Licensing of a Drastic Innovation with Product Differentiation*

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Abstract

We analyze the licensing of a drastic innovation when products are differentiated due to consumer and/or product heterogeneity. We show that an industry insider prefers to divest its production arm and license the new technology as an industry outsider, in which case it can replicate multiproduct monopoly profit. We derive the optimal contracts and the optimal number of licenses by assuming a logit demand system. Optimal number of licenses, quite strikingly, increases when the technology has a higher relative value than a commercialized alternative. This result stands in sharp contrast with the literature on the licensing of a homogenous good.

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JEL Classification: D45, K11, L11, L13, L21, L41.

Introduction

Technological innovation is probably the most important way we generate value. Innovators, naturally, need to see a profit opportunity, a way to extract some of this value, in order to innovate in the first place. As a result, a number of mechanisms, such as patents, copyrights or trade secrets, have evolved to give the right incentives to the innovators.

Quite often, innovation takes place outside the industry, for example in universities. Given that they lack their own production possibilities, technology transfer from universities has recently become a hot topic of debate.[1] Moreover, the transfer of technology has emerged as an important issue in the business realm. Wesley, Cohen and Walsh (2000) report that between 1983-1995 patent awards grew by 78% in the U.S. based on the Carnegie Mellon Survey on Industrial R & D. There are ten industries in which 40% or more of the respondents report licensing revenue as a motive for patenting. Evidently, technology transfer is seen as a source of revenue in some industries.

The bilateral relationship between the owner of a right and another agent, who is willing to use this right, is an important kind of economic activity. This paper presents a framework that highlights technology licensing as a paradigm to address several important questions related to how and to whom to license. We consider a scenario where an upstream developer of an innovation has to decide on whether or not to enter the downstream segment of the market where, using this innovation as input, the production of a final good takes place. Additionally, the upstream firm has to choose the number of competing downstream firms to whom it will sell its license, and the terms at which the sales will take place.

We frame these issues using the following set-up: First, we introduce a model of

[1] Jensen and Thursby (2001), based on "The 1996 Survey of the Association of University Technology Managers", report that the number of licenses that have been executed increased 75% between 1991 and 1996, with a total of 13,087 licenses being executed over the entire period. Moreover, the respondents of the survey have stated licensing revenues as a major objective.

product differentiation where we consider licensing by an industry insider. The upstream firm will choose optimal two-part licensing contracts which involve a down payment and running royalties. We assume that a single firm owns the rights to a drastic innovation.[2] There is a large number of potential licensees whose outside option is to stay out or to take part in the competitive production of a commercialized alternative. This implies that the value of not being a licensee is zero.[3]

A first and quite general result we derive is that the owner of a license can implement the multiproduct monopoly outcome, which delivers the highest possible level of profits, by means of a two-part licensing contract. More interestingly, we also show that an industry insider prefers to divest its production arm and become an industry outsider. This result generalizes that of Sandonis and Fauli-Oller (2006) for drastic innovations. This is due to the commitment problem the innovating firm faces: An industry insider cannot commit to charge itself a royalty that is higher than her marginal cost plus the opportunity cost of not selling through a licensee. As a result, implementing the multiproduct monopoly outcome is not possible. This result holds for some quite general specifications of demand, and product differentiation can be due to consumer and product heterogeneity.

Next, we introduce a flexible demand model—logit. By means of an outside option, we allow for commercialized alternatives which may account for the already existing technology. Assuming a particular demand system makes it possible to derive the optimal number of licenses as a function of market size, relative attractiveness of the new technology compared to the commercialized alternatives, fixed costs of production, marginal costs, and the degree of product differentiation or heterogeneity of consumer tastes. We also derive the optimal licensing contracts as a by product. The optimal number of licenses decreases in fixed costs of production, marginal costs, and the level of substitutability, while it increases in the market size and the relative attractiveness of the new technology.

[2] We have in mind the same type of innovation which Rey and Salant (2009) call an essential intellectual property.

[3] The framework actually allows for any fixed level of outside option.

Licensing is an integral part of technological innovation. Here, licensing means the transfer of the rights to a technology in exchange for monetary compensation. Many other kinds of economic activity are terribly similar in nature. Often governments limit access to a market by giving operation licenses, such as for airlines, buses, and taxicabs. In telecommunications, operators compensate each other for calls terminated on each others' networks. One could even view wholesale-retailer relationships as a kind of licensing, where the manufacturer sells the retailer the right to further market a product.

There is a vast literature on the licensing of a new technology. Katz and Shapiro (1985) show in a Cournot-duopoly model that it may be profitable for minor innovations to be licensed with a fixed fee. They also show that with two-part contracts the monopoly outcome can be replicated. Any innovation that increases industry profits when licensed will be licensed. There is a series of papers—Kamien and Tauman (1986), Kamien, Oren and Tauman (1992), Katz and Shapiro (1986), Erutku and Richelle (2007) — that aim to pinpoint optimal licensing contracts. These papers study the licensing strategies of an exclusive holder of a cost reducing technology in a Cournot oligopoly model. The licensor is an industry outsider, e.g. a research lab. A common result is that the market structure critically depends on the nature of the innovation. They also find that the number of firms in the industry decreases as the innovation becomes more significant. Moreover, drastic innovations are licensed to only a single firm.[4]

The main point of departure of this paper from the above mentioned ones is its analysis of licensing in a differentiated products industry. There are three papers which are closely related to ours. Hernàndez-Murillo and Llobet (2006) study licensing of a cost reducing technology when there is product differentiation and when firms are heterogenous with respect to their production technologies. For the downstream market, they adopt a monopolistic competition framework, where a representative agent consumes all from a

[4] Erutku and Richelle (2007) obtain a contrasting result where even for drastic innovations the innovation is always licensed to more than a single firm.

continuum of products. They show that a patent holder will always employ a royalty in addition to a fixed fee. Not surprisingly, a similar result is also obtained in this paper. A royalty can be used as an instrument to control the marginal costs of the final products and thus to control the final price levels. They find that the optimal number of licenses will increase as the level of product differentiation decreases. This result is the complete opposite of one of our findings. Their model differs from ours in two important ways: In their model, the downstream firms have heterogenous and privately observed uses for the innovation, and they incur no fixed costs of production. One can easily show that if the downstream firms did not have heterogenous uses for the innovation and incurred no fixed costs, the innovator would actually choose to sell infinitely many licenses. In some sense, this is the common ground between our paper and theirs.

