# OPTIMAL PRICING STRATEGIES FOR CAPACITY LEASING
## BASED ON TIME AND VOLUME USAGE IN TELECOMMUNICATION NETWORKS

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OPTIMAL PRICING STRATEGIES FOR CAPACITY LEASING BASED ON TIME AND VOLUME USAGE IN TELECOMMUNICATION NETWORKS

ABSTRACT

In this study, we use a monopoly pricing model to examine the optimal pricing strategies for “pay-per-time”, “pay-per-volume” and “pay-per both time and volume” based leasing of data networks. Traditionally, network capacity distribution includes short/long term bandwidth and/or usage time leasing. Each consumer has a choice to select volume based, connection-time based or both volume and connection-time based pricing. When customers choose connection-time based pricing, their optimal behavior would be utilizing the bandwidth capacity fully, which can cause network to burst. Also, offering the pay-per-volume scheme to the consumer provides the advantage of leasing the excess capacity to other potential customers serving as network providers. However, volume-based strategies are decreasing the consumers’ interest and usage, because the optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only enough bytes for time-fixed tasks (for real time applications), causing quality of the task to decrease, which in turn creating an opportunity cost. Choosing pay-per time and volume hybridized pricing scheme allows customers to take advantages of both pricing strategies while decreasing (minimizing) the disadvantages of each, because consumers generally have both time-fixed and size-fixed task such as batch data transactions. However, such a complex pricing policy may confuse and frighten consumers. Therefore, in this study we examined the following two issues: (i) what (if any) are the benefits to the network provider of providing the time and volume hybridized pricing scheme? and (ii) would this offering schema make an impact on the market size? The main contribution of this study is to show that pay-per both time and volume pricing is a viable and often preferable alternative to the only time and/or only volume-based offerings for a large number of customers, and that judicious use of such pricing policy is profitable to the network provider.

Keywords: telecommunication networks, time based /volume based / time and volume hybridized network pricing, optimization
1. Introduction and Related Literature

Customers can use information networks for a variety of reasons including video conferencing, voice over TCP/IP and data applications, which we call collectively as tasks. In general, two types of tasks are performed using data networks: time-fixed and size-fixed tasks (Kasap et al., 2007). A task is time-fixed (real-time, size compressible) if its size can be changed without disrupting its completion but the transmission time cannot be compressed or extended. Most audio/video tasks are time-fixed. A task is size-fixed (time compressible) if all bits have to be transmitted but the duration is not fixed. Most data applications such as file transfer, database transactions are of this category. All types of applications use some level of network bandwidth during an arbitrary period of time. Traditionally, network capacity distribution includes short/long term bandwidth and/or usage time leasing.

Earlier researches show that the connection billing model has a great effect on how people consume online data and how satisfied they are with the connection and online services. Today there are various billing models for quality-differentiated Internet access as a function of bandwidth, traffic volume, applications, and pricing structure (which is based either on time, volume, a combination of both, or a flat-rate), bandwidth efficiency optimizations guaranteed QoS, uniform pricing, multipart pricing and nonlinear pricing on total call time or the number of packets exchanged (Masuda and Whang, 2006; Kasap et al., 2007). However, there are several other pricing strategies (Keon and Anandalingam, 2005) that are examined by a large number of sources including “Paris-metro pricing (similarly time of day pricing)”, “priority pricing”, “smart-market pricing”, “top-percentile pricing” (Levy et al, 2006). Some parts of these pricing methodologies are usage-based, and other part of that are volume-based strategies.

In recent years, the description of network service provider’s has been changing rapidly. In order to manage traffic congestions and scarce resources in the midst of changing consumer preferences for variety of services, network providers have been experimenting with various billing strategies, which vary significantly among different countries. Volume-dependent pricing schemas have been used by some providers in the past, wherein the customers are billed based on the volume of the content downloaded or uploaded. There have been some sources claiming that this pricing strategy has lost its popularity in recent years. These volume-based strategies have been decreasing the consumers’ interest and usage due to the fact that they may seem too complex to costumers. These complexities may confuse and frighten consumers (Stiller et al., 2001). Some researches argue that the greatest advantage of volume-based pricing for operators is that the uncertainty and risk of consumption remains with the customer, rather
than the operator, which can pro rata bills based on the data volumes transferred (Biggs and Kelly, 2006). The pay-per-volume context is evaluated in many respects in recent research studies (Kim, 2006; Bouras and Sevasti, 2004; Altman et al., 1999; Levy et al., 2006; Anderson et al., 2006).

The other main broadband pricing strategies practiced by operators is a time-based pricing strategy, called pay-per-time. According to this point of view, consumers are charged based on per unit of time they spend online. There are variety of service types to serve different classes of customers needed variety of connection speed (Jain and Kannan, 2002; Altman et al., 1999; Biggs and Kelly, 2006).

When providers offer connection time based, volume based and both connection time and volume based pricing schemes, each consumer is allowed to select only one of them. If customers have more time-fixed tasks in their agenda, they would select connection time based pricing. Hence, they can plan/schedule their task and increase the volume of the task by increasing transmission rate in order to realize better audio/visual quality. Also they incur the same cost since the connection time has not been changed even transmission rate has been increased; therefore, customers can manage costs more effectively. Additionally, when customers choose connection time based pricing, their optimal behavior would be utilizing the bandwidth capacity fully (they can transmit another task simultaneously such as they can perform bulk data/file transfer while carrying out video conference), therefore their behavior can cause network to burst. Offering the pay-per-volume based pricing to the consumer can prevent bursting. The main advantage of pay-per-volume pricing scheme is independently from how long the task takes to manage; the customer would pay for the total volume used. In addition, if customers have more size-fixed tasks in their agenda, they can plan/schedule their task and manage costs more effectively, because most of the size fixed tasks are divisible, and the customer can schedule a task in different reasonable times.

However, the optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only just enough bytes for time-fixed tasks, and this situation can cause quality of the task to decrease, so it can create an opportunity cost. Choosing pay-per both time and volume based pricing scheme help customer to take advantages of only pay-per-time and only pay-per-volume pricing strategies while decreasing (minimizing) the disadvantages of each, because consumers generally have both time-fixed and size-fixed task. That’s why in this study we examine the following issues: (i) what are the extra benefits to the network provider for providing the time and volume hybridized pricing scheme? and (ii) would this offering schema make an impact on the amount of demand (number of customers enter the market)?

The rest of the paper is organized as follows: in the next section, we discuss the model assumptions and present the formulation. Then, we present an optimization model and determine the optimal pricing
strategy. In Section three the numerical study and sensitivity analysis of various system parameters are provided. A comparison of the optimal pricing policy without the hybrid pricing scheme is given in Section four. Finally, we conclude and discuss future research ideas in Section five.

2. Model Assumptions and Formulation

In this section, we discuss the model assumptions and formulate the optimization models for different pricing scenarios. In order to examine the optimal pricing strategies for pay-per-time, both pay-per-time and volume, and pay-per-volume leasing of data networks, we adapt the monopoly pricing model of Gurnani and Karlapalem (2001) to network pricing strategies. Gurnani and Karlapalem (2001) developed a monopoly pricing model to examine the optimal pricing strategies for selling and pay-per-use licensing of packaged software over the Internet. Unlike their model, our model has three different situations, and in all situation customers can lease the network but in different pricing strategies. Also, we assume that the marginal cost of network services is constant, and therefore all prices are assumed to be net of marginal cost. Pricing decisions with multiple network providers or multiple competing network capacity would result in a game-theoretic problem formulation, which is beyond the scope of this paper.

