Welfare Improving Product Bans

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Comments are welcome

Abstract

We formulate a model of vertical differentiation to evaluate the welfare effects of removing a low quality product from the market. The mechanism through which a welfare improvement might arise is simple: Once the low quality low cost alternative is banned, entry into the high quality segment becomes more likely. This in turn may lead to a significant reduction in the price of the high quality product. We find that such a ban might improve aggregate welfare when consumers value the higher quality more, the marginal cost of producing high quality is lower, the price of low quality is higher, and the price sensitivity for high quality is not too high.

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1 Introduction

On June 15, 1999 the European Commission passed a directive (the 'ban'), requiring that egg laying hen cages be phased out by 2012. According to the Directive, producers will have the choice to go "free-range or to use "enriched" cages where stocking densities are lower and that must have a nest, litter, perch and clawing board.[1] The directive is mainly a result of the lobbying efforts of animal welfare activists. Traditional cages increase the risk of disease, bone breakage and mortality among as hens do not have sufficient space to engage in a minimal amount of physical activities. European egg farmers, obviously, strongly oppose the directive. Several market studies mention the high costs of converting exiting farms into those that comply with the directive as well as the higher marginal costs of producing non-conventional eggs. Currently, 20% to 30% of all eggs produced in Europe come from free-range farms, and they are priced significantly higher than conventional eggs. Griffith and Nesheim (2008) find that household willingness to pay for organic and free range products is heterogenous, but significantly positive for the majority. In the foreground of all this is an intense public policy debate regarding the potential welfare costs of the ban to Europe.

A similar policy change has occurred in California, U.S. recently. In the November 2008 elections Californians have voted overwhelmingly in favor of a ban on the use of conventional cages in egg production.[2] The California ban stirred up heated debates between animal rights groups, industry members and the like. In fact, neighboring states like Idaho and Nevada are actively trying to lure Californian egg farmers to their states.[3] An economic impact study prepared by Sumner et al. (2008) predicts that the egg production in California will be eliminated until 2015, the date when the ban becomes effective.

Although such bans would definitely improve animal welfare in Europe and the U.S., they will lead to several changes that might have opposing effects on consumer welfare. On the one hand, under the assumption that conventional and non-conventional eggs are substitutes, the removal of a substitute good from the market is likely to increase the price of non-conventional eggs. On the other hand, as more traditional farms convert to non-conventional egg production and supply increases, there will be a downward pressure on the price. It is conceivable that the second effect

[1] See Appelby (2003) for a historic account of the events that led to the ban in 1999.

[2] The official title of measure was Proposition 2, or the Standards for Confining Farm Animals, and it passed with a % 63.5 percent of the votes.

[3] A detailed account of the California ban can be found at `http://ballotpedia.org/wiki/index.php/California_Proposition_2_(2008)`.

dominates the first one, such that the market price of non-conventional eggs decreases. Although a reduction in the price of non-conventional eggs does not guarantee an increase in consumer welfare, it is a necessary condition that, under certain conditions and combined with an overall higher willingness to pay for non-conventional eggs, could lead to such a change. Nevertheless, the impact on egg producers in areas affected by the ban is likely to be negative. For example, Sumner et al. (2008) predict most of production for the California market to shift to the neighboring states which have not imposed such restrictions while they do not predict significant investment in California in facilities that satisfy the restrictions imposed by the ban. It is likely then that the ban will have an impact on the organization of egg production.

The motivation for such measures both in Europe and US is based on moral grounds, and these measures are strongly supported by animal welfare groups. In both cases, the industry opposes the ban vehemently, producing reports with predictions which barely fall short of qualifying for doomsday scenarios. They commonly predict loss of jobs and loss of production facilities to areas without restrictions. Given the lack of economic justification for the bans, and predicted reorganization of industries that would negatively impact the states which enact such policies, it is curious whether such policies, which ban certain production technologies in favor of more costly ones, can ever be a wise policy option. An important caveat to note here is the higher willingness to pay of consumers for eggs produced by non-conventional methods, as reported by Griffith and Nesheim (2008). This suggests a perceived quality difference between eggs produced by means of conventional eggs and those produced by more humane ways envisioned by the ban policies.

Framed in this context, it is clear that the question is more general than the ban on certain egg production technologies in favor of others. The question is whether the diffusion or consumption of a new, more expensive, yet higher quality product can be encouraged by banning the production of its lower quality lower cost alternative. Furthermore, the follow up question is whether such a policy move could ever prove desirable from the perspective of consumers and the society at large.

More concretely, we ask the following question in this paper: In a vertically differentiated market where the low quality product is widely available and the high quality product has a limited supply, could the removal of the low quality product increase welfare as it encourages entry into the high quality segment of the market? As can be seen from our research question, we do have a mechanism in mind on how a welfare improvement might arise. The idea is simple: Once the low quality low cost alternative is banned, entry into the high quality segment becomes more likely. As a result of entry, the price of the high quality good is likely to fall. If this decline in the price is sufficiently large, it might yield a significant increase in consumer surplus so much so that it offsets the additional entry costs that firms need to incur, resulting in an overall increase in welfare. Whether this mechanism works, and under what conditions it might work however is not obvious.