Sandonis and Fauli-Oller (2006) study a differentiated products duopoly and investigate the incentives of an external innovator to merge with an insider firm. When the patentee is an industry outsider, a potential licensee's outside option depends on the offered licensing contracts and therefore inducing this licensee to accept a licensing contract may become costly. In the case of large innovations, any gain from taking the outside option for a potential licensee is low, thus it becomes optimal for the patentee to remain an industry outsider. On the other hand, for small innovations they show that the outside option of a potential licensee is high, and therefore being an industry insider becomes a more favorable option for the innovator. We depart from their model by making the number of licenses an endogenous decision variable.

Rey and Salant (2009) set up a model of licensing an innovation where the licensees offer differentiated products. They look at the impacts of different licensing policies, such as fixed fee licensing and royalties, of one or more innovators. They find that with a single owner of a drastic innovation, the innovator is indifferent between becoming an insider or an outsider. This result is in contrast with our finding that an outsider innovator can implement the monopoly outcome and that the innovator would always prefer to be an outsider. This contrast can be explained by two factors: First, in their model, they do not allow for two-part tariffs as a licensing policy. Second, the demand model they use (Salop circle) is restrictive in its nature, since at equilibrium the entire market is always covered by the set of licensees. An additional byproduct is that as the value of the innovation increases, the optimal number of licenses weakly (non-monotonically) decreases. In our logit model there are always some consumers who do not opt for one of the new technology products, and as a result we find that the number of licenses increases in the value of the innovation.

In section 1, we introduce the model and compare licensing by an industry insider with that by an outsider. The logit demand is introduced in section 2. We derive the optimal licensing contracts for a given number of licences, the optimal number of licenses, and the socially optimal number of licenses in section 3. Section 4 concludes.

1 The Model

Consider a monopolist, M, who is the sole owner of a production technology, and has sunk the costs of development. M has also obtained a patent, and therefore replicating the technology is not legally possible. This technology can be used as an input to produce differentiated goods, and there are many firms who are ready to produce given access to the technology. Let us denote M as firm 1 and the other firms as $\{2, \ldots, N^{max}\}$, with $N^{max} \gg 2$. The demand for each firm's product is denoted by $m_k(p_1, \ldots, p_{N^{max}})$. When only N firms are active, the demand is given by

$$m_k(p_1, \ldots, p_N) = \lim_{p_{N+1}, \ldots, p_{N^{max}} \to \infty} m_k(p_1, \ldots, p_{N^{max}}) \tag{1}$$

with $N \leq N^{max}$.

M has a marginal cost c_j^I of transferring the input technology to firm j. Each firm incurs a marginal cost c_j^F to produce the final good, for $j = 1, \ldots, N^{max}$. Moreover, each firm has a fixed operating cost of C_j, $j = 1, \ldots, N^{max}$.

In the case when M licenses its technology to firm j, it uses a contract which is formed by a fixed fee, f_j and a per unit royalty r_j. Price competition takes place given the licensing contracts.

M can decide to be an active producer, in which case it is referred to as an insider. If it chooses not to produce, then it remains an industry outsider. Let $\mathbf{x}^N = (x_1, \ldots, x_N)$. Then, the profit of firm 1, M, when it produces is given by

$$\begin{aligned}\Pi_1^{Insider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N) &= (p_1 - c_1^I - c_1^F)m_1(\mathbf{p}^N) - C_1 \\ &+ \sum_{j=2}^{N} \left[(r_j - c_j^I)m_j(\mathbf{p}^N) + f_j\right],\end{aligned}$$

and when M only licenses, by

$$\Pi_1^{Outsider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N) = \sum_{j=1}^{N} \left[(r_j - c_j^I)m_j(\mathbf{p}^N) + f_j\right].$$

Furthermore, the profit of firm j when it produces is given by

$$\Pi_j(\mathbf{p}^N, r_j, f_j) = (p_j - r_j - c_j^F)m_j(\mathbf{p}^N) - f_j - C_j, \qquad j = 2..N,$$

otherwise it is fixed and normalized to zero. This normalization is not an innocent one. It inherently assumes that the new technology is replacing an old one which is competitively supplied. Thus the innovation in this context is assumed to be drastic.[5]

Given the sequence of events, it is straightforward to formalize the decision problem faced by the owner of the technology, both when it is an insider and when it is an outsider, assuming that the downstream firms compete in prices. Existence of a pricing equilibrium requires that certain conditions on the demand function hold. In the remainder of the text, we assume that the demand functions are such that $\Pi_k(\mathbf{p}^N, r_k, f_k)$, $\Pi_1^{Outsider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N)$, and $\Pi_1^{Insider}(\mathbf{p}^N, \mathbf{r}^N, \mathbf{f}^N)$ are quasi-concave in the relevant variables. Therefore, first order conditions are sufficient to characterize best responses, and hence, the equilibrium.

[5] If the innovation is thought to reduce marginal costs, this assumption becomes even stronger. It requires that even the monopoly price of the new technology is sufficient to drive the rest of the firms out of business.

Below, we show that an industry outsider would prefer to divest its production arm and license the technology as an industry outsider. This result is due to the commitment problem faced by the owner of the technology when it is an industry insider. If the outside option of a potential licensee depends on the licensing contracts this result may not hold. Sandonis and Fauli-Oller (2003) shows that for minor innovations it may be better to be an industry insider. However, also in their linear demand differentiated products duopoly model, a drastic innovation is licensed by an industry outsider.