Each consumer has the chance of selecting one of three pricing schemas: “pay-per-time” pricing scheme, \( \Omega_1 \); “pay-per both time and volume” pricing scheme (we call it hybrid), \( \Omega_2 \); or “pay-per-volume” based pricing scheme, \( \Omega_3 \). Further, the demand for \( \Omega_1 \), \( \Omega_2 \) and \( \Omega_3 \) are given as \( q_1 \), \( q_2 \) and \( q_3 \) respectively. Let \( c' \) be the unit cost of using the network resources based on total transmission time, \( c^\times \) be the unit transmission cost of data carried over the network per byte per unit time, and \( \lambda(c' + c^\times) \) be the unit cost of using the network resources based on both total transmission time and total volume used, where \( 0 \leq \lambda \leq 1 \).

The traditional model used to categorize consumers assumes that there is a continuum of consumers indexed by the reservation prices \( r \in [0,1] \), with \( r \) distributed uniformly in the unit interval. The reservation price reflects the maximum amount that a consumer is willing to pay in exchange for a given service (Salop 1979; Dewan et al., 1999; Gurnani and Karlapalem, 2001).

In our model, we consider the case where the consumer has the choices of pay-per-time, pay-per both time and volume and pay-per-volume based pricing. Therefore, not only consumers’ reservation prices effects their network leasing decision but also the benefit derived from network usage effects their decision. When we talk about network usage, we consider two primary dimensions: time and volume.
since leasing decision contains contract duration and maximum bandwidth. While using the network, customers occupy some capacity (bandwidth) of the network throughout the connected time. Therefore, we adopt a three dimensional classification schema where each consumer indexed by \((x,t,r)\), where \(r \in [0,1]\), as before, represents the reservation price, \(x \in [0,1]\) represents the bandwidth usage (or volume used) of network capacity for the customers, and \(t \in [0,1]\) represents the effective usage of the transmission time for customers. These measures are assumed to be uniformly distributed over the unit time interval. With this new classification, we can consider consumers with different combinations of high/low reservation prices and usage levels of time and volume as depicted in Figure 1.

![Diagram](image)

**Figure 1:** Consumer indexing as per volume, time and reservation price

Sending more bytes during an audio/video conference can increase the quality of the conference, that’s why choosing the pay-per-time pricing scheme can be more advantageous for these type of tasks. So, we can assume that the benefit of using the network is higher for the consumers choosing pay-per-time pricing scheme compared to selecting per-per-volume pricing. In addition, the optimal behaviors of the customers who choose the pay-per-time pricing scheme generally encourages them to use all available bandwidth so that they can increase the quality of task by increasing the size of it or they can transmit another task simultaneously, and this situation can cause networks to burst. In order to prevent the bursting of networks, providers generally set pay-per-time unit price higher to discourage some of their
customers from choosing pay-per-time pricing scheme. The optimal behaviors of the customers who choose the pay-per-volume pricing scheme generally encourages them to send only just enough bytes for time fixed tasks, however this situation can cause quality of the task to decrease, and so it can create an opportunity cost. That’s why the reservation price of the customers for the pay-per-volume pricing strategy would be lower. Since sending just enough bytes for time fixed tasks can create an opportunity cost, customer would prefer to increase the transmission rate (send more byte) in order to increase the quality of the task. In order to attract customers having higher reservation price than customers preferring pay-per-volume pricing strategy, hybrid pricing scheme is being offered. By choosing hybrid, in other words pay-per both time and volume pricing scheme, customers can complete time-fixed tasks with less cost compared to pay per time only pricing scheme since hybrid pricing is cheaper. However, the quality of tasks is less since customer cannot increase transmission rate too much because of pay per volume part of hybrid scheme. Therefore, the quality of task via hybrid pricing would be higher than the one with pay per volume scheme while it would be lower than the one with pay per time scheme. Hence, customers, whose reservation price is higher than those preferring pay per volume scheme and lower than those preferring pay per time scheme, would prefer hybrid pricing.

Most data applications such as file transfer, database transactions are size-fixed tasks. The optimal behavior of the customer who choose the pay-per-time pricing scheme would push them to perform the task as soon as possible. However, the customer who chooses pay-per-volume pricing has task scheduling flexibility, because most of the size fixed task are divisible, and the customer can schedule a task in different reasonable times. Moreover, the providers will set a lower pay-per-volume unit cost to encourage customers to choose volume based pricing since they generally want to prevent bursting. So, we can easily assume that $c^x x < c'$. Choosing pay-per both time and volume based pricing scheme may help customer to take the advantages of only pay-per-time and only pay-per-volume pricing strategies while decreasing the disadvantages of them, because consumers generally have both time-fixed and size-fixed task. So, we can easily assume that $c^x x \leq \lambda (c' + c^x x) \leq c'$, where $0 \leq \lambda \leq 1$.

### 2.1. Consumer surplus function

For the consumer who leases the network capacity with the pay-per-time pricing strategy ($\Omega_t$), the total utility is given by $(r+kx+wt)$ for $x > 0$ and $t > 0$, where $r$ is the reservation price/utility of the consumer, $k$ is the benefit of transmitting unit volume, and $w$ is the benefit of using the network during
the unit time. We also assume that consumers with zero usage do not derive any utility. Note that, the consumer leases the network capacity and uses the network for transmitting and completing his tasks; the overall utility depends on the level of total connection time and total volume used. Therefore, the consumer surplus function for using the network with the pay per connection time pricing strategy is given by utility minus the total charge, that is

\[ v_1(x,t,r) = r + kx + wt - c' \]  \hspace{1cm} (1)

Similarly, for the consumer who uses the network capacity with the hybrid pricing scheme \((\Omega_2)\), the total utility is given by \((\beta r + kx + \theta wt)\) for \(x > 0, t > 0, 0 \leq \alpha \leq 1\) and \(0 \leq \theta \leq 1\). The reservation price of pay per volume option would be \(\beta r\), where \(0 \leq \beta \leq 1\). The parameter \(\beta\) models the difference in benefit of the network leased through time based pricing against hybrid pricing scheme. The smaller the value of \(\beta\), the higher is the difference in the benefit of the billing choices. Since the consumer who selects the hybrid pricing scheme may not use excess capacity to transmit simultaneous tasks, the utility derived is lower as compared to the consumer who selects connection time based pricing scheme. That is, the parameter \(\beta\) models the consumers’ inclination to choose hybrid pricing scheme rather than selecting connection time based pricing. In this case, the consumer pays both per-time and per-volume price \(\lambda(c' + c^s)\), and therefore surplus is given by

\[ v_2(x,t,r) = \beta r + kx + \theta wt - \lambda(c' + c^s x) \]  \hspace{1cm} (2)

For the consumer who uses the network capacity with the pay-per-volume strategy \((\Omega_3)\), the total utility is given by \((\alpha r + kx + \gamma wt)\) for \(x > 0, t > 0, 0 \leq \alpha \leq 1\) and \(0 \leq \gamma \leq 1\). The reservation price of pay per volume option would be \(\alpha r\), where \(0 \leq \alpha \leq 1\). The parameter \(\alpha\) models the difference in benefit of the network leased through time based pricing and volume based pricing against hybrid pricing scheme. The smaller the value of \(\alpha\), the higher is the difference in the benefit of the billing choices. Since the consumer who selects volume based pricing cannot use excess capacity to transmit simultaneous tasks, the utility derived is lower as compared to the consumer who selects connection time based or hybrid pricing schemes. That is, the parameter \(\alpha\) models the consumers’ inclination to choose volume based pricing rather than selecting the other two pricing schemes. In this case, the consumer pays a per-volume price \(c^s\), and therefore surplus is given by
Depending on the prices set by the provider and their own task profile, consumers would pay per-time price, hybrid price or pay per-volume price or decide not to enter the market. For the network provider the objective is to maximize the revenue by suitably choosing the prices. In the next section, we develop an optimization problem formulation for the network provider.

