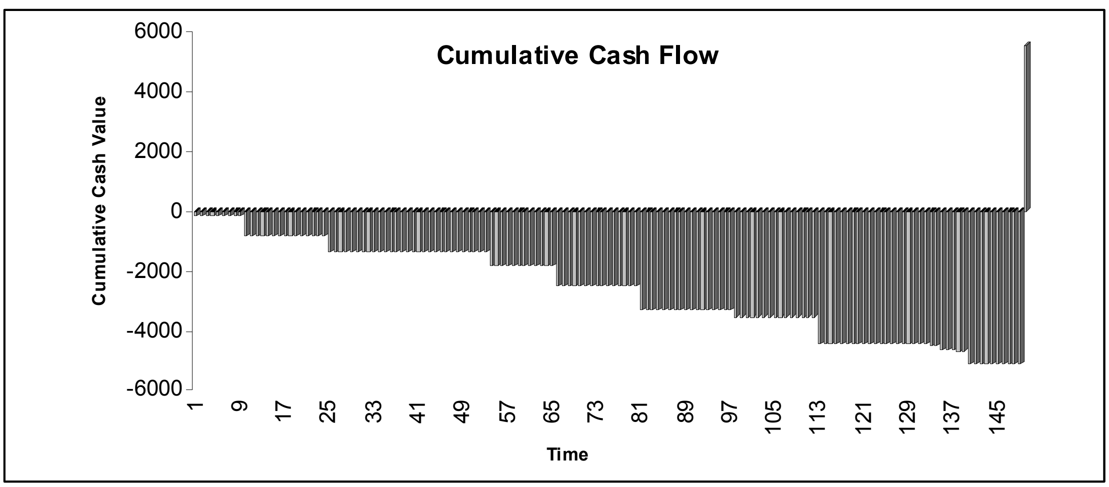


Figure 3a. The cumulative cash flow profile for the LSP case

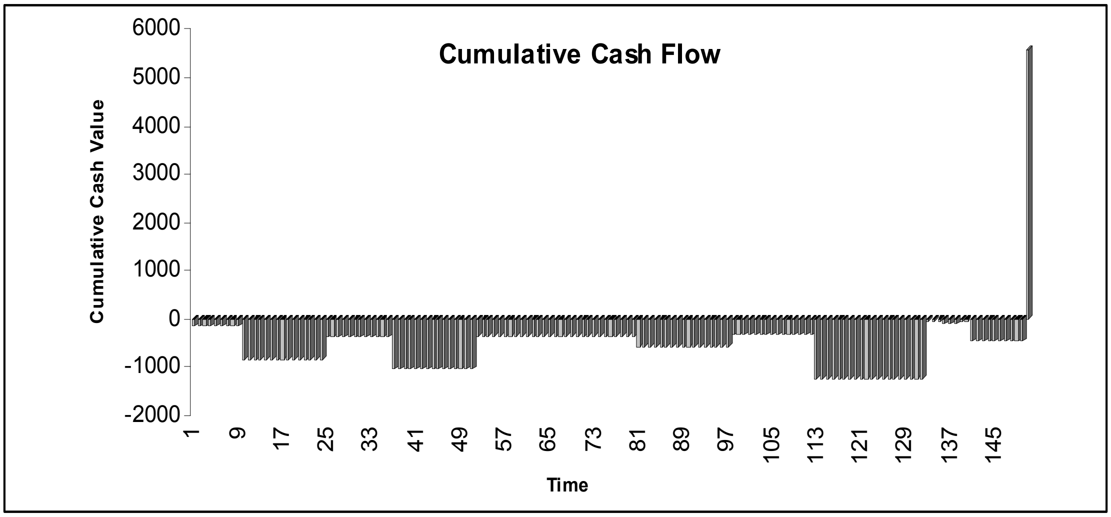


Figure 3b. The cumulative cash flow profile for the PEO case

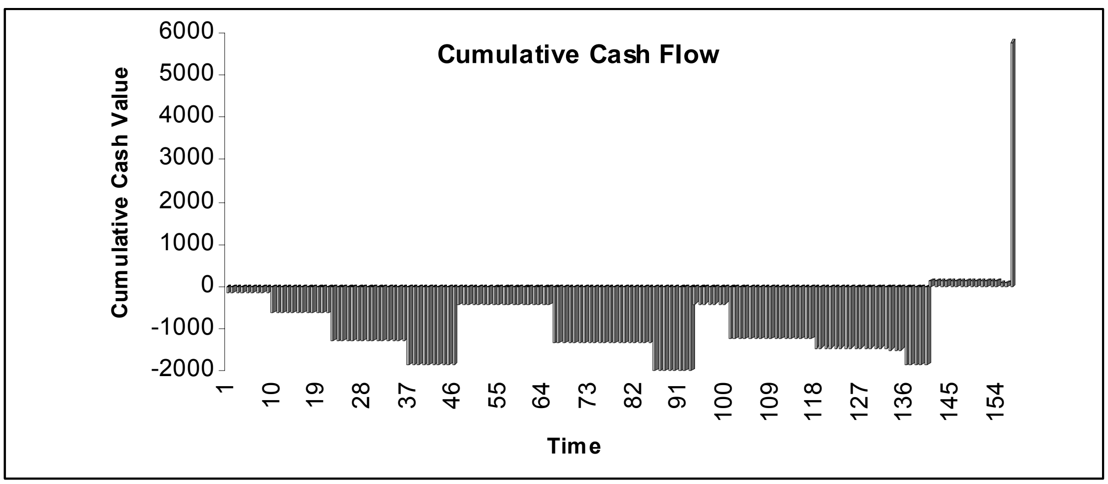


Figure 3c. The cumulative cash flow profile for the ETI case

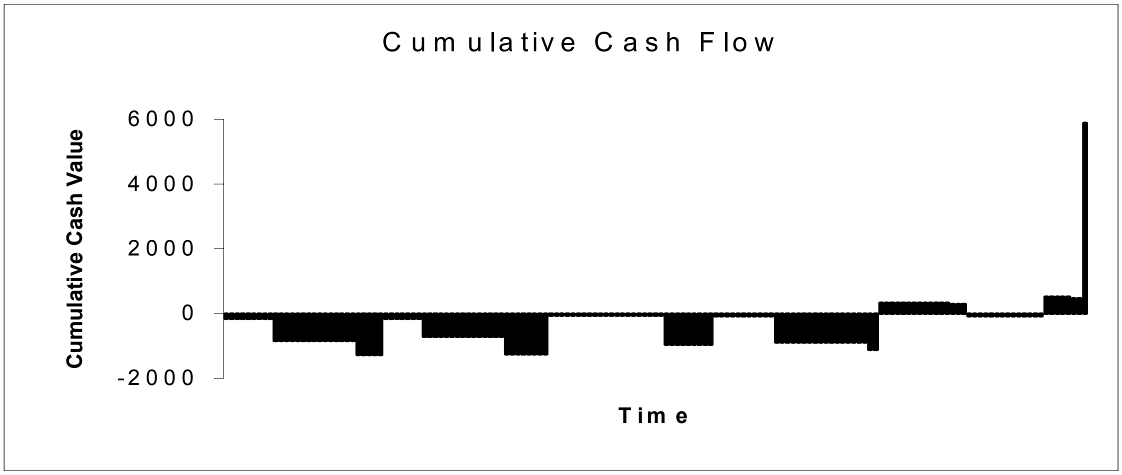


Figure 3d. The cumulative cash flow profile for the PP case