Proposition 1. *When M licenses at all, it prefers only to license and not to produce an output good.*

Proof. See Appendix.

The proof of Proposition 1 relies on an argument where the problems faced by an outsider and an insider are similar constrained optimization problems where the insider licensor faces one additional constraint. It follows that the profit of the outsider is always as large as that of the insider. Notice that, this result is general and applies to any kind of differentiated demand system which satisfies (1). The reason behind this result is that M cannot commit to a transfer price other than its cost of producing the input plus the opportunity cost of not selling through its licensee's when it is an industry insider. On the other hand, whenever it is an outsider, it is free to choose this transfer price. Thus the implication is that under two-part licensing contracts, the owner of the technology is going to divest itself from its production arm if it ever finds licensing attractive. Rey and Tirole (1999) report on AT&T's spinning off its manufacturing arm, forming the independent entity called Lucent Technologies.

The result in Proposition 1 holds for any given number of licensees, N. Consequently, this result is independent of the optimal number of licenses. Suppose that the innovator finds it more profitable to be an insider in addition to selling N licenses (leading to a total of $N+1$ final good producers) to being an outsider. Then, Proposition 1 implies that it is even more profitable to sell $N+1$ licenses and being an outsider. In other words, regardless of the number of licenses that are sold, M would prefer to be an outsider.

If the licensor was allowed to offer more sophisticated contracts, even an industry insider could obtain the multiproduct monopoly profits. Maurer and Scotchmer (2006) suggest that a licensing contract which involves a downpayment and a royalty which decreases in the output of the patentee may solve this commitment problem and result in monopoly profits. However, such contracts would probably be deemed anti-competitive and will not be allowed by anti-trust agencies. As stated above, the equivalence of the insider and outsider problems only holds when the innovator can ignore the effect of its own output price on the prices of the other firms. With a licensing contract that involves only a fixed component, the two problems would be equivalent only when the innovator incurs no costs of transferring its technology to the licensees.[6] In the case when the licensing contracts involve only royalties, the two problems will obviously be not equivalent.

This line of reasoning leads to an implication for the incentives to innovate. If two-part licensing contracts are allowed, then incentives are stronger for an innovator that has no production arm. However, if licensing contracts involve fixed fees only, the licensor will be indifferent between becoming an insider or an outsider. In this case, that decision has no bearing on the incentives to innovate.

2 Logit Demand

In this section, we introduce a parsimonious demand system which allows us to answer a few questions with more specificity. Assume that there is a consumer population of size S. Each consumer demands one unit of the produced brands or an outside alternative.

[6]Rey and Salant (2009) show this to be the case. However, note that with fixed fees the fully-integrated monopoly outcome still cannot be implemented.

Brand j produced by firm j yields the net utility

$$\bar{U}_{ij} = \bar{V}_j - \alpha p_j + \epsilon_{ij},$$

for consumer i and the outside good yields

$$\bar{U}_0 = V_0 + \epsilon_{i0}.$$

Assume also that ϵ_{ij} is independent for all i and j, and double exponential distributed with unit variance.[7] The price sensitivity is measured by α. As $\alpha \to \infty$, product differentiation disappears. $\bar{V}_j$ measures the stand alone value of a product which essentially can be viewed as the population mean of the valuation of product j, and ϵ_{ij} measures the deviation of each individual's personal valuation from this mean.

The presence of an outside option allows us to take the existing technologies into account. If the existing technology is mature and not protected by patents, it is natural to expect that it is supplied competitively. Moreover, it is possible to characterize the level of the innovation relative to this existing outside option. [8]

The expected market share of product i when there are $N+1$ alternatives in total is given by

$$m_i(\mathbf{p}^N) = \frac{\exp(\bar{V}_i - \alpha p_i)}{\exp(V_0) + \sum_{j=1}^{N} \exp(\bar{V}_j - \alpha p_j)} \tag{2}$$

$$= \frac{\exp(V_i - \alpha p_i)}{1 + \sum_{j=1}^{N} \exp(V_j - \alpha p_j)} \tag{3}$$

with $V_j = \bar{V}_j - V_0$ for $j = 1, \ldots, N$. The demand for each good is then given by the product of the market share with the market size S. Namely, $d_i(\mathbf{p}^N) = Sm_i(\mathbf{p}^N)$ where $d_i(\cdot)$ represents the total demand for product i. Observe that this demand system satisfies

[7] This is an innocent normalization since the underlying preferences are the same when V_j and α are measured in units of this variance.

[8] Given the linear form of consumer utilities, a process innovation that reduces production costs is isomorphic to a product innovation that increases the stand alone value. This isomorphism is immediately apparent if one redefines the price of each firm as a mark-up over marginal cost.

property (1). For a detailed derivation of the expected market shares see Anderson, de Palma and Thisse (1992). Using this demand system, it is possible to solve the licensing problem in a closed form which is presented in the next section.

3 Optimal Number of Licenses and Contracts

We assume that there is complete information about the potential licensee's costs as well as the mean product valuations. By proposition 1 we know that if M is going to license, it will not produce. Therefore, it is sufficient to solve the problem for an industry outsider. Optimal licensing in this framework involves choosing the optimal number of licensees, N, as well as the optimal contracts (f_j, r_j) for $j = 1, \ldots, N$. To achieve this, M can figure out the optimal licensing policies given N, and then compare profits for each N in order to choose the optimal number of licensees. In the next subsection, we provide the optimal licensing contracts for a given N, and then derive the optimal number of contracts.

3.1 Optimal Licensing Contracts to N Firms

First, consider the problem of the monopolist producing all N brands by itself. The optimal prices are found by solving

$$\max_{p_1,\ldots,p_N} \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F) m_j(\mathbf{p}^N) S - C\right]$$

and they satisfy

$$p_j = c_j^I + c_j^F + \frac{1}{\alpha(1 - \sum_{i=1}^{N} m_i(\mathbf{p}^N))} \tag{4}$$

Note that all products are sold at the same mark-up over cost. A detailed derivation of this result can be found in Anderson, De Palma and Thisse (1992).[9] Thus, the only reason for a multiproduct monopolist to charge different prices would be differences in the

[9]They derive this result for the nested-logit model which is a generalization of the approach adopted here.

costs of production. However, even when the costs, and hence the prices, are the same, the sales of each product may differ due to differences in the stand alone values, V_j.

