

# A New Functional Observer to Estimate Velocity, Acceleration and Disturbance for Motion Control Systems

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**Abstract**—This paper presents a novel functional observer which can observe the velocity, acceleration and disturbance information of a motion control system with higher accuracy and less noise in comparison to classical observers. The observer uses the input current and position information and the nominal parameters of the plant. The novelty of the observer is based on its functional structure that can intrinsically estimate and compensate the un-measured inputs (like disturbance acting on the system) using the measured input current. The experimental results of the proposed estimator verifies its success in estimating the velocity, acceleration and disturbance with better precision than classical observers.

## I. INTRODUCTION

In recent years, realization of high level applications of motion control systems are becoming more and more desired. Research is being dominated by various applications of force feedback/control, haptics, bilateral control, teleoperation and time delayed control systems. For any kind of application related to those research areas, one of the primary needs is to have a clear and accurate measurement of position, velocity and even acceleration.

Generally in motion control systems, the measurements available to the controller are the input current to the system and position information from the encoder. The problem to obtain the velocity and acceleration data with high precision and low noise while maintaining a very large bandwidth sits in the middle of all high level motion control applications. The standard approach is to use the first and second order derivatives of position information of an incremental encoder and process the resulting data through a low pass filter. However this approach brings two disadvantages which are impossible to overcome simultaneously. With this classical structure, one either has to acquire a fast but very noisy data, or has to have a less noisy but sluggish data [1], [2]. The payoff between those two cases is determined by the frequency of the filter. In either case, the degradation in the performance of controllers might be problematic.

Many researchers analyzed this problem and performed to come up with fast and accurate estimators using dif-

ferent methodologies. In [3] Kalman Filter is used to estimate the velocity and disturbance in low speed range. Although this approach is a good way to clear the noise in estimation, the computational cost might be problematic for cases where fast response in estimation is desired. Another study, which relies on the use of Extended Kalman Filter, implements the velocity estimation with current and DC voltage inputs of an induction motor [4]. A more recent example of Extended Kalman Filtering on velocity estimation can be found in [15]. Some researchers used the direct output of well known disturbance observer to estimate the velocity. In [5] the disturbance torque and the input current is used to observe the speed of the system. A similar methodology is performed by implementing disturbance observer based full state observer algorithm to recover the dead time problem in estimation of low speed motion [6]. However, although disturbance observer is proven to be very useful for robust motion control [7], the observer structure intrinsically requires the velocity information of the plant which is again done by classical filtered differentiation. Besides, since the disturbance observer gives non-zero value for a scenario where there is non-zero current input and zero position change, this kind of approach might give a non-zero velocity value which can mislead the controllers using this information. In their study, Patten et al. proposed a structure to observe velocity based on optimal state estimation using input torque and position information [8]. Their work basically originates through closing the loop for velocity estimator. This way, even though the estimation result is accurate for low speeds, it is not fast enough to recover rapid fluctuations in velocity. In a recent study by Berducat et al. the speed information is obtained via an adaptive two level observer using estimation of friction model [9]. In [10] a novel approach is tried and the authors used adaptive fuzzy logic to realize the velocity observer. In this method, the fuzzy controller adapts the disturbance acting on the plant and hence it can perform very good in annihilating the noise in the estimation. However, this approach can lose reliability where there is rapid change of disturbance acting on the system. [11] presents another

speed estimation method based on a model reference adaptive scheme that can recover mechanical inertia time for changing load. More information about velocity and acceleration estimators can be found in [12], [13], [14], and [16].

