Entanglement of hard-core bose gas in degenerate levels under local noise

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ABSTRACT

Quantum entanglement properties of the pseudo-spin representation of the BCS model is investigated. In case of degenerate energy levels, where wave functions take a particularly simple form, spontaneous breaking of exchange symmetry under local noise is studied. Even if the Hamiltonian has the same symmetry, it is shown that there is a non-zero probability to end up with a non-symmetric final state. For small systems, total probability for symmetry breaking is found to be inversely proportional to the system size.

Fermions undergoing BCS type pairing interaction can be treated as hard-core bosons in real or momentum space depending on whether they interact strongly or weakly. In both cases, lattice points (or energy levels) are occupied by pairs or else they are empty. Therefore, the system can be described by pseudo-spin variables [1]. Quantum entanglement and superconducting order parameter of such systems have been found to be closely related [2]. In case of degenerate energy levels, wave functions take a particularly simple form and hence all possible eigenstates can be written down easily [3]. Such a degeneracy can for example be due to parabolic energy bands in mesoscopic and nanoscopic superconducting particles [4].

Recently, it has been shown that exchange symmetry of some entangled states can be spontaneously broken under local noise even if the Hamiltonian has the same symmetry [5]. Considering the symmetry of the initial state and the Hamiltonian, this is a very interesting result. Since BCS model of superconductivity can be written in terms of pseudo-spins or (in the language of quantum information theory) in terms of qubits, one might ask consequences of this kind of symmetry breaking on superconducting state.

Our starting point is a single, d-fold degenerate energy level with N spin 1/2 fermions. Assuming that fermions in the same state are paired, the model Hamiltonian becomes

$$H_0 = -g \sum_{n,n'=1}^{d} a^\dagger_{n'\uparrow} a^\dagger_{n'\downarrow} a_{n\downarrow} a_{n\uparrow}, \tag{1}$$

where $a^\dagger_{n\sigma}$ ($a_{n\sigma}$) creates (annihilates) a fermion in state n with spin σ. Introducing pseudo-spins $\vec{s}_n$ defined by $s_{n+} = a^\dagger_{n\uparrow} a^\dagger_{n\downarrow}$ and $s_{nz} = (1/2)\left(a^\dagger_{n\uparrow} a_{n\uparrow} + a^\dagger_{n\downarrow} a_{n\downarrow} - 1\right)$, we can rewrite the Hamiltonian as

$$H_0 = -gS_+S_-, \tag{2}$$

where $\vec{S} = \sum_n \vec{s}_n$ is the total spin and $S_\pm$ are the corresponding raising and lowering operators. Energy eigenvalues are given by

$$E(s) = -(g/4)(N-s)(2d-s-N+2), \tag{3}$$

where seniority number $s = d - 2S$ can take values $s = 0, 2, 4, \ldots, N$ when the number of fermions N is even. The ground state, which is invariant under Cooper pair exchange, has the maximum total-S value $d/2$. Degeneracy of each state can be found easily as [6]

$$\Omega(s) = \sum_{i=0}^{s/2} \left[\binom{d-s+2i}{i} - \binom{d-s+2i}{i-1} \right] \binom{d}{s-2i} 2^{s-2i}. \tag{4}$$

Now, let us consider an exchange symmetric local external noise described by, for $d = 2$,

$$H_1(t) = w_1(t)(s_{1z} \otimes I) + w_2(t)(I \otimes s_{2z}), \tag{5}$$

where $w_1(t)$ and $w_2(t)$ are stochastic noise fields that lead to statistically independent Markov processes satisfying

$$\langle w_n(t) \rangle = 0, \tag{6}$$

$$\langle w_n(t) w_n(t') \rangle = a_n \delta(t - t'). \tag{7}$$

In case of real space pairing, such a noise might originate from an external disturbance localized in space in a region of Cooper pair size. Generalization to arbitrary d is obvious. Let the system be in state $\rho(t=0)$, say the ground state. The time evolution of the system's density matrix can be obtained as

$$\rho(t) = \langle U(t) \rho(0) U^\dagger(t) \rangle, \tag{8}$$

where ensemble averages are evaluated over the two noise fields $w_1(t)$ and $w_2(t)$ and the time evolution operator, $U(t)$, is given by

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$$U(t) = \exp\left[-i\int_0^t dt' H(t')\right]. \tag{9}$$

In Ref. [5], using Kraus representation [7–9]

$$\rho(t) = \sum_{\mu=1}^{M} K_\mu(t)\rho(0)K_\mu^\dagger(t), \tag{10}$$

general time evolution has been evaluated and it has been shown that not all possible final states

$$\frac{K_\mu(t)\rho(0)K_\mu^\dagger(t)}{tr(K_\mu(t)\rho(0)K_\mu^\dagger(t))} \tag{11}$$

have exchange symmetry. The same result has been reproduced for quantum noise using the decoherence Hamiltonian [10]

$$H_1 = s_{1z}\sum_{k=1}^{N_1} \hbar\omega_{1k}\sigma_{1kz} + s_{2z}\sum_{k=1}^{N_2} \hbar\omega_{2k}\sigma_{2kz}, \tag{12}$$

where z-component operators s_{1z} and s_{2z} are coupled to bath spins represented by σ_{nkz}. Here $n = 1, 2$ labels the baths and $k = 1, 2, 3, \ldots, N_n$ labels the individual spins in the baths. For exchange symmetric case parameters of the two baths are taken to be identical.

In conclusion, exchange symmetry of a state, undergoing an external noise having the same symmetry, can be spontaneously broken as a result of decoherence due to local interactions. A natural question is the maximum probability of finding a symmetric possible final state as the system evolves in time. Even though we don't have a general analytical solution yet, our calculations show that total probability of conservation of exchange symmetry is $1/d$ at least for small systems. Evaluation of probabilities for possible broken symmetry states for higher d values will shed light on stability of superconducting state.

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