The highest possible profit in the industry is attained by prices given by (4), as these internalize all the cross product effects. The immediate question then is why one should license at all. It could be that the monopolist cannot credibly sell differentiated goods, or the monopolist might be financially constrained and does not have the funds to cover the necessary fixed costs. Alternatively, the monopolist might be forced to license by regulation.

Given N, M has to solve Program $1'$ which reduces to

$$max_{(r_j),j=1..N} \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F)m_j(\mathbf{p}^N)S - C_j\right] \tag{5}$$

such that

$$p_j \quad = \quad r_j + c_j^F + \frac{1}{\alpha(1 - m_j(\mathbf{p}^N))}, \qquad j = 1, .., N. \tag{6}$$

It is apparent that an industry outsider can attain the profits of a multiproduct monopolist by an appropriate choice of the royalty rate. This result is summarized in the next proposition.

Proposition 2. *An industry outsider can obtain the multiproduct monopoly profits by choosing the royalty rate to be*

$$r_j \quad = \quad c_j^I + \frac{\sum_{i \neq j} m_i(\mathbf{p}^N)}{\alpha(1 - m_j(\mathbf{p}^N))(1 - \sum_{i=1}^{N} m_i(\mathbf{p}^N))}, \qquad j = 1, \ldots, N, \tag{7}$$

which leads to equilibrium prices as given in (4). The fixed fee charged to each firm, f_j, is then given by

$$f_j = (p_j - r_j - c_j^F)m_j(\mathbf{p}^N)S - C_j. \tag{8}$$

Proof. Observe that the highest possible level of profits are obtained by the multiproduct monopolist with the prices given by (4). Substituting these prices in (6) yields

$$c_j^I + c_j^F + \frac{1}{\alpha(1 - \sum_{i=1}^{N} m_i(\mathbf{p}^N))} = r_j + c_j^F + \frac{1}{\alpha(1 - m_j(\mathbf{p}^N))}.$$

Solving for r_j produces the necessary royalty rates. ■

Observe that the royalty rates in (7), are firm specific; they depend on any marginal costs that might be incurred in the process of transferring the technology, as well as the stand alone value of a particular product through the market shares that will be realized in equilibrium.

One advantage of adopting the logit approach is that it allows us to characterize the market shares in terms of the model primitives in closed form. However, in order to achieve this goal, it will necessary to introduce an auxiliary concept. Lambert's W function is defined as

$$W(x) = \{z | z \exp(z) = x\} \tag{9}$$

For x real and positive, $W(x)$ is a positive valued, increasing, and concave function of x. We refer the interested reader to Corless et. al. (1996) for a more detailed information on the Lambert W function. Given this definition, we characterize the equilibrium market shares in terms of the model parameters in the next proposition.

Proposition 3. *Equilibrium market shares are given by*

$$m_j^* = \frac{K_j}{K} \frac{W(Ke^{-1})}{1 + W(Ke^{-1})}, \qquad j = 1, \ldots, N, \tag{10}$$

where $W(\cdot)$ *is the Lambert W function,* $K_j = \exp(V_j - \alpha(c_j^I + c_j^F))$, *and* $K = \sum_{i=1}^{N} K_i$. *Moreover, the market share of firm* j *increases in* V_j, *and decreases in* α, c_j^I *and* c_j^F.

Proof. See Appendix.

The equilibrium market shares have the expected properties. Products which provide higher surplus are the ones with higher market shares. Characterization of the market shares in Proposition 3, allows us to compute royalty rates, fixed fees and retail prices using (7), (8), and (4). However, when the costs and stand alone values are arbitrary, it is not possible to make statements on how these quantities will change as one of the model primitives changes because of the dependence of each on all of the market shares.

Naturally, when the marginal cost and the stand alone value of each potential licensee is different, the question of to whom to license becomes more involved. The number of optimal licenses will depend on the distribution of these values across potential licensees. Moreover, it is likely that in a more realistic situation the patent owner will have incomplete information at best, thus the implementation of a licensing mechanism will require more complicated arguments than those offered so far. We leave this topic for future research and continue with placing a few restrictions on the model.

One special case arises when firms are symmetric. This implies a symmetric equilibrium outcome. For this purpose, we assume that $V_j = V$, $c_j^I = c^I$, $c_j^F = c^F$ and $C_j = C$ for all j in the remainder of the paper. The assumption of symmetry is commonly used in the literature, and is also reasonable from an ex-ante perspective. The product differentiation in this case is only due to consumer heterogeneity as product heterogeneity is ruled out. In this case, $K_j = k = \exp(V - \alpha(c^I + c^F))$ for all j, and $K = Nk$, and thus the equilibrium market share in (10) reduces to

$$m^* = \frac{1}{N}\frac{W(Nke^{-1})}{1 + W(Nke^{-1})}. \tag{11}$$

The comparative statics on m^* are the same as in Proposition (10), meaning that in equilibrium, a new technology which is more valuable relative to the existing one will be diffused more.

Proposition 4. *If all the licensees are symmetric, the equilibrium retail price of each product is given by*

$$p^* = c^I + c^F + \frac{1}{\alpha(1 - Nm^*)}. \tag{12}$$

which increases in V, c^I, c^F and decreases in α.

Proof. See Appendix.

Equilibrium retail prices have intuitive properties. Symmetry also implies that all the licensees receive the same licensing contract, that is $r_j = r$ and $f_j = f$ for all j. The symmetric licensing contract is characterized in the next proposition.