## II. PROBLEM DEFINITION

The aim of this paper is to construct a functional observer that can be used to estimate the velocity, acceleration and disturbance information of a motion control system. The objectives are both to achieve estimation that is as accurate as possible and to provide bandwidth that is as large as possible. The structure given in Fig. 1 represents a classical motion control system. Here,  $i^{ref}$

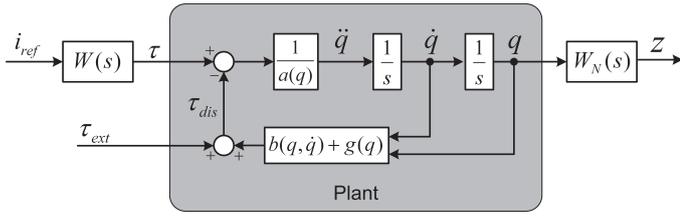


Fig. 1. Structure of the system with ideal observers

is the current input and  $\tau_{dis}$  stands for the disturbance acting on the system. The feedback terms  $b(q, \dot{q})$  and  $g(q)$  represent the viscous friction and gravity acting on the system respectively. The torque input to the system can be formulated by a simple gain;

$$\tau(s) = W(s)i^{ref}(s) \quad (1)$$

without loss of generality, one can lump the non-idealities in the input torque formulation to the disturbance term and come up with a constant gain. This way the system input becomes,

$$\tau(s) = K_n i^{ref}(s) \quad (2)$$

with  $K_n$  being the nominal torque constant that maps the input current to the input torque. The second order plant can be represented by,

$$P(s) = \frac{1}{a(q)s^2} \quad (3)$$

where,  $q$  is the generalized coordinate of motion and  $a(q)$  stands for the plant inertia. The output of this structure can be given as,

$$q(s) = P(s) \{W(s)i^{ref}(s) - \tau_{dis}(s)\} \quad (4)$$

where,  $P^{-1}(s) = a_n s^2$  and  $a_n$  represents the nominal inertia of the system. In order to acquire measurements of the system, one has to incorporate the plant output,  $q$  with a transfer function. In the structure shown in Fig. 1,  $z$  is the variable of interest that is related to the plant output by the ideal (not necessarily realizable) transfer function  $W_N(s)$ , i.e.;  $z = W_N(s)q(s)$ . If output  $z$  cannot be directly measured, then  $W_N(s)$  stands for the

ideal transfer function of the observer that needs to be designed. However, a system as depicted in Fig. 1 may not be physically realizable if  $W_N(s)$  is an improper transfer function like  $\eta_1 s^2 + \eta_2 s + \eta_3$  (i.e. a linear combination of acceleration, velocity and position). For such cases, one can utilize an approximate structure as shown in Fig. 2 and come up with an estimate of the output  $z$ . In this second structure, assuming that the error due to disturbance is characterized by transfer function  $W_L(s)$ , one can write the error due to unmeasured inputs as  $\Delta z = W_L(s)\tau_{dis}(s)$ . In designing the observer, the main criteria is to select the error in the estimated variable  $\hat{z}$  to have a desired value of zero. Now the problem can be formulated as follows: For the system given in Fig. 2, using the nominal plant parameters and measurable outputs, find transfer functions  $W_1(s)$  and  $W_2(s)$  that will best approximate the variable of interest  $z$ .

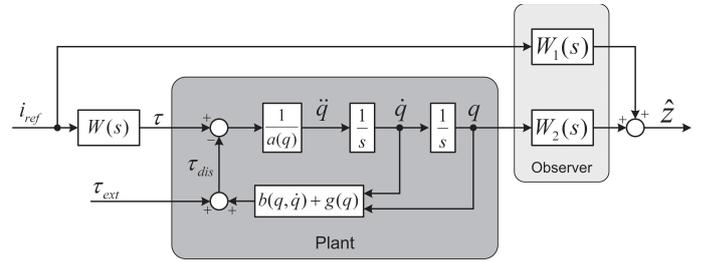


Fig. 2. Structure of the system with observers

## III. OBSERVER CONSTRUCTION

Using equations (2), (3), (4) and the structures shown in Fig. 1 and Fig. 2, one can write the actual and the estimated values of  $z$  as follows:

$$\begin{aligned} z(s) &= W_N(s)q(s) + W_L(s)\tau_{dis}(s) \\ z(s) &= W_N(s)P(s) \{W(s)i^{ref}(s) - \tau_{dis}(s)\} + W_L(s)\tau_{dis}(s) \end{aligned} \quad (5)$$

$$\begin{aligned} \hat{z}(s) &= W_2(s)q(s) + W_1(s)i^{ref}(s) \\ \hat{z}(s) &= W_2(s)P(s) \{W(s)i^{ref}(s) - \tau_{dis}(s)\} + W_1(s)i^{ref}(s) \end{aligned} \quad (6)$$