2.2. Optimization problem formulation

The objective for the network provider is to maximize revenue by selecting the pay-per-time pricing, \( c' \), the pay-per-hybrid pricing, \( \lambda(c' + c^x) \), the pay-per-volume pricing, \( c^x \). As a function of these prices, consumers would make their own optimal selection. Note that \( q_1 \) is the demand generated by consumers who prefer to pay per usage time, \( q_2 \) is demand due to pay-per-hybrid pricing scheme, \( q_3 \) demand due to pay-per-volume. Then, the objective function for the vendor is to maximize the (normalized) revenue, \( \prod \):

\[
\text{Max } c'q_1 + \lambda q_2 + c^x(q_3 + \lambda q_2)
\]

In order to derive the expressions for \( q_1 \), \( q_2 \) and \( q_3 \), we consider the following cases. First, we consider the case when \( k \geq c^x \) that is, the per-used volume, the per-hybrid price and the per-used time benefits for the consumer exceeds the access price per-volume usage, so all the customers in the market prefer using the network. In the second case, we assume that \( k \leq c^x \), that is some customers neither choose pay-per-time, pay-per-hybrid price nor pay-per-volume pricing scheme, so prefer not to use networks. The optimal solution for the provider is the maximum of these two cases.

Case 1: \( k \geq c^x \) - All consumers prefer entering the market.

In this case, it is easy to see from (3) that \( v_3(,) \) is nonnegative. As a result, the consumer is better off choosing the pay-per-volume pricing scheme as compared to not to entering the market. Therefore, the entire consumer population is covered in this case, that is, \( q_1 + q_2 + q_3 = 1 \), see Figure 2.
Most probably the consumers choosing the pay-per-time pricing scheme do not have low reservation prices. Because of this reason, the surface representing the frontier at which the consumer is indifferent between pay-per-time and pay-per-hybrid pricing schemes cannot pass from the points where \( r = 0 \). Since it is expected that the optimum behavior of pay-per-time customers will be transmitting as much task as they can in unit time with fully utilization of network, it generally results in network bursting. In order to prevent network bursting, we minimize the number of pay-per-time pricing scheme customers while maximizing the revenue of the network provider. Therefore, in Figure 2, the region representing the demand in consumer index cube for pay-per-time pricing scheme can be a triangular pyramid. Hence, in our model the volume of pay-per-time customers cannot exceed \( \frac{1}{6} \) of the total population, from eq. (8). And also, we recommend pay-per-hybrid pricing scheme to maintain revenue of the provider. As well, the frontier at which consumer is indifferent between pay-per-time and pay-per-hybrid pricing scheme should be a surface passing from points on the edges at the upper part of the unit cube in order to minimize the number of customers selecting pay-per-time pricing scheme. Consequently, the surface connecting the points are \((x, 1, 1), (1, t, 1)\) and \((1, 1, r)\). Using (1) and (2), from the indifference equations, we get:
\[1 + kx + w - c' = \beta + kx + \theta w - \lambda (c' + c^x)\]

\[1 + k + wt - c' = \beta + k + \theta wt - \lambda (c' + c^x)\]

\[r_i + k + w - c' = \beta r_i + k + \theta w - \lambda (c' + c^x)\]

From the above three indifference equations, we obtain \(x_i\), \(t_i\), and \(r_i\) as follows.

\[x_i = \frac{c' (1 - \lambda) - w (1 - \theta) - (1 - \beta)}{\lambda c^x} \quad (5)\]

\[t_i = \frac{c' (1 - \lambda) - \lambda c^x - (1 - \beta)}{w (1 - \theta)} \quad (6)\]

\[r_i = \frac{c' (1 - \lambda) - \lambda c^x - w (1 - \theta)}{(1 - \beta)} \quad (7)\]

Then, from the volume of the triangular pyramid shown in Figure 2, we get:

\[q_i = \frac{1}{2} \frac{1}{3} (1 - x_i) (1 - t_i) (1 - r_i) \quad (8)\]

\[q_i = \frac{[-\lambda c^x - c' (1 - \lambda) + w (1 - \theta) + (1 - \beta)]^3}{6w \lambda (1 - \theta) (1 - \beta) c^x} \quad (9)\]

The providers should set a lower pay-per-volume unit cost to encourage customers to choose volume based pricing, in order to prevent bursting of their networks. Optimal behaviors of the pay-per-volume pricing scheme’s customers generally encourage them to send only just enough bytes for time-fixed tasks and to take an opportunity cost. Therefore, in order to cover their opportunity cost consumers choosing pay-per-volume pricing schemes would not be intending to pay a premium price, so most probably they have low reservation prices. Thus, the consumers choosing pay-per-volume pricing scheme would be in the lower part of the unit cube in Figure 2. Customers prefer pay-per-hybrid pricing scheme under the conditions as denoted in equation (10)

\[v_2(x, t, r) - v_3(x, t, r) > 0\]
\[(\beta - \alpha) r + (\theta - \gamma) w t + \left(c^x x - \lambda \left(c^t + c^x x\right)\right) > 0 \quad \text{(10)}\]

We assume that \(c^x \leq \lambda (c^t + c^x x) \leq c^t\), where \(0 \leq \lambda \leq 1\), so the frontier surface between pay-per-volume and pay-per-hybrid pricing schemes cannot cross from bottom \(x\) edges of the cube, where \(t\) and \(r\) equal to zero since the above equation is always nonnegative due to assumption. Therefore there is no indifference point on the bottom edge of the cube where \(t\) and \(r\) equal to zero. The indifference point on \(x\) axis will be an imaginary point \((x_2, 0, 0)\) outside of the cube.

As denoted in Figure 2, the surface connecting the points: \((0, t_2, 0)\), \((0, 0, r_2)\), \((1, t_3, 0)\) and \((1, 0, r_3)\), represents the frontier at which the consumer is indifferent between pay-per-volume and pay-per-hybrid pricing schemes. Using (2) and (3), from the indifference equations, we get:

\[\theta w t_2 - \lambda c^t = \gamma w t_2\]
\[\beta r_2 - \lambda c^t = \alpha r_2\]
\[\theta w t_3 - \lambda c^t - \lambda c^x = \gamma w t_3 - c^x\]
\[\beta r_3 - \lambda c^t - \lambda c^x = \alpha r_3 - c^x\]

From the above four indifference equations, we obtain \(t_2, r_2, t_3,\) and \(r_3\) as follows.

\[t_2 = \frac{\lambda c^t}{(\theta - \gamma) w}\quad \text{(11)}\]
\[r_2 = \frac{\lambda c^t}{(\beta - \alpha)}\quad \text{(12)}\]
\[t_3 = \frac{\lambda c^t - (1 - \lambda) c^x}{(\theta - \gamma) w}\quad \text{(13)}\]
\[r_3 = \frac{\lambda c^t - (1 - \lambda) c^x}{(\beta - \alpha)}\quad \text{(14)}\]
Then, from the volume of the triangular pyramid shown in Figure 2, we get

\[
\frac{|x_2| - 1}{|x_2|} = \frac{t_z}{t_2} = \frac{r_3}{r_2}
\]

\[
x_2 = \frac{\lambda c'}{(1 - \lambda)c^x}
\]

(15)

\[
q_3 = 1 - \frac{1}{2} \frac{x_2 t_2 r_2}{2} - \frac{1}{2} (x_2 - 1) t_3 r_3
\]

and

(17)

\[
q_3 = \frac{\left[ (\lambda c')^3 - (\lambda c' - (1 - \lambda)c^x)^3 \right]}{6w(\beta - \alpha)(\theta - \gamma)(1 - \lambda)c^x}
\]

(16)