Proposition 5. *When all licensees are symmetric, the equilibrium royalty rate for each licensee is given by*

$$r^* = c^I + \frac{(N-1)m^*}{\alpha(1-m^*)(1-Nm^*)}, \tag{13}$$

which increases in V, c^I and decreases in α, c^F. The equilibrium fixed fee is given by

$$f^* = \frac{m^*}{\alpha(1-m^*)} S - C. \tag{14}$$

Furthermore, f^* increases with V, S and decreases with α, c^I, c^F and C.

Proof. See Appendix.

It is interesting to note that when licensees are incurring higher marginal costs to transform the licensed technology to the final product, royalty rates tend to be lower. They are also lower in those industries where consumers are less heterogeneous in their tastes for products. A new technology, which allows firms to produce products that are more valuable relative to the commercialized alternatives (outside option), is licensed with a higher royalty rate and a fixed fee.

3.2 Optimal Number of Licenses

The equilibrium profit of M when it licenses to N firms is given by

$$\Pi(N) = \frac{m^*}{\alpha(1-Nm^*)} NS - NC$$

If we were to treat N as a continuous variable temporarily, it is easy to verify that

$$\frac{\partial \Pi(N)}{\partial N} = \frac{m^*}{\alpha} S - C, \tag{15}$$

since

$$\frac{\partial m^*}{\partial N} = -m^{*2}(2 - Nm^*). \tag{16}$$

Whenever (15) is positive at $N = 1$, the owner of the patent will prefer licensing or producing more than one product which is assumed in the following.

Furthermore, (16) implies concavity of the profits in N, thus, solving (15) is sufficient for a maximum. Observe that N can take integer values only, and therefore the optimal number of licenses must be one of the two closest integers to the value of N which satisfy

$$\frac{1}{N}\frac{W(Nke^{-1})}{1+W(Nke^{-1})}=\frac{\alpha C}{S}.$$

The solution is summarized in the next proposition.

Proposition 6. *The optimal number of licenses is given by one of the integers closest to*

$$N^*=\frac{S}{\alpha C}\left(1-\frac{1}{W(\frac{kS}{\alpha C})}\right). \tag{17}$$

Moreover, N^* increases with V and S, while it decreases with α, c^I, c^F and C.

Proof. See Appendix.

The fact that the optimal number of licenses increases with V, or similarly decreases with c^F, implies that a new technology which is drastically better than the alternatives will be licensed to a greater number of firms. This result stands in sharp contrast with the homogenous goods Cournot-oligopoly licensing literature. The underlying factor is the value of variety in the present model, such that when there are more brands, more consumers opt for one of the products rather buying the commercialized alternative—a typical feature for differentiated products.

Empirical evidence for such practices is vast and often referred to as *puzzling*. A good example is the liberal licensing policies adopted by Phillips and Sony in the case of Audio CD technology. Even though there may have been additional reasons, such as network effects, for increasing CD sales[10], there is no question that CD technology is a large leap from the vinyl-records or audio cassettes for delivering printed music. Moreover, either of these technologies were widely available and produced by many firms at the time of the introduction of the CD technology in the early eighties. According Grindley and Mc Bride(1992), there were more than 30 licensees of the CD technology by 1981.

[10]Both Phillips and Sony owned two of the largest record companies at the time.

Both Phillips and Sony produce CD-players as well, however, considering the size of each company, it might be that the commitment problem they faced was not much of an issue, and therefore they did not divest their production arms.

On the other hand, as α increases the effect of consumer heterogeneity vanishes and products are perceived to be closer substitutes. The optimal number of licenses tends to decrease in this case. It is apparent from the (15) that, for sufficiently large α, it may be that only a single firm gets a license. In some sense, this result implies the continuity of the outcomes in the degree of heterogeneity of consumer tastes. When consumers are homogenous, that is when $\alpha \to \infty$, the optimal number of licenses is one, which would be the case under price competition in a homogenous products market.

3.3 Social Planner's Problem

In this section, we are going to set up the social planner's problem while ignoring the impact of the outcome on the incentives to innovate. A social planner chooses the number of licenses and the final sales price of the products given a drastic innovation. Even though abstracting from the effects of the said policy choices on the incentives to innovate might seem unrealistic, one can think of a situation where the planner commissions the technological innovation, pays an upfront fee to obtain the rights and then chooses the number of licenses and the terms in which the products will be sold.[11] We restrict our attention to the case where all firms are symmetric. It is relatively straightforward to show that our results carry over to the more general case.

Note that at the optimal policy the social planner will always distribute the licenses for free. We will first start by showing that the optimal final product sales policy involves marginal cost pricing. Next, we are going to solve for the socially optimal number of licenses. The number of licences that maximizes the social welfare function turns out to be always greater than the number that maximizes the monopolist's profit.

[11] It could also be that the social planner itself comes up with the innovation.

Proposition 7. *The socially optimal sales price of each final product equals the combined marginal costs of transfer and production, $c^I + c^F$.*

Proof. See Appendix.

The result is quite intuitive. For a given number of licenses, an increase in the final sales price leads to a loss in consumer surplus some of which is captured by the firms as profits. The rest, however, is dead-weight-loss—the loss in surplus due to the reduction in consumption. By a similar argument a sales price below the combined marginal costs is never optimal.

Proposition 8. *The socially optimal number of licenses must be one of the two integers closest to the value of N which satisfy*

$$\bar{N} = \frac{S}{\alpha C} - \exp(-V + \alpha(c^I + c^F)). \tag{18}$$

Furthermore, the socially optimal number of licenses is always greater than privately optimal number of licenses.

Proof. See Appendix.

While determining the optimal number of licenses to sell, the monopolist faces a tradeoff. As the number of firms increases, consumer valuations for the final products and hence the amount that can be extracted from each firm increase. On the other hand, competition between firms becomes more intense as their number grows. According to Proposition 8, it is the second effect that is more dominant for the monopolist.