For simplicity of analysis, we can drop the  $s$  dependent long expressions. From (5) and (6), one can write the error in the estimation as follows:

$$z - \hat{z} = \{PW(W_N - W_2) - W_1\} i^{ref} - \{P(W_N - W_2) - W_L\} \tau_{dis} \quad (7)$$

The difference between desired output  $z$  and its estimated value  $\hat{z}$ , as expressed in (7) depends on both control input and the disturbance. In order to push this estimation error to zero, coefficients of both current ( $i^{ref}$ ) and disturbance ( $\tau_{dis}$ ) should be imposed to have zero value. Letting those coefficients be equal to zero and continuing further, one

finds the following two equations for the transfer functions  $W_1$  and  $W_2$ ;

$$\begin{aligned} W_1 &= WW_L \\ W_2 &= W_N - P^{-1}W_L \end{aligned} \quad (8)$$

The implicit assumption made in (2) saying that the torque can be transmitted to the plant with a constant gain (i.e.  $W(s) = K_n$ ) results in  $W_1(s)$  being equal to a scalar multiple of  $W_L(s)$ . This result is very important since it implies that the error due to disturbance is compensated by the current input during estimation. In other words, the observer, while using position information and transfer function  $W_2(s)$  to acquire the estimated value, also uses the current information and transfer function  $W_1(s)$  along with the nominal parameters of the plant to cancel the effect of disturbance in estimation.

In order to solve for  $W_1(s)$  and  $W_2(s)$  we can define a generalized error for  $W_L(s)$ . Since the disturbance acting on the system pass through a second order dynamics, we can formulate this generalized error as follows;

$$W_L(s) = \frac{g^2 s(\gamma s + \delta)}{a_n(s + g)^2} \quad (9)$$

where,  $a_n$  and  $K_n$  represent the nominal inertia and nominal torque constant of the plant respectively. Using this error, the expression for  $P$  from (3) and equation (8), generalized forms for the transfer functions  $W_1(s)$  and  $W_2(s)$  can also be defined;

$$W_1(s) = \frac{K_n g^2 s(\gamma s + \delta)}{a_n (s + g)^2} \quad (10)$$

$$W_2(s) = W_N(s) - \frac{g^2 s^3(\gamma s + \delta)}{(s + g)^2} \quad (11)$$

In both of the equations (10) and (11), the coefficients  $g$ ,  $\gamma$  and  $\delta$  should be selected in design process. In order to design the parameters, we have to refer to the format of the ideal transfer function  $W_N(s)$ . Let the ideal transfer function be  $W_N(s) = \alpha s^2 + \beta s$  or in other words let us assume that a linear combination of velocity and acceleration is to be estimated. Substituting  $W_N(s)$  into (11), one can obtain;

$$W_2(s) = (\alpha s^2 + \beta s) - \frac{g^2 s^3(\gamma s + \delta)}{(s + g)^2} \quad (12)$$

Expanding further results in,

$$W_2(s) = \frac{C_4 s^4 + C_3 s^3 + C_2 s^2 + C_1 s}{(s + g)^2} \quad (13)$$

where,

$$\begin{aligned} C_4 &= \alpha - g^2 \gamma \\ C_3 &= 2g\alpha - g^2 \delta + \beta \\ C_2 &= 2g\beta + g^2 \alpha \\ C_1 &= g^2 \beta \end{aligned}$$