We assume \( k \geq c^x \) Also, since \( 0 \leq x_1 \leq 1, \ 0 \leq t_1 \leq 1, \ 0 \leq r_1 \leq 1, \ 0 \leq t_3 \leq t_2 \leq 1 \) and \( 0 \leq r_3 \leq r_2 \leq 1 \) we get:

\[
c' (1 - \lambda) \geq w(1 - \theta) + (1 - \beta)
\]

(18)

\[
c' (1 - \lambda) \leq w(1 - \theta) + (1 - \beta) + \lambda c^x
\]

(19)

\[
c' (1 - \lambda) \geq (1 - \beta) + \lambda c^x
\]

(20)

\[
c' (1 - \lambda) \geq w(1 - \theta) + \lambda c^x
\]

(21)

\[
\lambda c' \geq (1 - \lambda)c^x
\]

(22)

\[
(1 - \lambda)c^x \geq 0
\]

(23)

\[
\lambda c' \leq (\theta - \gamma)w
\]

(24)

\[
\lambda c' \leq (\beta - \alpha)
\]

(25)

\[
k \geq c^x
\]

(26)
On substituting $q_1$, $q_2$ and $q_3$ from (9), (16) and (17) respectively into (4), the optimization problem is

$$
\begin{align*}
\text{Max}_{(c',c^\lambda)} & \prod_i c_i' \\
& = c' \left[ \lambda + \frac{(1-\lambda)\left[ \lambda c^\gamma - c'(1-\lambda) + w(1-\theta) + (1-\beta) \right]}{6w\lambda(1-\theta)(1-\beta)c^\gamma} \right] \\
& - \lambda \left[ \lambda c^\gamma - c'(1-\lambda) + w(1-\theta) + (1-\beta) \right] \left[ \frac{6w(1-\theta) \left[ \lambda c^\gamma - c'(1-\lambda) + w(1-\theta) + (1-\beta) \right]}{6w \lambda(1-\theta)(1-\beta)c^\gamma} \right] \\
& + \lambda \left[ \lambda c^\gamma - c'(1-\lambda) + w(1-\theta) + (1-\beta) \right] \left[ \frac{6w(1-\theta) \left[ \lambda c^\gamma - c'(1-\lambda) + w(1-\theta) + (1-\beta) \right]}{6w \lambda(1-\theta)(1-\beta)c^\gamma} \right]
\end{align*}
$$

Subject to (18) – (26).

**Case 2: $k \leq c^\gamma$ - Some consumers prefer not entering the market.**

In this case, we note that some consumers (with low reservation price $r$ and high usages $k$ and $w$) who choose the pay-per-volume pricing scheme could have negative surplus and would therefore not enter the market. Therefore for this case, we could have $q_1 + q_2 + q_3 < 1$, see Figure 3.

Under the conditions as denoted in equation (27) some customers prefer not entering to the market, because their reservation prices and their benefit of using network are not high enough to cover their bills.

$$
v_3(x,t,r) < 0
\tag{27}
$$

$$
ar + kx + \gamma wt - c^\gamma x < 0
$$

From the above equation we realize that because of the assumption, it is always negative on $x$ edge at the lower end of the cube when $t$ and $r$ equals to zero. Therefore, the surface connecting $(0,0,0)$, $(1,t_4,0)$ and $(1,0,r_4)$ in Figure 3, represents the frontier at which the consumer is indifferent between choosing pay-per-volume pricing scheme and not entering the market. Then from the indifference equation, we get:

$$
k + \gamma wt_4 - c^\gamma = 0
$$

$$
\alpha r_4 + k - c^\gamma = 0
$$

From the above two indifference equations, we obtain $t_4$, and $r_4$ as follows.
Then, from the volume of the triangular pyramid shown in Figure 3, we get:

\[ q_4 = \frac{1}{3} r_4 I_4 \]

\[ q_4 = \frac{(e^x - k)^2}{6\alpha\gamma w} \] (30)

Also, \( q_1 \) and \( q_2 \) for case 2 is the same as defined in (8) and (17) in case 1, and therefore, \( q_3 \) for case 2 will be calculated as follows.
\[ q_3 = 1 - q_1 - q_2 - q_4 \]  

(31)

We assume \( k \leq c^x \). Also, since \( 0 \leq x_1 \leq 1 \), \( 0 \leq t_1 \leq 1 \), \( 0 \leq r_1 \leq 1 \), \( 0 \leq t_2 \leq 1 \), \( 0 \leq r_2 \leq 1 \), \( 0 \leq t_3 \leq 1 \) and \( 0 \leq r_3 \leq 1 \), we get:

\[ c^x \geq k \]  

(32)

\[ c^x w (\theta - \lambda) \leq \lambda c^x w + k w (\theta - \gamma) \]  

(33)

\[ (1 - \lambda) c^x \geq \lambda c^x - (\theta - \gamma)w \]  

(34)

\[ (\beta - \alpha \lambda) c^x \leq \alpha \lambda c^x + (\beta - \alpha)k \]  

(35)

\[ (1 - \lambda) c^x \geq \lambda c^x - (\beta - \alpha) \]  

(36)

On substituting (9), (17) and (31) into (4), the optimization problem is

\[
\max_{c^x, c^w} \prod_{c^x, c^w} \left[ \frac{(1 - \lambda) \left[ \lambda c^x - (1 - \lambda) c^x + (1 - \theta) w + (1 - \beta) \right]}{6w \lambda (1 - \theta) (1 - \beta) c^x} - \lambda \left[ \frac{(\lambda c^x - (1 - \lambda) c^x)^3}{6w (\beta - \alpha) (\theta - \gamma) (1 - \lambda) c^x} \right] \right] + \\
\left[ \frac{(1 - \lambda) \left[ \frac{(\lambda c^x)^3 - (\lambda c^x - (1 - \lambda) c^x)^3}{6w (\beta - \alpha) (\theta - \gamma) (1 - \lambda) c^x} \right]}{6w (\beta - \alpha) (\theta - \gamma) (1 - \lambda) c^x} \right] \right] \\
\left[ \frac{(c^x - k)^2}{6w \lambda (1 - \theta) (1 - \beta) c^x} - \lambda \left[ \frac{(\lambda c^x - (1 - \lambda) c^x + (1 - \theta) w + (1 - \beta))^3}{6w \lambda (1 - \theta) (1 - \beta) c^x} \right] \right]
\]

Subject to \( 18 - 26 \) and \( 32 - 36 \)

3. Numerical Study and Sensitivity Analysis

The numerical study is performed with a program written in GAMS. We let \( w = 1 \), \( \gamma = 0.10 \), \( \theta = 0.90 \) and select two values of \( k \), \( k = 0.4 \) (low), \( k = 1.4 \) (high) for different values of \( \lambda = 0.50 \), \( \lambda = 0.48 \), \( \alpha = 0.10 \) (low), \( \alpha = 0.17 \) (high), \( \beta = 0.90 \) (high) and \( \beta = 0.86 \) (low). Note that \( k \) represents the benefit of transmitting unit volume, \( \beta \) and \( \alpha \) represent consumers’ inclination to hybrid pricing scheme and pay per volume pricing strategy as compared to pay per time pricing strategy. \( \lambda \) controls the price variation in hybrid scheme. Pricing will be close to volume based scheme when \( \lambda \) value is low. It will be close to time based scheme when \( \lambda \) value is high. The results for both cases are given in Table 1.
Table 1: Numerical results for \( k \)