It is also worth pointing out that at the optimal policy formulated above, each firm needs to be subsidized for an amount equal to its fixed costs of production. This is what we call the *'first-best' solution*, where the social planner is not concerned with balancing its budget.

4 Conclusion

We analyze patent licensing when products are differentiated due to consumer and/or product heterogeneity. We consider two part licensing contracts, which involve a fixed downpayment and running royalties that are quite common in practice. The first result, which holds quite generally, is that an industry insider prefers to divest its production arm and license the new technology as an industry outsider. This is due to a commitment problem faced by the industry insider, which can only credibly commit to a transfer price(self-royalty) of marginal cost plus the opportunity cost of not licensing. When the patent owner remains outside the industry, it is free to choose this royalty, and in fact, is able to replicate multiproduct monopoly profits by using two part licensing contracts.

Given that a patent owner is better off as an industry outsider, the licensing problem involves choosing a fixed fee, a royalty rate and the optimal number of licenses. We derive optimal contracts given a certain number of licenses, and then obtain the optimal number of licenses by assuming a logit demand system. We characterize equilibrium market shares in terms of the model primitives, which in turn allow us to further characterize equilibrium prices and licensing contracts under the assumption of symmetric firms.

The optimal contracts have intuitive properties: Both the royalty rate and the fixed fee increase in the relative value of the new technology. Both decrease in the marginal cost of transforming the input technology to a final good. Interestingly, any costs that the patent owner might incur due to the transfer of the technology increase the royalty rate while they decrease the fixed fee. The fixed fee is larger for a larger market. The retail prices increase in the relative value of the technology, while they decrease in the substitutability of the products and the marginal costs of production.

Finally, we derive the optimal number of licenses, which quite strikingly increase when the technology has a higher relative value than a commercialized alternative. This result stands in sharp contrast with the literature on the licensing of a homogenous good. The main force behind this result is the fact that variety has value, that is, more consumers purchase the new technology when there are more products employing it. Furthermore, the number of licences that maximizes social welfare is always greater than the number that maximizes the monopolist's profit. In our logit model, the value of increased variety to the monopolist is diminished by the loss in revenues due to the increased competition between the licensees.

This paper contributes to the licensing literature by adding to the relatively scarce research on licensing with product differentiation. We also believe that it serves as a solid benchmark for a more general case with multiple competing innovations. One valuable extension to the current model would be to add a first stage innovation game where the competition between potential innovators and their incentives to innovate would be dependent on the nature of second stage licensing contracts and the number of licenses they sell. This is left as future research.

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Appendix

Proof of Proposition 1

Assume that M licenses to N firms. The pricing game takes place given (f_j, r_j), for $j = 2, \ldots, N$. When M is an outsider, the equilibrium is determined by the simultaneous solution of

$$p_j = r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N.$$

On the other hand, when M is an insider the equilibrium is determined by the solution of

$$\begin{aligned} p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 2..N, \\ p_1 &= c_1^I + c_1^F - \frac{m_1(\mathbf{p}^N)}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \\ &\quad - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N) \end{aligned}$$

Given these relationships, M selects optimal contracts (f_j, r_j). When it is an outsider the optimal contracts are selected as a solution to

Program 1

$$\max_{(f_j, r_j), j=1..N} \sum_{j=1}^{N} \left[(r_j - c_j^I) m_j(\mathbf{p}^N) + f_j \right]$$

such that

$$\begin{aligned} p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N \\ f_j &\leq \Pi_j(\mathbf{p}^N, r_j, f_j) - C_j, \qquad j = 1, .., N \end{aligned}$$

Substituting the second set of constraints yields

Program 1′ :

$$A = \max_{(r_j), j=1..N} \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F) m_j(\mathbf{p}^N) - C_j \right]$$

such that

$$p_j = r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N$$

When M is an insider on the other hand, the optimal contracts are determined by

Program 2

$$\begin{aligned} \max_{(f_j, r_j), j=2,\ldots,N} \quad & (p_1 - c_1^I - c_1^F) m_1(\mathbf{p}^N) - C_1 \\ & + \sum_{j=2}^{N} \left[(r_j - c_j^I) m_j(\mathbf{p}^N) + f_j \right], \end{aligned}$$

such that

$$\begin{aligned} p_1 &= c_1^I + c_1^F - \frac{m_1(\mathbf{p}^N)}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \\ & \quad - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N) \\ p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_k(\mathbf{p}^N)}, \qquad j = 2, .., N \\ f_j &\leq \Pi_j(\mathbf{p}^N, r_j, f_j) - C_j, \qquad j = 2, .., N \end{aligned}$$

Let

$$r_1 = c_1^I - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N)$$

and substitute the last set of constraints in Program 2 to yield

Program 2′

$$B = \max_{(r_j), j=1,\ldots,N} \quad \sum_{j=1}^{N} \left[(p_j - c_j^I - c_j^F) m_j(\mathbf{p}^N) - C_j \right],$$

such that

$$\begin{aligned} p_j &= r_j + c_j^F - \frac{m_j(\mathbf{p}^N)}{\frac{\partial}{\partial p_j} m_j(\mathbf{p}^N)}, \qquad j = 1, .., N \\ r_1 &= c_1^I - \frac{1}{\frac{\partial}{\partial p_1} m_1(\mathbf{p}^N)} \sum_{j=2}^{N} (r_j - c_j^I) \frac{\partial}{\partial p_1} m_j(\mathbf{p}^N) \end{aligned}$$

Comparing Program 1′ and Program 2′, it is apparent that we have the same optimization problem with one more constraint in the second one. Thus, the maximum of the second program must be less than or equal to the first one. ■

Proof of Proposition 3

The equilibrium market shares satisfy

$$\frac{m_j(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} = \exp(V_j - \alpha c_j - \frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)}), \tag{19}$$

where $c_j = c_j^I + c_j^F$. Rearranging (19) yields

$$\frac{m_j(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} \exp(\frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)}) = K_j. \tag{20}$$

Summing (20) over j results

$$\frac{\sum_{i=1}^N m_i(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} \exp(\frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)}) = K. \tag{21}$$

Let

$$y = \frac{\sum_{i=1}^N m_i(\mathbf{p}^N)}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)},$$

then it is easy to verify that

$$\frac{1}{1-\sum_{i=1}^N m_i(\mathbf{p}^N)} = 1 + y.$$

Therefore, (21) can be re-written as

$$y\ e^y = \frac{K}{e}. \tag{22}$$

Thus, $y = W(Ke^{-1})$, and also observe that

$$m_j(\mathbf{p}^N)(1+y)exp(1+y) = K_j.$$

The equilibrium market share of firm j is then given by (10).