Since for a physical system, the estimator will have at most second degree derivative, we can set the coefficients of  $s^4$  and  $s^3$  terms ( $C_4$  and  $C_3$ ) be equal to zero, which gives;

$$\alpha - g^2 \gamma = 0 \quad (14)$$

$$\gamma = \frac{\alpha}{g^2} \quad (15)$$

$$2g\alpha - g^2 \delta + \beta = 0 \quad (16)$$

$$\delta = \frac{\beta + 2g\alpha}{g^2} \quad (17)$$

Substituting (15) and (17) into (10) and (11) gives the following set of transfer functions:

$$W_1 = \frac{K_n \alpha s^2 + (\beta + 2g\alpha)s}{a_n (s + g)^2} \quad (18)$$

$$W_2 = g s \frac{(g\alpha + 2\beta)s + g\beta}{(s + g)^2} \quad (19)$$

$$W_N = \alpha s^2 + \beta s \quad (20)$$

Now, the only design parameters are  $\alpha$  and  $\beta$  which is determined from the structure of the ideal observer  $W_N$ . The functional observer can be realized using just two first order filters as depicted in Fig. 3. This structure

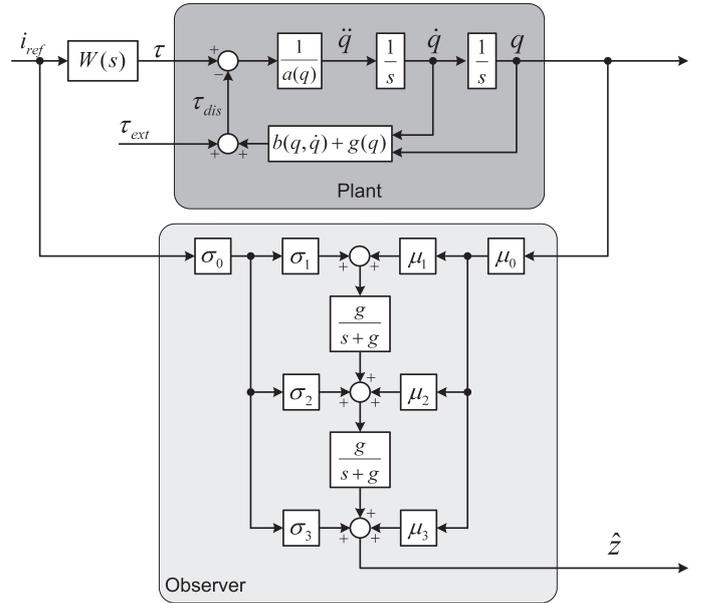


Fig. 3. Structure of the observer

mathematically imposes the following two equations.

$$\begin{aligned} W_1 &= \frac{\alpha s^2 + (\beta + 2g\alpha)s}{a_n (s + g)^2} \\ &= \sigma_0 \left( \sigma_3 + \frac{\sigma_2 g}{(s + g)} + \frac{\sigma_1 g^2}{(s + g)^2} \right) \end{aligned} \quad (21)$$

$$\begin{aligned}
W_2 &= \frac{(g^2\alpha + 2g\beta)s^2 + g^2\beta s}{(s+g)^2} \\
&= \mu_0 \left( \mu_3 + \frac{\mu_2 g}{(s+g)} + \frac{\mu_1 g^2}{(s+g)^2} \right) \quad (22)
\end{aligned}$$

The values for gains  $\sigma_i$  and  $\mu_i$  ( $i = 0, 1, 2, 3$ ) can be found by substituting the necessary numbers for  $\alpha$  and  $\beta$  to the ideal observer  $W_N(s)$ .

