TIME AND VOLUME BASED OPTIMAL PRICING STRATEGIES FOR TELECOMMUNICATION NETWORKS

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ABSTRACT

In the recent past, there have been several initiatives by major network providers such as Turk Telekom lead the industry towards network capacity distribution in Turkey. In this study, we use a monopoly pricing model to examine the optimal pricing strategies for "pay-per-volume" and "pay-per-time" based leasing of data networks. Traditionally, network capacity distribution includes short/long term bandwidth and/or usage time leasing. Each consumer has a choice to select volume based pricing or connection time based pricing. When customers choose connection time based pricing, their optimal behavior would be utilizing the bandwidth capacity fully therefore it can cause network to burst. Also, offering pay-per-volume scheme to the consumer provides the advantage of leasing the excess capacity for other potential customers for network provider.

We examine the following issues in this study:
(i) What are the extra benefits to the network provider for providing the volume based pricing scheme? and
(ii) Does the amount of demand (number of customers enter the market) change?

The contribution of this paper is to show that pay-per-volume is a viable alternative for a large number of customers, and that judicious pricing for pay-per-volume is profitable for the network provider.

Keywords: telecommunication networks, time based network pricing, volume based network pricing, optimization

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Introduction and Related Literature

Customers can use networks for video conferencing, voice over TCP/IP and data applications, which we will call tasks. As (Kasap *et al.* 2007) stated, in general, two types of tasks are performed using data networks. A task is time-fixed (real-time, size compressible) if its size can be changed without disrupting its completion but the transmission time cannot be compressed or extended. Most audio/video tasks are time-fixed. A task is size-fixed (time compressible) if all bits have to be transmitted but the duration is not fixed. Most data applications such as file transfer, database transactions are of this category. All types of applications use bandwidth and manage time. Traditionally, network capacity distribution includes short/long term bandwidth and/or usage time leasing.

In the recent past, there have been several initiatives by major network providers such as Turk Telekom lead the industry towards network capacity distribution in Turkey. In this study, we use a monopoly pricing model to examine the optimal pricing strategies for "pay-per-volume" and "pay-per-time" based leasing of data networks.

Earlier researches show that the connection billing model has a great effect on how people consume online data and how satisfied they are with the connection and online services. Today there are various billing models for quality-differentiated Internet access as a function of bandwidth, traffic volume, applications, and pricing structure which is based either on time, volume, a combination of both, or a flat-rate, bandwidth efficiency optimizations guaranteed QoS, uniform pricing, multipart pricing and nonlinear pricing on total call time or the number of packets exchanged (Masuda and Whang, 2006; Kasap *et al.*, 2007).

However, there are several other pricing strategies (Keon and Anandalingam, 2005) that are examined in lots of sources. Some of them are "Paris-metro pricing (similarly time of day pricing)", "priority pricing", "smart-market pricing", "top-percentile pricing" (Levy *et al.*, 2006), some parts of these pricing methodologies are usage-based, and other part of that are volume-based strategies.

In today's billing strategies vary in different countries. Volume-dependent charges have been used by some providers in the past. It's the system that costumers are billed according to content downloaded. There are some resources claimed that it has lost popularity in recent years. Volume-based strategies are decreasing the consumers' interest and usage regarding that. It could seem to costumers so complex. These complexities frighten consumers (Stiller *et al.*, 2001). Despite that aspect, some researches argue that the greatest advantage of volume-based pricing for operators is that the uncertainty and risk of consumption remains with the customer, rather than the operator, which can pro rata bills according to the data volumes transferred (Biggs and Kelly, 2006). The pay-per-volume context is evaluated in many kind of aspect within some papers (Kim, 2006; Bouras and Sevasti, 2004; Altman *et al.*, 1999; Levy *et al.*, 2006).