For proving the second part of the proposition, it is useful to first note that

$$\begin{aligned} \frac{\partial}{\partial K_j} m_j^* &= \left[\frac{W(Ke^{-1})}{(1+W(Ke^{-1}))^3}\frac{K_j}{K^2} + \frac{W(Ke^{-1})}{(1+W(Ke^{-1}))}\frac{\sum_{i\neq j} K_i}{K^2}\right] \\ &= \frac{1}{K}\left[(1-\sum_{i=1}^N m_i^*)^2 m_j^* + \sum_{i\neq j}^N m_i^*\right] > 0, \end{aligned} \tag{23}$$

that is, the market share of firm j is increasing in K_j. The rest of the comparative statics directly follows.$\blacksquare$

Proof of Proposition 4

Imposing symmetry implies that the equilibrium outcome is also symmetric. Then, the equilibrium market share of each firm is given by

$$m^* = \frac{1}{N}\frac{W(Nke^{-1})}{1+W(Nke^{-1})}.$$

Now a change in one of the model primitives leads to a change in the surplus of all the firms, $log(k)$. Consequently, it is easy to verify that

$$\frac{\partial}{\partial k}m^* = \frac{1}{k}m^*(1-Nm^*)^2 > 0.$$

Also note that $\partial k/\partial V = k > 0$, $\partial k/\partial c^H = -\alpha k > 0$, for $H \in \{I,F\}$ and $\partial k/\partial \alpha = -(c^I + c^F)k > 0$. Therefore,

$$\frac{\partial}{\partial V}p^* = \frac{N}{\alpha(1-Nm^*)^2}\big(\frac{1}{k}m^*(1-Nm^*)^2\big)k = \frac{Nm^*}{\alpha} > 0,$$

$$\begin{aligned}\frac{\partial}{\partial \alpha}p^* &= -\frac{1}{\alpha^2(1-Nm^*)} + \frac{N}{\alpha(1-Nm^*)^2}\big(\frac{1}{k}m^*(1-Nm^*)^2\big)(-(c^I+c^F)k)\\ &= -\frac{1}{\alpha^2(1-Nm^*)} - \frac{Nm^*(c^I+c^F)}{\alpha} < 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial c^H}p^* &= 1 + \frac{N}{\alpha(1-Nm^*)^2}\big(\frac{1}{k}m^*(1-Nm^*)^2\big)(-\alpha k)\\ &= 1 - Nm^* > 0,\end{aligned}$$

for $H \in \{I,F\}$.$\blacksquare$

Proof of Proposition 5 It is easy to verify that (7) and (8) reduce to (13) and (14) when symmetry is imposed. To derive the comparative statics, first note that

$$\frac{\partial}{\partial m^*}r^* = \frac{(N-1)(1-Nm^{*2})}{\alpha(1-m^*)^2(1-Nm^*)^2} > 0,$$

and

$$\frac{\partial}{\partial m^*} f^* = \frac{S}{\alpha(1-m^*)^2} > 0.$$

Then

$$\frac{\partial}{\partial V} r^* = \frac{\partial r^*}{\partial m^*}\frac{\partial m^*}{\partial k}(k) > 0,$$

$$\frac{\partial}{\partial c^F} r^* = \frac{\partial r^*}{\partial m^*}\frac{\partial m^*}{\partial k}(-\alpha k) < 0$$

$$\begin{aligned}\frac{\partial}{\partial c^I} r^* &= 1 + \frac{\partial r^*}{\partial m^*}\frac{\partial m^*}{\partial k}(-\alpha k)\\ &= \frac{(N-1)(1-m^*-(N-1)m^{*2})}{(1-m^*)^2} > 0,\end{aligned}$$

since $1-m^*-(N-1)m^{*2} > 1-m^*-(N-1)m^* > 0$, and

$$\frac{\partial}{\partial \alpha} r^* = -\frac{(N-1)m^*}{\alpha^2(1-m^*)(1-Nm^*)} + \frac{\partial r^*}{\partial m^*}\frac{\partial m^*}{\partial k}(-(c^I+c^F)k) < 0.$$

Similarly, the comparative statics can be found found for f^* as

$$\frac{\partial}{\partial V} f^* = \frac{\partial f^*}{\partial m^*}\frac{\partial m^*}{\partial k}(k) > 0,$$

$$\frac{\partial}{\partial S} f^* = \frac{m^*}{\alpha(1-m^*)} > 0,$$

$$\frac{\partial}{\partial C} f^* = -1 < 0,$$

$$\frac{\partial}{\partial c^H} f^* = \frac{\partial f^*}{\partial m^*}\frac{\partial m^*}{\partial k}(-\alpha k) < 0$$

for $H \in \{I, F\}$, and

$$\frac{\partial}{\partial \alpha} f^* = -\frac{m^* S}{\alpha^2(1-m^*)} + \frac{\partial f^*}{\partial m^*}\frac{\partial m^*}{\partial k}(-(c^I+c^F)k) < 0.$$

■

Proof of Proposition 6 Let $L = \alpha C/S$, then it is easy to verify that

$$W(Nke^{-1}) = \frac{NL}{1-NL}.$$

By the definition of the Lambert's W function we have

$$\begin{aligned}\frac{NL}{1-NL}\exp(\frac{NL}{1-NL}) &= Nke^{-1},\\ \frac{1}{1-NL}\exp(\frac{1}{1-NL}) &= \frac{k}{L}.\end{aligned}$$