A summary of the coefficients for velocity, acceleration and disturbance estimation is given in Table I

TABLE I  
PARAMETERS OF THE FUNCTIONAL OBSERVER FOR DIFFERENT CONFIGURATIONS

$W_N$	$0s^2 + s$	$s^2 + 0s$	$K_n i^{ref} - a_n s^2$
$(\dot{z})$	$(\dot{q})$	$(\ddot{q})$	$(\tau_{dis})$
$\sigma_0$	$\frac{K_n}{g a_n}$	$\frac{K_n}{a_n}$	$-K_n$
$\sigma_1$	-1	-1	-1
$\sigma_2$	1	0	0
$\sigma_3$	0	1	0
$\mu_0$	$g$	$g^2$	$-a_n g^2$
$\mu_1$	1	1	1
$\mu_2$	-3	-2	-2
$\mu_3$	2	1	1

#### IV. EXPERIMENTS

The experimental setup consisted of a Hitachi-ADA series linear motor and driver. The linear motor had an incremental encoder with  $1\mu m$  precision. MATLAB-Simulink environment was used as the implementation software and real time processing was enabled by a D-Space DS1103 card. A picture of the experimental setup is provided in Fig. 4. The verification of the proposed estimator is done by different experiments for velocity, acceleration and disturbance. The following subsections discuss the details and results of the experiments for different observer configurations.

##### A. Estimation of Velocity

In order to compare the velocity estimation results two different sets of experiments are made. In the first set, a constant velocity reference was imposed and this constant reference was changed from  $0.002m/s$  to  $0.003m/s$  and  $0.001m/s$  consecutively during motion. In the second set, a trapezoidal velocity reference is imposed to the plant and the response is recorded. The rising and falling edges of the reference had a  $\frac{0.005}{3}m/s^2$  slope with a peak constant velocity of  $0.005m/s$ . The velocity estimation results for

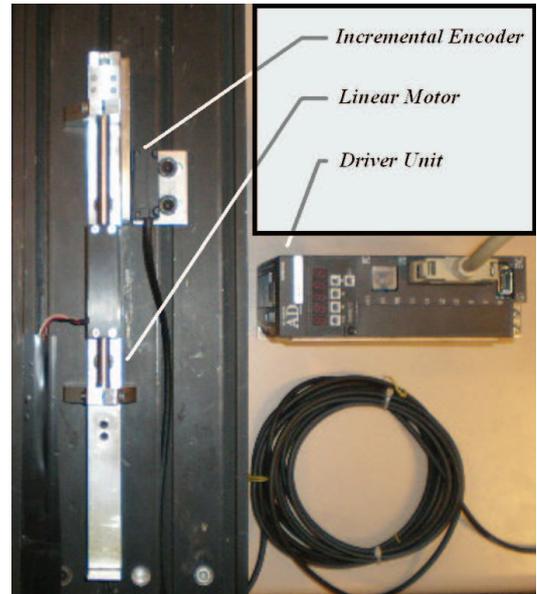


Fig. 4. Picture of the Experimental Setup

those experiments are provided in Fig. 5 and Fig. 6 respectively. In each figure, A-Type response is the response of the proposed functional observer and B-Type response is the response of classical first order differentiator used with a low pass filter. The frequency for the low pass filters of both the functional observer and the classical observer was selected to be 500 Hz. As the graphs show, the performance of the observer in estimating the velocity is much better than the classical observer. The reduction in the noise level and the rise in precision proves the success of functional observer. Besides the graphical data, the statistical data related to the experiments also provide insight about the improvement of the method. The standard deviations in the constant velocity region for the first experiment came out to be  $\sigma_1 = 0.0792$  and  $\sigma_2 = 0.1374$  for functional and classical observer outputs respectively. Similarly for the second experiment, the standard deviations were  $\sigma_1 = 0.0694$  for functional observer and  $\sigma_2 = 0.1306$  for the classical observer. From those results, is clear that the proposed observer can estimate the same velocity with almost half of the noise that can be measured in classical observers.