The welfare and demand of consumers are estimated for seeking the effective bandwidth with guaranteed QoS in many sources (Courcoubetis *et al.*, 2000; Ganesh *et al.*, 2007; Naiksatam *et al.*, 2007; Charmantzis *et al.*, 1996; Keon *et al.*, 2003). Apart from these resources, to evaluate the characteristic of telecommunication demand in a competitive market, there are several investigations to make welfare analysis by aggregating demand (Aldebert *et al.*, 2004; Dhebar and Oren, 1986; Ganesh *et al.*, 2007; Altman *et al.*, 1999).

The main other broadband pricing strategies practiced by operators is a time-based pricing strategy, called pay-per-time. According to this point of view, consumers spent online time is charged per unit. There is various kind of service to serve different classes of speed (Jain and Kannan, 2002; Altman *et al.*, 1999; Biggs and Kelly, 2006).

When providers offer connection time and volume based pricing schemes, each consumer has a chance to select one of them. If customers have more time-fixed tasks in their agenda and select connection time based pricing, they can plan (schedule) their task and increase the volume of the task by increasing transmission rate in order to realize better audio/visual quality. Also they incur the same cost since the connection time has not been changed even transmission rate has been increased; therefore, customers can manage costs more effectively. In addition, when customers choose connection time based pricing, their optimal behavior would be utilizing the bandwidth capacity fully (they can transmit another task simultaneously), therefore their behavior can cause network to burst. Offering the pay-per-volume based pricing to the consumer can prevent bursting. The main advantage of pay-per-volume pricing scheme is independently from how long the task takes to manage; the customer would pay for the total volume used. In addition, if customers have more size-fixed tasks in their agenda, they can plan (schedule) their task and manage costs more effectively, because most of the size fixed tasks are divisible, and the customer can schedule a task in different reasonable times. However, the optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only just enough bytes for time-fixed tasks, and this situation can cause quality of the task to decrease, so it can create an opportunity cost. Therefore, if the provider set a reasonable pay-per unit volume price to compensate this opportunity cost, it can encourage some of the potential customers, who normally prefer not to use the network, to enter the market. That's why we examine the following issues in this study: (i) what are the extra benefits to the network provider for providing the volume based pricing scheme? And (ii) does the amount of demand (number of customers enter the market) change?

In the next section, we discuss the model assumptions and present the formulation. Then, we present an optimization model and determine the optimal pricing strategy. Finally, we conclude and discuss future research.

Model Assumptions and Formulation

In this section, we discus the model assumptions and formulate the model. In order to examine the optimal pricing strategies for pay-per-time and pay-per-volume leasing of data networks, we adapt the monopoly pricing model for software dissemination of Gurnani and Karlapalem (2001) to network pricing strategies. Gurnani and Karlapalem (2001) developed a monopoly pricing model to examine the optimal pricing strategies for selling and pay-per-use licensing of packaged software over the Internet. Different from their model, customers can lease the network with different pricing strategies as time and volume based. Also, we assume that the marginal cost of network services is constant, and therefore all prices are assumed to be net of marginal cost. Pricing decisions with multiple network providers or multiple competing network capacity would result in a game-theoretic problem formulation, which is beyond the scope of this paper.

Each consumer has the chance of selecting "pay-per-time" pricing scheme or "pay-per-volume" pricing scheme offered. We define Ω_1 as the "pay-per-time" pricing scheme, Ω_2 as the "pay-per-volume" pricing scheme offered to the customer. Further let q_1 be the demand for Ω_1, q_2 be the demand for Ω_2, and c^t be the cost of using network per unit time and c^x be the unit transmission cost of data carried over network per unit time.

The traditional model used to categorize providers assumes that there is a continuum of consumers indexed by the *reservation prices* $r \in [0,1]$, with r distributed uniformly in the unit interval.