\[
\begin{array}{cccccccccc}
\lambda & \alpha & \beta & k & c' & c^* & q_1 & q_2 & q_3 & q_{total} & \text{Revenue} \\
\hline
\text{Case 1} & 0.48 & 0.10 & 0.90 & 0.4 & 0.754 & 0.400 & 0.000 & 0.945 & 0.055 & 1.000 & 0.545 \\
\text{Case 1} & 0.48 & 0.10 & 0.90 & 1.4 & 1.667 & 1.400 & 0.000 & 0.817 & 0.183 & 1.000 & 1.459 \\
\text{Case 2} & 0.48 & 0.10 & 0.90 & 0.4 & 0.771 & 0.419 & 0.000 & 0.944 & 0.050 & 0.994 & 0.560 \\
\text{Case 2} & 0.48 & 0.10 & 0.90 & 1.4 & 1.667 & 1.408 & 0.000 & 0.818 & 0.181 & 0.999 & 1.462 \\
\text{Case 1} & 0.50 & 0.10 & 0.90 & 0.4 & 0.800 & 0.400 & 0.000 & 0.927 & 0.073 & 1.000 & 0.585 \\
\text{Case 1} & 0.50 & 0.10 & 0.90 & 1.4 & 1.667 & 1.400 & 0.024 & 0.786 & 0.190 & 1.000 & 1.483 \\
\text{Case 2} & 0.50 & 0.10 & 0.90 & 0.4 & 0.825 & 0.425 & 0.000 & 0.924 & 0.066 & 0.999 & 0.605 \\
\text{Case 2} & 0.50 & 0.10 & 0.90 & 1.4 & 1.667 & 1.400 & 0.024 & 0.786 & 0.190 & 1.000 & 1.474 \\
\text{Case 1} & 0.48 & 0.17 & 0.90 & 0.4 & 0.754 & 0.400 & 0.000 & 0.940 & 0.060 & 1.000 & 0.545 \\
\text{Case 1} & 0.48 & 0.17 & 0.90 & 1.4 & 1.521 & 1.400 & 0.013 & 0.834 & 0.153 & 1.000 & 1.403 \\
\text{Case 2} & 0.48 & 0.17 & 0.90 & 0.4 & 0.771 & 0.419 & 0.000 & 0.938 & 0.058 & 0.996 & 0.561 \\
\text{Case 2} & 0.48 & 0.17 & 0.90 & 1.4 & 1.521 & 1.400 & 0.013 & 0.834 & 0.153 & 1.000 & 1.398 \\
\text{Case 1} & 0.48 & 0.10 & 0.86 & 0.4 & 0.831 & 0.400 & 0.000 & 0.926 & 0.074 & 1.000 & 0.577 \\
\text{Case 1} & 0.48 & 0.10 & 0.86 & 1.4 & 1.583 & 1.400 & 0.012 & 0.822 & 0.165 & 1.000 & 1.429 \\
\text{Case 2} & 0.48 & 0.10 & 0.86 & 0.4 & 0.853 & 0.424 & 0.000 & 0.923 & 0.067 & 0.990 & 0.594 \\
\text{Case 2} & 0.48 & 0.10 & 0.86 & 1.4 & 1.583 & 1.404 & 0.013 & 0.822 & 0.164 & 0.998 & 1.372 \\
\end{array}
\]

The managerial implications for the results are as follows. In Table 1, we notice that \( c^x = k \) in Case 1. In other words, the optimal price for transmitting unit volume equals to the benefits of transmitted volume for pay per volume pricing scheme for Case 1. Intuitively, since all customers enter the market, it is optimal for the network provider to charge the highest per volume price without having the risk of losing any of the customers. When \( c^x = k \), customers selecting the pay per volume scheme \( q_3 \) do not get any volume transmission utility, however, they still achieve a non-negative surplus by entering the market.

In order to determine the effect of \( k \) on the optimal tariff policy, we perform a sensitivity analysis using values of \( \lambda = 0.48 \), \( \alpha = 0.1 \) and \( \beta = 0.9 \) as a reference scenario. Results of the reference scenario are given in Figure 4, 5a and 5b, where we observe that optimal revenues increase in \( k \). Then, firstly we increase the value of \( \lambda \) from \( \lambda = 0.48 \) to \( \lambda = 0.5 \). The results are similar to the scenario when \( \lambda = 0.5 \). However, since \( \lambda \) is lower in this scenario, we observe that \( q_1 + q_2 + q_3 < 1 \) for a wider range of the values of \( k \).
Figure 4: Optimal tariffs and optimal revenues for $\lambda = 0.48$, $\alpha = 0.1$ and $\beta = 0.9$

Note that a smaller value of $\lambda$ implies that the consumer has a higher preference to choose pay-per-hybrid pricing scheme as compared to the other schemes. However, it is hard to tell what would be the optimal $c^t$ and $c^x$ for low values of $\lambda$, since the provider could decrease price levels and maximize its revenue by covering more population or increase price levels by preventing the customers passing from a higher level price scheme to a lower level price scheme.

Figure 5a: Consumer demand for $\lambda = 0.48$, $\alpha = 0.1$ and $\beta = 0.9$
The scenario results for $\lambda = 0.5$ are given in Figures 6a and 6b. As can be seen, the revenue for the provider increases with the increasing values of $k$. This is somewhat intuitive because as the benefit per-volume usage to the consumer gets higher, the provider would charge a higher price and generate more revenue. We know from Figure 2 where the entire population is covered ($q_1 + q_2 + q_3 = 1$) and from Table 1 that $c^3 = k$ for Case 1. In Case 2, for low values of $k$, we notice that $c^3 > k$ and $q_1 + q_2 + q_3 < 1$. However, for higher values of $k$, we have, $c^3 = k$ and $q_1 + q_2 + q_3 = 1$ as in Case 1. Intuitively, for low values of $k$, some consumers with low reservation prices would not enter the market as they have negative surplus values. As $k$ increases, more consumers enter the market and we obtain $q_1 + q_2 + q_3 = 1$. Again, as the utility per transmitted volume increases, the provider can charge more.

We also observe that in both cases, $q_2$ is higher and, $q_1$ and $q_3$ is lower for the base scenario ($\lambda = 0.48$) than for the scenario $\lambda = 0.5$ for all the values of $k$. In the base scenario for the value of $\lambda = 0.48$ pay-per hybrid pricing scheme is cheaper than for the scenario $\lambda = 0.5$, and the consumer has a higher preference to choose the pay-per hybrid pricing scheme because of price advantage of this pricing scheme. In Case 1, since all consumers enter the market, it is optimal for the provider to charge highest per-volume transmission price without the risk of losing any of the consumers. Since $c^3 = k$, for both $\lambda$ scenarios, even though the pay-per volume usage consumers ($q_3$) do not get any per-volume transmission gain, they still achieve a non-negative surplus by entering the market. In Case 2, while $\lambda$
having smaller values as in the base scenario ($\lambda = 0.48$) the provider sets $c^+$ at a lower level for the customers with low benefit from volume transmission in order to gain nonusers. However for the customers having higher $k$ values, trying to gain nonusers could not be an optimal behavior because more customers choose pay-per hybrid pricing scheme for low values of $\lambda$, and the provider could compensate more revenue loss by setting $c^+$ at a higher level. Also, we observe that for $k = 1.4$, when $\lambda$ is small as in the base scenario, $c^+$ becomes higher in both cases. Taking the price advantage of pay-per hybrid pricing scheme and gaining more revenue from pay-per hybrid pricing customers is the optimal behavior to minimize revenue loss since number of customers switching from pay per time scheme to hybrid scheme is less than customers switching from pay per volume to hybrid scheme. Hence more customers would choose hybrid pricing scheme.