##### B. Estimation of Acceleration

The acceleration estimation results are tested with a different experiment. In acceleration experiment, consecutive positive and negative pulse references are given to the system and the estimation responses are observed. The amplitude of the pulse reference was selected to be  $10m/s^2$  acting on the system for a period of  $0.05s$ . The results of the proposed observer are compared to the results obtained from a double filtered second order differentiator (i.e. classical acceleration observer). In order to have a better comparison of the observed accelerations, one needs

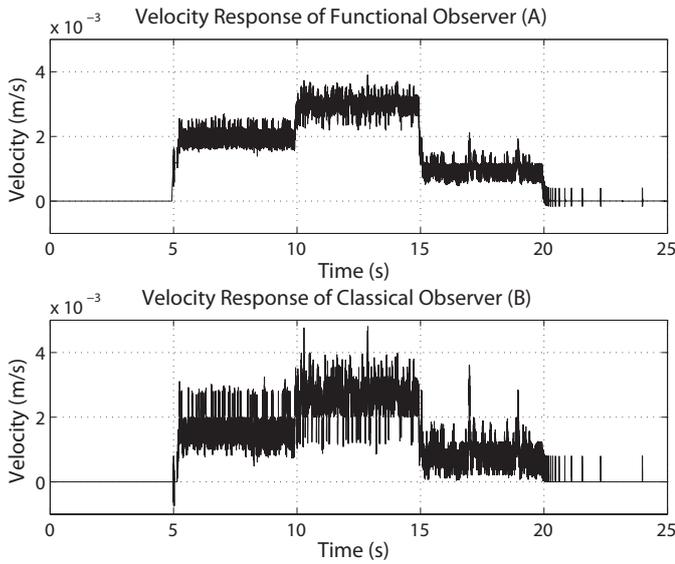


Fig. 5. Comparison of Velocities from Functional Observer (A) and Classical Observer (B) Under Constant Velocity Reference

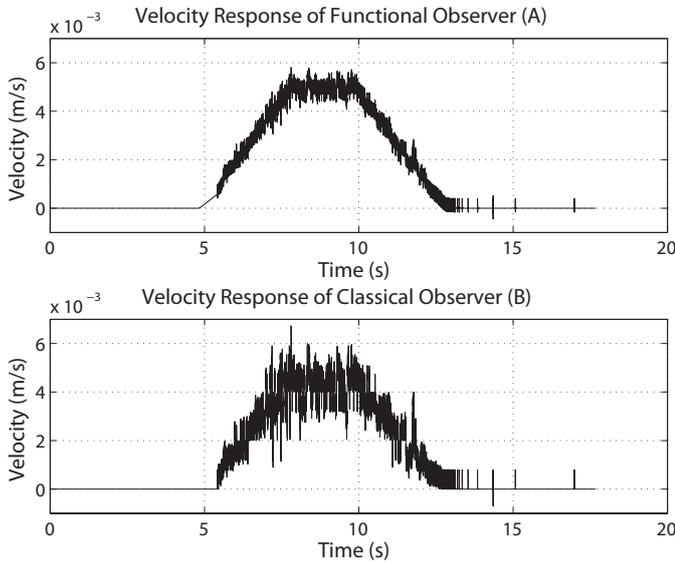


Fig. 6. Comparison of Velocities from Functional Observer (A) and Classical Observer (B) Under Trapezoidal Velocity Reference

the actual acceleration response of the system. For that purpose, the position response of the same experiment is double differentiated in an offline setting and put to the same plot. For numerical differentiation, the three-point estimation approach is used.

The acceleration estimates of the functional observer and classical observer are given in Fig. 7 along with the actual acceleration response. For both observers, the low-pass filter gains are selected to be 200 Hz. When the results are compared, it becomes obvious that the tracking performance of the functional observer is much better than that of the classical observer. Those graphs show

the effectiveness of the implemented methodology, namely using current input in estimation to eliminate the unmeasured disturbances.