In our model, we consider the case that the consumer has the choices of pay-per-time and pay-per-volume based pricing. Therefore, not only consumers' reservation prices effects their network leasing decision but also the benefit derived from network usage effects their decision. When we talk about network usage, we must consider two dimensions. While using the network, customers occupy some capacity of the network throughout the connected time. Therefore, we adopt a three dimensional classification where each consumer indexed by (x,t,r), where $r \in [0,1]$, as before, represents the reservation price, $x \in [0,1]$ represents the usage level of network capacity for the customers, and $t \in [0,1]$ represents the usage level of the transmission time for customers, and they are also assumed to be uniformly distributed over the unit interval. With this new classification, we can consider consumers with different combinations of high/low reservation prices and high/low usage levels of time and volume as in Figure 1.

Sending more bytes during an audio/video conference that is time-fixed task can increase the quality of the conference, that's why choosing the pay-per-time pricing scheme can be more advantageous for these type of tasks. So, we can assume that the benefit of using the network is higher for the consumers choosing pay-per-time pricing scheme compared to selecting per-per-volume pricing. In addition, the optimal behaviors of the customers who choose the pay-per-time pricing scheme generally encourages them to use all available bandwidth so that they can increase the quality of task by increasing the size of it or they can transmit another task simultaneously, and this situation can cause networks to burst. In order to prevent the bursting of networks, providers generally set pay-per-time unit price higher to discourage some of their customers from choosing pay-per-time pricing scheme. The optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only just enough bytes for time fixed tasks, however this situation can cause quality of the task to decrease, and so it can create an opportunity cost. Therefore the reservation price of the customers for the pay-per-volume pricing strategy, and the benefit of using the network for the customers choosing pay-per-volume pricing scheme would be lower.

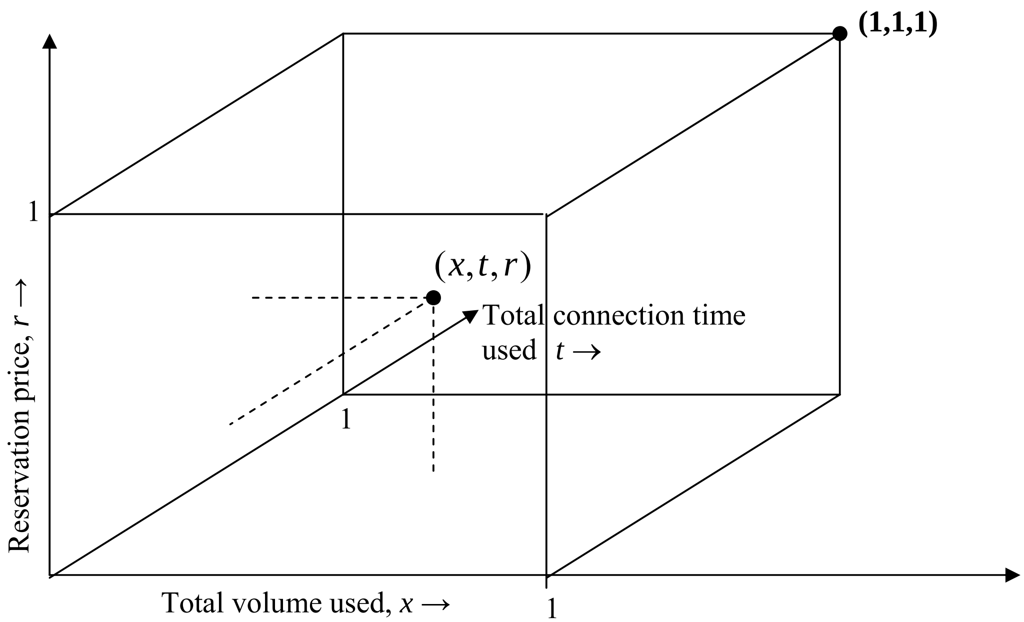

Figure 1. Consumer index

Most data applications such as file transfer, database transactions are size-fixed tasks The optimal behavior of the customer who choose the pay-per-time pricing scheme would push them to perform the tasks as soon as possible. However, the customer who chooses pay-per-volume pricing has task scheduling flexibility, because most of the size fixed tasks are divisible, and the customer can schedule a task in different reasonable times. Therefore, we assume that there is no difference in the benefit of transmitting data for the both pricing scheme. Moreover, the providers will set a lower pay-per-volume unit cost to encourage customers to choose volume based pricing since they generally want to prevent bursting. So, we can easily assume that $c^x x < c^t$.