![Figure 6a: Consumer demand for $\lambda = 0.5$, $\alpha = 0.1$ and $\beta = 0.9$](image)

For $\alpha = 0.17$, the results are given in Figures 7a and 7b. Again, we observe that the revenues for the provider increase with the increasing values of $k$. The results are similar to the scenario where $\lambda = 0.48$ and $\alpha = 0.1$. However, since $\alpha$ is larger in this scenario, we observe that $q_1 + q_2 + q_3 = 1$ for a narrower range of the values of $k$. 
We observe that for higher values of $\alpha$, decreasing price levels results in more customers changing their pricing schemes, and network provider gaining more customers from the entire population. Also, from the results of the sensitivity analysis it can be seen that decreasing both $c'$ and $c^+$ is preferable for the higher values of $k$, $k = 1.4$, because as more and more customers have higher benefit of transmitting unit volume, they would prefer pay-per-time and pay-per-hybrid pricing schemes and more nonusers would enter the market. However, for the smaller values of $k$, $q_3$ is higher and $q_1$ and $q_2$ is lower for the
scenario $\alpha = 0.17$ compared to the scenario $\alpha = 0.1$ in both cases. This is intuitive because the larger value of $\alpha$ the smaller is the difference in the benefit of billing choices, and as a consequence of this situation more customers choose the cheapest option. For the consumers’ profile having low $k$ values and high tendency to choose volume based pricing rather than selecting the other two pricing schemes, preferred behavior of the provider would be to maintain the levels of $c^t$ and $c^{x}$. Because, volume transmission benefit of the consumers is low, not many nonusers would intend to enter the market just because of low price levels. Therefore maintaining its price levels is the preferred choice for the provider.

![Graph](image)

Figure 7b: Consumer demand for $\lambda = 0.48$, $\alpha = 0.17$ and $\beta = 0.9$

Note that the parameter $\beta$ models the difference in benefit of the network leased through hybrid based pricing against other two pricing schemes. The smaller is the value of $\beta$, the higher the difference in the benefit of pay-per time and pay-per hybrid billing choices. However, the higher is the value of $\beta$, the higher the difference in the benefit of pay-per volume and pay-per hybrid billing choices.

In Figures 8a and 8b, we observe that for smaller values of $\beta$ ($\beta = 0.86$) decreasing price levels of $c^t$ and $c^{x}$ results in customers having high $k$ value ($k = 1.4$) to change their pricing schemes to the expensive one. In this situation, preferred behavior of the provider is to take some risk of network bursting, in other words, have some pay-per-time customers who have highest profit margin. However, for the smaller values of $k$, $q_3$ is higher and, $q_2$ is lower for the scenario $\beta = 0.86$ than for the scenario...
\(\beta = 0.9\) in both cases. This is intuitive since smaller the value of \(\beta\), smaller the difference in the benefit of pay-per hybrid pricing and pay-per-volume pricing choices, and as a consequence of this situation more customers choose the cheapest option. For the consumers’ profile having low \(k\) values and high tendency to choose volume based pricing rather than selecting the other two pricing schemes, optimal behavior of the provider would be to decrease the levels of \(c'\) and \(c^x\) in Case 2. Because, volume transmission benefit of the consumers is low, not many nonusers would intend to enter the market just because of the low price levels. Therefore decreasing its price levels is the best choice for the provider in order to prevent too much migration from pay-per hybrid pricing scheme to pay-per-volume pricing scheme.

We also performed sensitivity analysis to assess the benefits of unit connection time, \(w\). We let \(k = 0.1\) and select two values of \(w\), \(w = 0.35\) (low), \(w = 3.5\) (high) for different values of \(\lambda = 0.50\), \(\lambda = 0.6\), \(\alpha = 0.10\) (low), \(\alpha = 0.20\) (high), \(\beta = 0.90\) (high), \(\beta = 0.80\) (low), \(\gamma = 0.1\) (low), \(\gamma = 0.2\) (high), \(\theta = 0.9\) (high) and \(\theta = 0.8\) (low). Differently from the sensitivity analysis of \(k\), in sensitivity analysis of \(w\) we observe the behavior of the provider and customers for different values of \(\gamma\) and \(\theta\). Note that \(\gamma\) and \(\theta\) represent differences in consumers’ benefit of unit connection time for different pricing strategies. The results for both cases are given in Table 2.

![Figure 8a: Consumer demand for \(\lambda = 0.48\), \(\alpha = 0.10\) and \(\beta = 0.86\)](image)
For Review

Figure 8b: Consumer demand for $\lambda = 0.48$, $\alpha = 0.10$ and $\beta = 0.86$

The managerial implications for the results are as follows. In Table 2, we notice similar results as we have in the sensitivity analysis of $k$ where $c^* = k$ in Case 1. Since all customers enter the market, it is optimal for the network provider to charge the highest per volume price without the risk of losing any of the customers. When $c^* = k$, customers selecting the pay per volume scheme ($q_3$) do not get any volume transmission utility, however, they still achieve a non-negative surplus by entering the market. Also, we observe that the revenue for the provider increases with the increasing values of $w$. This is intuitive because as the benefit in per-connected time to the consumer gets higher, the provider would charge a higher price and generate more revenues.

The scenario results when $\lambda = 0.5$ are given in Figures 9, 10a and 10b. We know from Figure 2 where the entire population is covered and from Table 2 that $c^* = k$ for Case 1. Differently from $k$ analysis, we notice that for all values of $w$, $c^* > k$ and $q_1 + q_2 + q_3 < 1$ in Case 2. Note that with our model, we are looking for the answers for the question “what are the optimum pricing strategies to prevent network bursting with certainty?” For this reason, we develop our model to limit the percentage of pay-per-time customers. Since it is expected that the optimum behavior of pay-per-time customers will be transmitting as much task as they can in unit time with fully utilization of network, it generally results in network bursting. Hence, in our model the volume of pay-per-time customers cannot exceed $1/6$ of the total population, from eq. (8). For very low values of $k$, that is $k = 0.1$, it does not matter what the value of $w$ is since populations having low reservation prices also have low benefit in per-connection time.
Table 2: Numerical results for $w$

<table>
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<th>Case</th>
<th>$\lambda$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>$w$</th>
<th>$c'$</th>
<th>$c^*$</th>
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Figure 9: Optimal tariffs and optimal revenues for $\lambda = 0.5$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$
In order to determine the effect of $w$ on the optimal tariff policy, we perform a sensitivity analysis using two values of $\lambda$: $\lambda = 0.50$ and $\lambda = 0.60$. The results are given for $\lambda = 0.6$ in Figure 11, 12a and 12b. Note that a higher value of $\lambda$ implies that the hybrid pricing will be close to the time based scheme. The consumer has a lower preference to choose pay-per hybrid pricing scheme as compared to the other schemes since hybrid price neither so cheaper than time based price nor so close to the level of volume based price. Intuitively, the provider gains more revenue by increasing its price levels, $c^v$ and $c^t$, for higher values of $\lambda$. While $\lambda$ is getting bigger in Case 2, in spite of losing some customers, total revenue
of the provider would also increase because of increasing percentage of pay-per time and pay-per volume customers as well as the increase in pay-per time and pay-per volume price levels. This is an interesting result because pay-per volume price level is lower than pay-per hybrid price level. However, $c^* x$ get closer (becomes almost equal) to $\lambda (c^* + c^* x)$ while $\lambda$ is getting bigger, hence the provider gets an advantage to gain more revenue with pay-per hybrid pricing scheme.