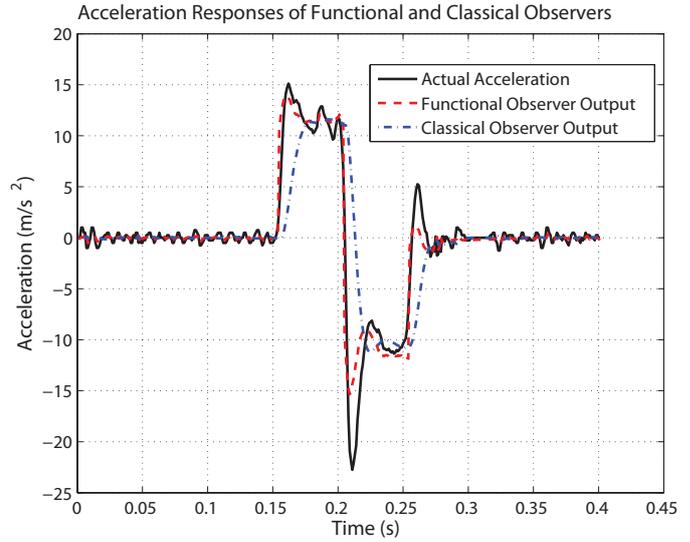


Fig. 7. Comparison of Accelerations from Functional Observer (A) and Classical Observer (B) Under Constant Velocity Reference

### C. Estimation of Disturbance

For comparison of disturbance estimation responses, a constant velocity reference tracking experiment is done. During the experiment, output of classical disturbance observer is compared to that of functional observer. Figure 8 show the disturbance estimation results for the proposed functional observer and classical disturbance observer respectively. Like the velocity observers, the functional disturbance observer is capable of making the same estimation with almost half of the noise obtained from classical disturbance observer.

## V. DISCUSSION

The performance of functional observer especially in more accurate velocity estimation makes it preferable over conventional filtered derivative. Smoother velocity estimation bring the advantage of acquiring a better precision in many systems involving motion control. Moreover, the estimation in velocity is as fast as the classical estimator. This means, although the noise in estimation is reduced dramatically, the bandwidth almost remains the same, which is also another advantage of the proposed structure.

Besides velocity, much faster acceleration estimation can be made with the proposed functional observer in comparison to filtered double differentiators. Although the acceleration information is usually not directly used, in many settings it is integrated to obtain a feed forward term. Having faster response in acceleration estimation would decrease the integration error resulting in a better controller performance.

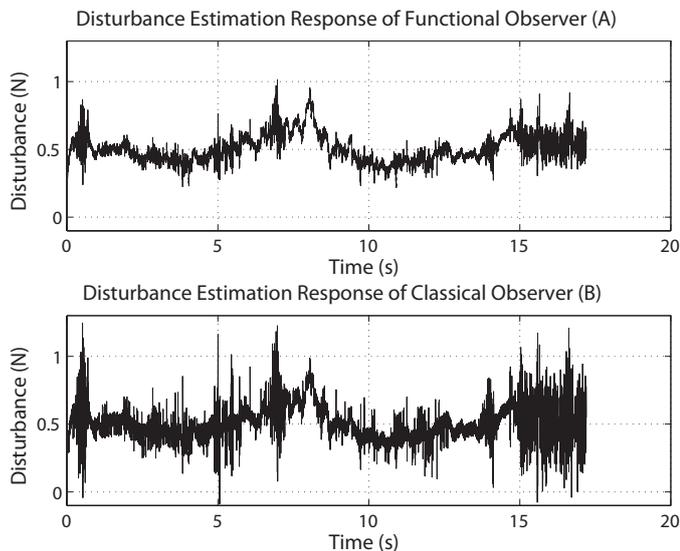


Fig. 8. Comparison of Disturbances from Functional Observer (A) and Classical Observer (B) Under Constant Velocity Reference

On the other hand, since the speed of the disturbance observer is very crucial in the robustness of the system, instead of using a double filtered estimation, the use of classical disturbance observer might still perform better in control loop. Smoothed disturbance estimation from the observer can be a better candidate for external torque/force reconstruction.

## VI. CONCLUSION

In this paper, a functional observer is presented. The observer is capable of estimating the velocity, acceleration and disturbance information of a motion control system only by a change in the configuration parameters. In addition to the position measurement, the estimator benefits from estimating and eliminating the disturbance effects by using the measured input current and plant's nominal parameters. The theoretical development of the estimator has been validated through experiments.

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