Consumer surplus function

For the consumer who leases the network capacity with the pay-per-time pricing strategy (Ω_1), the total utility is given by $(r + kx + wt)$ for $x > 0$ and $t > 0$, where r is the reservation price/utility of the consumer, k is the benefit of transmitting unit volume, and w

is the benefit of using the network during the unit time. We also assume that consumers with zero usage do not derive any utility. Note that, the consumers lease the network capacity and use the network for transmitting and completing their tasks; the overall utility depends on the level of total connection time and total volume used. Therefore, the consumer surplus function for using the network with the pay per connection time pricing strategy is given by utility minus the total charge, that is

$$v_1\left(x,t,r\right)=r+kx+wt-c^t \tag{1}$$

Similarly, for the consumer who uses the network capacity with the pay-per-volume strategy (Ω_2), the total utility is given by $(\alpha r+kx+\theta wt)$ for $x>0$, $t>0$, $0\le\alpha\le1$ and $0\le\theta\le1$. The parameter α and θ models the difference in benefit of the network leased through time based pricing against volume based pricing. The smaller the value of α and/or θ, the higher is the difference in the benefit of the billing choices. Since the consumer who selects volume based pricing can not use excess capacity to transmit simultaneous tasks, the utility derived is lower as compared to the consumer who selects connection time based pricing scheme. That is, the parameter α and θ model the consumers' inclination to choose volume based pricing rather than selecting connection time based pricing. In this case, the consumer pays a per-volume price c^x, and therefore surplus is given by

$$v_2\left(x,t,r\right)=\alpha r+kx+\theta wt-c^x x \tag{2}$$

Depending on the prices set by the provider and their on task profile, consumers would select per-time or per-volume pricing schemes or decide not to enter the market. For the network provider the objective is to maximize the profit by suitably setting the prices. In the next section, we develop an optimization problem for the network provider.

Optimization Problem

The objective for the network provider is to maximize profit by selecting the pay-per-time pricing, c^t and the pay-per-volume pricing, c^x. As a function of these prices, consumers would make their optimal selection.

Note that q_1 is the demand generated by consumers who prefer to pay per usage time, and q_2 is demand due to pay-per-volume. Then, the objective function for the vendor is to maximize the (normalized) profit, $\prod$:

$$\underset{(c^t,\,c^x)}{Max}\prod=c^t q_1+c^x q_2 \tag{3}$$

In order to derive the expressions for q_1 and q_2, we consider the following cases. First, we consider the case when $k\ge c^x$ that is, the per-used volume benefit for the consumer exceeds the access price per-volume usage, and so all the customers in the market prefer using the network. In the second case, we assume that $k\le c^x$, that is the some customers neither choose pay-per-time nor pay-per-volume pricing scheme, so prefer not to use networks. The optimal solution for the provider is the maximum of these two cases.

Case 1: $k \geq c^x$

In this case, it is easy to see from (2) that $v_2(.)$ is nonnegative. As a result, the consumer is better off choosing the pay-per-volume pricing scheme as compared to not to entering the market. Therefore, the entire consumer population is covered in this case, that is, $q_1 + q_2 = 1$, see Figure 2.

When $t = 0$, we can not talk about any task and network usage, also when $x = 0$, again we can not talk about any task and network usage. That's why the surface representing the frontier at which the consumer is indifferent between pay-per-time and pay-per-volume pricing schemes can not pass from the points where $x = 0$ or $t = 0$.