Figure 11: Optimal tariffs and optimal revenues for $\lambda = 0.6$, $\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$

Figure 12a: Consumer demand for $\lambda = 0.6$, $\alpha = 0.1$ and $\beta = 0.9$, $\gamma = 0.1$ and $\theta = 0.9$
For $\alpha = 0.20$, the results are given in Figures 13a and 13b. The results are similar to the scenario where $\lambda = 0.50$ and $\alpha = 0.1$. However, since $\alpha$ is larger in this scenario, we observe that the value of $(q_1 + q_2 + q_3)$ gets bigger. Intuitively, if the consumers’ inclination to pay per volume pricing strategy (as compared to pay per time pricing strategy) increases, the provider would use this chance to gain new customers from the whole population. Hence, having more pay-per-volume customers hedges the provider against network bursting.
In Figures 14a and 14b, we observe that for smaller values of $\beta$ ($\beta = 0.80$) increasing price levels of $c'$ and $c^s$ results in customers to change their pricing schemes to the cheaper one. In this situation optimal behavior of the provider is to take some risk of losing some customers with low profitability in order to increase profit margin of all billing options, and hence to increase its total revenue. However, for the smaller values of $\beta$, $q_3$ gets bigger and, $q_1$ and $q_2$ gets smaller in both cases. While value of $\beta$ getting smaller, the difference in the benefit of pay-per hybrid pricing and pay-per-time pricing choices gets higher. However, the smaller is the value of $\beta$, the smaller would the difference be in the benefit of pay-per hybrid pricing and pay-per-volume pricing choices. Thus, customers shift to cheaper billing choice, and most customers choose the cheapest option in both Cases. In order to increase its revenue, the preferred behavior of the provider would be to increase the levels of $c'$ and $c^s$, which contradicts the behavior observed in the sensitivity analysis of $k$, with respect to the decrease in $\beta$. In spite of the decrease in consumers’ inclination in pay-per hybrid pricing scheme, having such an alternative price option that is indexed to both $c'$ and $c^s$ enables the provider to take the advantage of increasing its revenue. Since, there were no hybrid price option when $c'$ increase, the pay-per time customers would have to choice pay-per volume option. Therefore, the provider would have to set $c^s$ in a much higher level in order to preserve its revenue. And as a result of this situation, the provider would lose more pay-per volume customers than the situation with including pay-per hybrid pricing scheme.
For $\gamma = 0.20$, the results are given in Figures 15a and 15b. The results are similar to the scenario where $\lambda = 0.50$ and $\gamma = 0.1$. However, since $\gamma$ is larger in this scenario, we observe that the value of $(q_1 + q_2 + q_3)$ is bigger, in Case 2. Intuitively, if the pay-per-volume consumer’s benefit of unit connection time increases, the provider would use this chance to gain new customers from the whole population. However, when the consumer’s benefit of unit connection time increases for pay-per-volume pricing option, it gets closer to the unit connection time benefit of pay-per-hybrid billing option. And, if the provider does not change the price levels, some pay-per-hybrid billing customers would shift to the
pay-per volume pricing option. But still remaining price levels can be an optimum behavior for the provider since, having more pay-per volume customers hedges the provider against network bursting.

Figure 15a: Consumer demand for $\lambda = 0.5$, $\alpha = 0.1$ and $\beta = 0.9$, $\gamma = 0.2$ and $\theta = 0.9$

In Figures 16a and 16b, we observe that for smaller values of $\theta$, $\theta = 0.80$, increasing price levels of $c'$ and $c^s$ results in pay per hybrid customers (having high $w$ value, $w = 3.5$) to change their pricing schemes to the other two options, in both Case 1 and Case 2. In this situation optimal behavior of the provider is to take some risk of network bursting, that is to have some pay-per-time customers who have highest profit margin in order to increase its revenue. However, for the smaller values of $w$, $q_3$ is higher and, $q_2$ is lower for the scenario $\theta = 0.80$ than for the scenario $\theta = 0.9$ in both cases. This is intuitive since for the smaller value of $\theta$, the smaller is the difference in the benefit of unit connection time for pay-per hybrid pricing and pay-per-volume pricing choices, and as a consequence of this situation more
customers choose the cheapest option. For the consumers’ profile having low $w$ values and high
tendency to choose volume based pricing rather than selecting the other two pricing schemes, again, the
optimal behavior of the provider would be to increase the levels of $c'$ and $c^*$. Because unit connection
time benefit of the consumers is low, not many nonusers would be interested on entering the market
despite the low price levels. Therefore, for the provider, increasing its price levels in order to prevent too
much migration from pay-per hybrid pricing scheme to pay-per volume pricing scheme is more profitable
than decreasing its price levels in order to gain more customers.

Figure 16a: Consumer demand for $\lambda = 0.5, \alpha = 0.1, \beta = 0.9, \gamma = 0.1$ and $\theta = 0.8$

Figure 16b: Consumer demand for $\lambda = 0.5, \alpha = 0.1$ and $\beta = 0.9, \gamma = 0.1$ and $\theta = 0.8$
4. Comparison the Pricing Model with No Hybrid Pricing Scheme

In this section we compare the results for the case where the provider offers only the pay per time and pay per volume pricing schemes. Let $q_t$ again be the total demand of the consumers choosing pay per time billing option, and $q_d$ again be the volume of the customers not entering the market. Note that the consumer selection for this scenario is similar to the ones in Figure 2 and Figure 3 for the Case 1 and 2 with $q_3$ only (since there is no hybrid scheme, there is no $q_2$). Hence, $q_3$ becomes $q_3 = 1 - q_t$ for Case 1, and $q_3 = 1 - q_t - q_d$ for Case 2. Let the surface connecting the points $(x,1,1)$, $(t,1,1)$ and $(1,1,r)$ represent the frontier at which the consumer is indifferent between pay-per-time and pay-per-volume pricing schemes. Using (1) and (3), from the indifference equations, we get:

$$1 + kx_t + w - c' = \alpha + kx_t + \theta w - c^* x_t$$

$$1 + k + wt_t - c' = \alpha + k + \theta wt_t - c^*$$

$$r_t + k + w - c' = \alpha r_t + k + \theta w - c^*$$

From the above three indifference equations, we obtain $x_t$, $t_t$, and $r_t$ as follows.

$$x_t = \frac{c' - w(1 - \theta) - (1 - \alpha)}{c^*}$$  \hspace{1cm} (37)

$$t_t = \frac{(c' - c^*) - (1 - \alpha)}{w(1 - \theta)}$$  \hspace{1cm} (38)

$$r_t = \frac{(c' - c^*) - w(1 - \theta)}{(1 - \alpha)}$$  \hspace{1cm} (39)

Then, from the volume of the triangular pyramid shown in Figure 2, we get:

$$q_t = \frac{1}{2} \cdot \frac{1}{3} \cdot (1 - x_t)(1 - t_t)(1 - r_t)$$

$$q_t = \frac{[c' - c^* + w(1 - \theta) + (1 - \alpha)]^3}{6w(1 - \theta)(1 - \alpha)c^*}$$  \hspace{1cm} (40)
and \( q_2 = 1 - q_1 \) \hspace{1cm} (41)

Also, since \( 0 \leq x_i \leq 1, \) \( 0 \leq t_i \leq 1, \) and \( 0 \leq r_i \leq 1, \) we get:

\[ c' \geq w(1 - \theta) + (1 - \alpha) \] \hspace{1cm} (42)

\[ c' \leq w(1 - \theta) + (1 - \alpha) + c^x \] \hspace{1cm} (43)

\[ c' \geq (1 - \alpha) + c^x \] \hspace{1cm} (44)

\[ c' \geq w(1 - \theta) + c^x \] \hspace{1cm} (45)

For Case 1, substituting \( q_1 \) and \( q_2 \) from (40) and (41) respectively into (3), the optimization problem becomes

\[
\text{Max}_{(c',c^x)} \prod \frac{(c' - c^x - 1 + \alpha)^3}{6c^x(1 - \alpha)} + \frac{6c'c^x(1 - \alpha) - (c' - c^x - 1 + \alpha)^3}{6c^x(1 - \alpha)}
\] \hspace{1cm} (46)

Subject to (42) – (45)