Therefore,

$$W(\frac{k}{L}) = \frac{1}{1-NL}, \tag{24}$$

and solving (24) for N yields the expression for N^*. In order to perform the comparative statics, first note that,

$$\frac{\partial}{\partial k}N^* = \frac{1}{W(k/L)(1+W(k/L))Lk} > 0,$$

and

$$\frac{\partial}{\partial L}N^* = -\frac{W(k/L)}{(1+W(k/L))L^2} < 0.$$

Furthermore, $\partial L/\partial S = -\alpha C/S^2$,$\partial L/\partial \alpha = C/S$, and $\partial L/\partial C = \alpha/S$. Therefore,

$$\begin{aligned}\frac{\partial}{\partial V}N^* &= \frac{\partial}{\partial k}N^* k > 0,\\ \frac{\partial}{\partial S}N^* &= \frac{\partial}{\partial L}N^*(-\frac{\alpha C}{S^2}) > 0,\\ \frac{\partial}{\partial C}N^* &= \frac{\partial}{\partial L}N^*(\frac{\alpha}{S}) < 0,\\ \frac{\partial}{\partial c^H}N^* &= \frac{\partial}{\partial k}N^*(-\alpha k) < 0,\end{aligned}$$

for $H \in \{I, F\}$, and

$$\frac{\partial}{\partial \alpha}N^* = \frac{\partial}{\partial k}N^*(-(c^I + c^F)k) + \frac{\partial}{\partial L}N^*(\frac{C}{S}) < 0.$$

■

Proof of Proposition 7

Consumer surplus, when there are N available brands that are sold at prices $p_1, \ldots, p_N$, is given by

$$CS(\mathbf{p}^N; N) = \frac{1}{\alpha}\log\left(1 + \sum_{i=1}^{N} e^{\bar{V}_i - \alpha p_i}\right)S$$

(see Anderson, de Palma and Thisse (1992) for the derivation). The consumer surplus function satisfies Roy's identity, i.e. $\frac{\partial}{\partial p_j} CS(\mathbf{p}^N; N) = -m_j(\mathbf{p}^N)S$. When the social planner decides to award licenses to N firms, the aggregate welfare is given by

$$W(\mathbf{p}^N; N) = CS(\mathbf{p}^N; N) + \sum_{j=1}^{N} (p_j - c^I - c^F) m_j(\mathbf{p}^N) S - NC.$$

The first order condition for setting the price of variant i is given by

$$\frac{\partial W(\mathbf{p}^N; N)}{\partial p_i} = -m_j(\mathbf{p}^N)S + m_j(\mathbf{p}^N)S + \sum_{j=1}^{N} (p_j - c^I - c^F) S \frac{\partial m_j(\mathbf{p}^N)}{\partial p_i} = 0, \qquad i = 1, \ldots, N.$$

Finally, $p_1 = p_2 = \ldots = p_N = c^I + c^F$ solves the system of N first order conditions simultaneously.

■

Proof of Proposition 8

When all available brands are sold at the same price that is equal to the marginal cost, $\bar{p} = p_1 = p_2 = \ldots = p_N = c^I + c^F$, the consumer surplus reduces to

$$CS(N) = \frac{1}{\alpha} \log \Big(1 + Nk\Big) S$$

where $k = \exp(V - \alpha(c^I + c^F))$. At these prices, aggregate welfare reduces to

$$W(N) = CS(N) - NC.$$

It is easy to verify that

$$\frac{\partial W(N)}{\partial N} = \frac{kS}{(1 + Nk)\alpha} - C = \frac{\bar{m}S}{\alpha} - C$$

where $\bar{m}$ is the market share of a variant when all firms charge a marginal cost price. Moreover, we have that

$$\frac{\partial^2 W(N)}{\partial N^2} = -\frac{k^2 S}{(1 + Nk)^2 \alpha} = -\frac{\bar{m}^2 S}{\alpha} < 0,$$

and therefore aggregate welfare is a concave function of the number of available variants when they are all sold at marginal cost prices. The number of licenses which maximizes aggregate welfare solves

$$\frac{k}{(1+Nk)\alpha} = \frac{C}{S}.$$

Next, it is straightforward to show that the socially optimal number of licenses is given by

$$\bar{N} = \frac{S}{C\alpha} - \frac{1}{k} = \frac{S}{C\alpha} - \exp(-V + \alpha(c^I + c^F)).$$

Rewriting the socially optimal number of licenses as

$$\bar{N} = \frac{S}{C\alpha}\left(1 - \frac{C\alpha}{kS}\right),$$

recall that the optimal number of licenses from the patentee's perspective is given by

$$N^* = \frac{S}{\alpha C}\left(1 - \frac{1}{W(\frac{kS}{\alpha C})}\right).$$

Define $\eta = \frac{kS}{C\alpha} > 0$. Then, it is straightforward to show that

$$\bar{N} - N^* \propto -\frac{1}{\eta} + \frac{1}{W(\eta)}.$$

Therefore, $\bar{N} - N^*$ has the same sign as $h(\eta) = eta - W(\eta)$. Note that

$$\frac{dh(\eta)}{d\eta} = 1 - \frac{W(\eta)}{(1 + W(\eta))\eta}$$

and

$$\frac{d^2h(\eta)}{d\eta^2} = \frac{W(\eta)^2(2 + W(\eta))}{(1 + W(\eta))^3\eta^2} > 0;$$

hence, $h(\eta)$ convex in η. It can be verified that $\lim_{\eta\to 0} h(\eta) = 0$. Then, $h(\eta)$ is an increasing function of η for $\eta > 0$. Furthermore, $h(0) = 0$. Therefore, $h(\eta) > 0$ for $\eta > 0$ which in turn implies that $\bar{N} - N^* > 0$, or equivalently $\bar{N} > N^*$. ■