Most probably the consumers choosing the pay-per-time pricing scheme do not have low reservation prices. Because of this reason, the surface representing the frontier at which the consumer is indifferent between pay-per-time and pay-per-volume pricing schemes can not pass from the points where $r = 0$. Therefore, the surface connecting the points; $(x_1,1,1)$, $(1,t_1,1)$ and $(1,1,r_1)$ represents the frontier at which the consumer is indifferent between pay-per-time and pay-per-volume pricing schemes, see Figure 2. Using (1) and (2), from the indifference equations, we get:

$$1 + kx_1 + w - c^t = \alpha + kx_1 + \theta w - c^x x_1$$
$$1 + k + wt_1 - c^t = \alpha + k + \theta w t_1 - c^x$$
$$r_1 + k + w - c^t = \alpha r_1 + k + \theta w - c^x$$

From the above three indifference equations, we obtain x_1, t_1, and r_1 as follows.

$$x_1 = \frac{c^t - w(1-\theta) - (1-\alpha)}{c^x} \quad (4)$$

$$t_1 = \frac{(c^t - c^x) - (1-\alpha)}{w(1-\theta)} \quad (5)$$

$$r_1 = \frac{(c^t - c^x) - w(1-\theta)}{(1-\alpha)} \quad (6)$$

Then, from the volume of the triangular pyramid shown in Figure 2, we get:

$$q_1 = \frac{1}{2}\frac{1}{3}(1-x_1)(1-t_1)(1-r_1)$$

$$q_1 = \frac{[c^x - c^t + w(1-\theta) + (1-\alpha)]^3}{6w(1-\theta)(1-\alpha)c^x} \quad (7)$$

and $q_2 = 1 - q_1$ (8)