The surface connecting \((0,0,0), (1,t_4,0)\) and \((1,0,r_4)\) represents the frontier at which the consumer is indifferent between choosing pay-per-volume pricing scheme and not entering the market (see Figure 3). Then from the indifference equation, we get:

\[ k + \theta wt_4 - c^x = 0 \]
\[ \alpha r_4 + k - c^x = 0 \]

From the above two indifference equations, we obtain \( t_4, \) and \( r_4 \) as follows.

\[ t_4 = \frac{c^x - k}{\theta w} \] \hspace{1cm} (47)

\[ r_4 = \frac{c^x - k}{\alpha} \] \hspace{1cm} (48)

Then, from the volume of the triangular pyramid shown in Figure 3, we get:
\[ q_4 = \frac{1}{3} r_4 t_4 \]

\[ q_4 = \frac{(c^x - k)^2}{6\alpha\theta w} \]  \quad (49)

Also, \( q_1 \) is the same as defined in (40), and therefore

\[ q_3 = 1 - q_1 - q_4 \]  \quad (50)

Since \( 0 \leq t_4 \leq 1 \) and \( 0 \leq r_4 \leq 1 \), the following conditions must hold:

\[ c^x \geq k \]  \quad (51)

\[ c^x \leq k + \theta w \]  \quad (52)

\[ c^x \leq k + \alpha \]  \quad (53)

For Case 2, substituting (40) and (50) into (3), the optimization problem becomes

\[ \text{Max}_{(c',c')} \prod = \left( c' - c^x \right) \left[ \frac{c^x - c' + w(1-\theta) + (1-\alpha)}{6w(1-\theta)(1-\alpha)c^x} \right] + c^x \left( 1 - \frac{(c^x - k)^2}{6\alpha\theta w} \right) \]

Subject to \((42) - (45)\) and \((51) - (53)\)

In order to make a comparison between with and without hybrid pricing scheme, we present the results in Table 3 which are partially obtained from numerical examples presented previously where, \( \alpha = 0.10 \), \( \gamma = 0.10 \) for the values of \( k = 0.4 \), \( k = 0.8 \) and \( k = 1.4 \).

We can notice that for smaller values of \( k \) \((k = 0.4)\) offering pay-per hybrid billing option with pay-per time and pay-per volume billing options is more profitable for the provider for both Case 1 and Case 2. Also, market size for the provider increases by offering pay-per hybrid pricing scheme for smaller values of \( k \) in Case 2. For larger values of \( k \) \((k = 0.8 \) and \( k = 1.4 \) \) revenue of the provider increases because, some of the pay-per hybrid pricing customers with high benefit of transmitting unit volume shift to the pay-per time pricing option in the absence of this pricing option. However, market size for the provider decreases when it does not offer pay-per hybrid pricing scheme to the customers. Therefore, by offering the pay-per hybrid pricing option, the provider is able to get more consumers to enter the market (see
Figure 6b). This illustrates that judicious use of pay-per hybrid scheme is not only beneficial to the provider, but it also increases the market size.

Table 3. Comparison of the results of the pricing strategies with and without pay-per hybrid pricing option

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<tr>
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<th>$\alpha$</th>
<th>$\gamma$</th>
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<th>$c'$</th>
<th>$c^\prime$</th>
<th>$q_1$</th>
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<th>$q_3$</th>
<th>$q_{total}$</th>
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<td>0.909</td>
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<td>-</td>
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<td>0.900</td>
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For the bigger values $\alpha$ ($\alpha = 0.17$) we notice that offering pay-per hybrid billing option with pay-per time and pay-per volume billing options is more profitable for the wider range of $k$ ($k = 0.4$, $k = 0.8$ and $k = 1.4$) in Case 1. Also, market size for the provider increases by offering pay-per hybrid pricing scheme for wider range of the values of $k$ in Case 2. For larger values of $k$ ($k = 0.8$ and $k = 1.4$), revenue of the provider increases, because more customers have a tendency to choose pay-per time pricing scheme than the pricing strategy without pay-per hybrid pricing scheme since their benefit of transmitting unit volume increases as the value of $\alpha$ gets bigger. Thus the provider increases levels of $c'$ and $c'$. As a result of high pay-per time and pay-per volume prices, less customers enter the market in the situation $\alpha = 0.17$ than in the situation $\alpha = 0.1$. Also, in Table 1 and 3 it can be seen that market size for the provider decreases when it does not offer pay-per hybrid pricing scheme to the customers while $\alpha$
getting bigger. Therefore by offering the pay-per hybrid pricing option, the provider is able to get more revenue and more consumers to enter the market for wider range of the values of $k$. Once again, this illustrates that judicious use of pay-per hybrid scheme is not only beneficial to the provider, but it also increases the market size.

In order to make a comparison between with and without hybrid pricing scheme, we present the results in Table 4 which are obtained from the numerical example presented previously where $\alpha = 0.10$, $\gamma = 0.10$ for the values of $w = 0.35$, $w = 1.0$ and $w = 3.5$.

We notice that for all values of $w$, offering pay-per hybrid billing option with pay-per time and pay-per volume billing options is more profitable for the provider for both Case 1 and Case 2. Also, market size for the provider increases by offering pay-per hybrid pricing scheme for all values of $w$ in Case 2.

Table 4. Comparison of the results of the pricing strategies with pay-per hybrid pricing and without pay-per hybrid pricing option for different values of $w$

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<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$w$</th>
<th>$c^t$</th>
<th>$c^v$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$q_3$</th>
<th>$q_{total}$</th>
<th>Revenue</th>
</tr>
</thead>
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<td>0.000</td>
<td>0.960</td>
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For the bigger values $\alpha$ ($\alpha = 0.2$) we notice that offering pay-per hybrid billing option with pay-per time and pay-per volume billing options is more profitable for all values of $w$, in both Case 1 and 2.
Also, market size for the provider increases by offering pay-per hybrid pricing scheme in Case 2. Revenue of the provider increases, because as it can be seen in Table 4 more customers would have a tendency to choose pay-per time pricing scheme than the pricing strategy without pay-per hybrid pricing scheme at $\alpha = 0.1$, since their benefit of transmitting unit volume increases as $\alpha$ gets bigger. Thus the provider maintains level of $c^+$ and decreases level of $c^-'$. As a result of low pay-per time price level more customers enter the market in the situation of $\alpha = 0.2$ than in the situation of $\alpha = 0.1$. Also, in Table 2 and 4 it can be seen that market size for the provider decreases when it does not offer pay-per hybrid pricing scheme to the customers as $\alpha$ gets bigger. Therefore by offering the pay-per hybrid pricing option, the provider is able to get more revenue and more consumers to enter the market for all values of $w$. This also illustrates that judicious use of pay-per hybrid scheme is not only beneficial to the provider, but it also increases the market size.

5. Conclusions and Future Research

Hybrid and pay per volume pricing scheme are the next generation strategies to prevent bursting of networks. These strategies facilitate leasing the excess capacity to other customers. Hybrid pricing scheme is useful for those customers having fewer tasks with low total volume and relatively high reservation price compared to customers who choose pay per volume scheme. Pay per volume pricing scheme is useful for those customers having generally size fixed tasks and/or low reservation price. Thus, the provider can increase revenue by setting higher price by offering pay-per-time pricing scheme to those customers who use data networks too often, having tasks with more volume or having more time-fixed tasks. The main focus of the study, as presented in this paper, is to show that hybrid scheme is a viable and often preferable pricing option to the network provider. This point is illustrated by the optimization models, maximizing provider’s revenue when hybrid pricing strategy exists, as it compared to the situation where it does not exist. In conclusion, this paper shows that (i) the offering hybrid pricing option can increase the profitability of the network provider, (ii) it can increase the overall market size, and (ii) when all three strategies are offered, it is possible to formulate an optimal pricing strategy that maximizes the provider’s revenue.

Acknowledgements

References