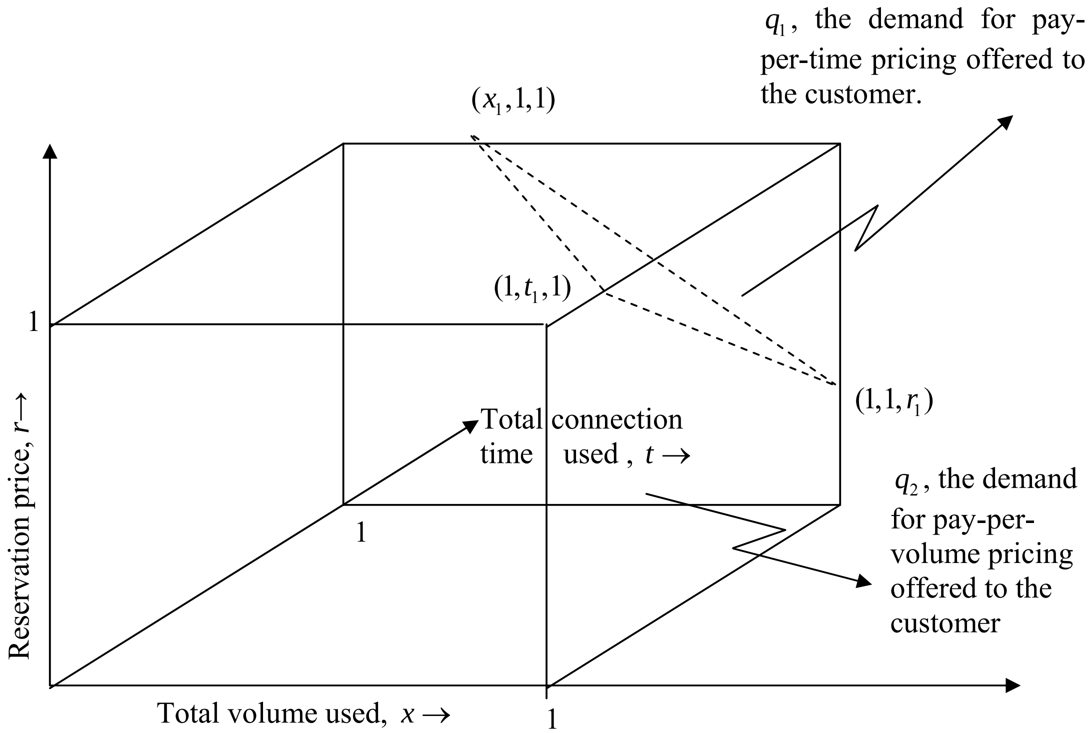

Figure 2. Consumer selections for Case 1

Also, since $0 \le x_1 \le 1$, $0 \le t_1 \le 1$ and $0 \le r_1 \le 1$, we get:

$$c^t \ge w(1-\theta)+(1-\alpha) \tag{9}$$

$$c^t \le w(1-\theta)+(1-\alpha)+c^x \tag{10}$$

$$c^t \ge (1-\alpha)+c^x \tag{11}$$

$$c^t \ge w(1-\theta)+c^x \tag{12}$$

On substituting q_1 and q_2 from (7) and (8), respectively into (3), the optimization problem is

$$\underset{(c^t,c^x)}{Max} \prod = \frac{(c^t-c^x-1+\alpha)^3}{6c^x(1-\alpha)} + \frac{6c^t c^x(1-\alpha)-(c^t-c^x-1+\alpha)^3}{6c^t(1-\alpha)} \tag{13}$$

Subject to (9) – (12)

Case 2: $k \le c^x$

In this case, we note that some consumers (with low reservation price r and high usages k and w) who choose the pay-per-volume pricing scheme could have negative surplus and would therefore not enter the market. Therefore for this case, we could have $q_1 + q_2 < 1$, see Figure 3.

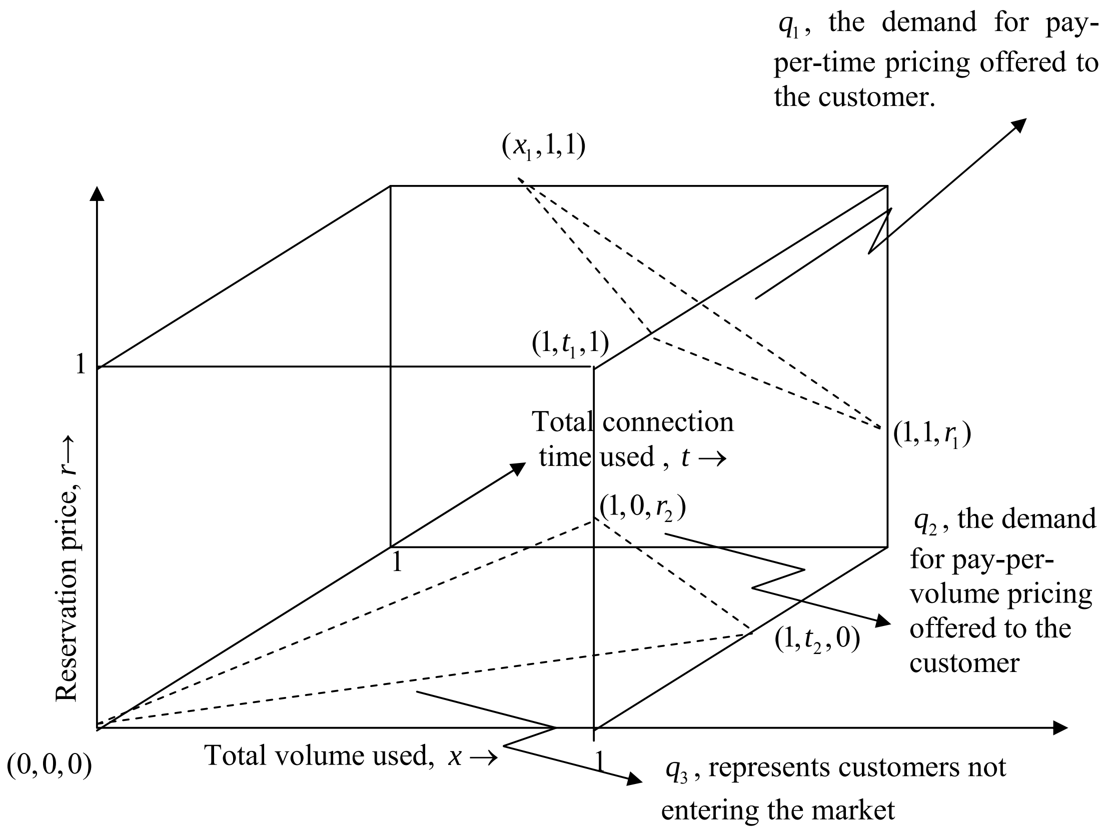

Figure 3. Consumer selections for Case 2

The surface connecting $(0,0,0)$, $(1,t_2,0)$ and $(1,0,r_2)$ represents the frontier at which the consumer is indifferent between choosing pay-per-volume pricing scheme and not entering the market, see Figure 3. Then from the indifference equation, we get:

$$k + \theta w t_2 - c^x = 0$$
$$\alpha r_2 + k - c^x = 0$$

From the above two indifference equations, we obtain t_2, and r_2 as follows.

$$t_2 = \frac{c^x - k}{\theta w} \tag{14}$$

$$r_2 = \frac{c^x - k}{\alpha} \tag{15}$$

Then, from the volume of the triangular pyramid shown in Figure 3, we get:

$$q_3 = \frac{1}{3}\frac{1}{2} r_2 t_2$$

$$q_3 = \frac{(c^x - k)^2}{6\alpha\theta w} \tag{16}$$

Also, q_1 is the same as defined in (7), and therefore

$$q_2 = 1 - q_1 - q_3 \tag{17}$$

Since $0 \le t_2 \le 1$ and $0 \le r_2 \le 1$, the following conditions must hold:

$$c^x \ge k \tag{18}$$

$$c^x \le k + \theta w \tag{19}$$

$$c^x \le k + \alpha \tag{20}$$

On substituting (7) and (17) into (3), the optimization problem is

$$\underset{(c^t, c^x)}{Max} \quad \Pi = c^t \frac{[c^x - c^t + w(1-\theta) + (1-\alpha)]^3}{6w(1-\theta)(1-\alpha)c^x} + \frac{36\alpha w^2 \theta(1-\theta)(1-\alpha)c^x - 6\alpha\theta w[c^x - c^t + w(1-\theta) + (1-\alpha)]^3 - 6w(1-\theta)(1-\alpha)c^x(c^x - k)^2}{36\alpha w^2 \theta(1-\theta)(1-\alpha)}$$

Subject to (18) – (20)

Conclusions

Pay-per-volume pricing scheme is the next generation strategy to prevent bursting of networks. Such a pricing strategy facilitates leasing the excess capacity to other customers, and is particularly useful for those customers having fewer tasks with low total volume or having generally size fixed tasks. Thus the provider can increase profit by setting higher price by offering pay-per-time pricing scheme to those customers, who really use data networks too often, having tasks with more volume or having more time fixed tasks. And the provider can charge other customers on pay-per-volume terms when they transmit data with large volume occasionally. The main focus of this paper is to show that pay-per-volume is an additional option available for the network provider. This is illustrated by the optimization models maximizing provider's profit when pay-per-volume pricing strategy exists, as against when it does not exist. In conclusion, this paper shows that (i) the pay-per-volume choice can increase the profitability of the network provider, and (ii) when both strategies are used, the optimal pricing strategies to maximize the provider's profit are presented.

The optimization models in this paper assumed linear pay-per-time and pay-per-volume pricing schemes. As future research, we plan to extend it by adding another pricing scheme, combination of both pay-per time and volume. Also, we plan to make numerical studies to compare pricing schemes and perform sensitivity analysis.

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