In principle, this phenomenon could be relevant in any type of industry with vertical product differentiation or with a steady rate of technological innovation, such as the software industry. It is of great economic importance to identify the economic factors that lead to an inefficient adoption of a higher quality (more generally, a higher valued) product. In this respect, the problem is similar to the phenomenon of 'excess inertia', which is relevant in network industries. However, we abstract from this effect as we try to show that even in the absence of network effects a market may be dominated by a lower value product for too long, or maybe indefinitely.

In order the address the question we raised above, we develop a model of vertical differentiation.[4] Consumers have a perceived valuation of both high and low quality products. Conditional on their valuations, each consumer has a log-linear demand function for the high and low quality products as a function of their prices. Consumers perceive all high quality variants as homogenous. The same holds true for all low quality variants. Consumers, however, differ in their respective valuations. Each consumer draws a bivariate vector of valuations from a modified bivariate exponential distribution taking into account the condition that the high quality valuation is higher than that of the low quality. Were both products to be sold at the same price, all consumers would obviously prefer the high quality product. When prices differ however, those consumer who value the high quality significantly more opt for the high quality product, while others prefer to purchase the low quality product.

We first derive an aggregate demand function for the high quality product when it competes with a low quality alternative that is supplied by a competitive fringe. We then implement the ban by setting the price of the low quality product to infinity, and derive the corresponding aggregate demand for the high quality product after a ban. Both aggregate demand functions have the same general exponential form. The effect of a ban on the aggregate demand for high quality is two-fold. First demand elasticity decreases as result of the ban. Second, provided that the high quality product's price exceeds the price of the low quality alternative, the demand function for the high quality product shifts outward as well. Both of these effects are likely to increase the

[4] We do not use the standard Shaked and Sutton (1982) model. Because of the inelastic unit demands in such a model, regardless of the change in the price of the high quality good, the contribution to the aggregate welfare from the consumers who purchase the high quality product before and after a ban is zero when production takes place using a constant returns to scale technology.

profitability of the high quality product.

We then move to the analysis of competition between firms in the high quality segment. Given the homogeneity of products in the high quality segment, we adopt a Cournot model to study the competition between firms. We assume that the high quality variants can be produced with a constant returns to scale technology, however, a firm who wishes to produce needs to incur a fixed cost of entry. Given the exponential nature of the aggregate demand function both before and after a ban, the equilibrium prices have a similar form, and are independent of the price of the low quality alternative, while they depend on the inherent price sensitivity of demand, the average perceived quality differential as well as the number of active firms. Our results indicate that as long as the number of active firms that produce the high quality product becomes larger than it was before the ban, and the average perceived quality differential are sufficiently high, the price of the high quality product decreases as a result of the ban.

We next restrict our attention to a specific scenario in order to obtain relatively simple analytical results. We consider a situation where the high quality segment is served by a single firm before the ban, while after the ban two firms serve the high quality segment in a free entry equilibrium.[5] We first derive the conditions under which the ban results in an increase in aggregate consumer surplus. We find that the average perceived quality differential has to be sufficiently high. This alone is not sufficient however. In addition, either the inherent price sensitivity of consumers has to be sufficiently low and/or the marginal cost of producing the high quality should not be too different from the price of the low quality alternative for the ban to lead to an increase in consumer surplus. The region of parameters where the ban yields an increase in aggregate welfare has similar properties, but is naturally smaller. The main reason for this is the requirement that the consumer surplus increase substantially in order to offset the loss of monopoly profits earned by the monopolist prior to a ban. Nevertheless, we find that there exists a nonnegligible region in the feasible parameter space where banning a low quality alternative results in an increase in aggregate welfare.

The next section introduces the basic model. In section 3, we investigate the welfare effects of a ban. Section 4 concludes.

[5] As the number of active firms before a ban increases, the difference between the before and after ban prices erodes, hence it becomes unlikely that a ban would produce consumer benefits. However, we also disregard the possibility that more than two firms supply the high quality after the ban. In such a case, the price of the high quality product would be more likely to fall.

2 Model

2.1 Demand

Consider a market with a continuum of consumers. Consumers choose among two products: a vertically differentiated good and the numeraire good. Suppose that the utility of a consumer is given by the following quasilinear utility function

$$u(x, y; s) = \frac{x(\delta + s + 1) - x\ln(x)}{\beta} + y$$

where $s > 0$ denotes the quality and x denotes the quantity of the non-numeraire good being consumed, y is the quantity of the numeraire good, and $\delta > 0$ and $\beta > 0$ are common taste parameters. Individual utility maximization with respect to a budget constraint leads to the following demand function for the non-numeraire good

$$d(p, s) = \exp(\delta + s - \beta p) \tag{1}$$

and a corresponding indirect utility function of

$$V(p, s) = \frac{\exp(\delta + s - \beta p)}{\beta}. \tag{2}$$

We assume that there are two possible variants of this product and that consumers choose only one of the two quality variants. This framework is in line with the motivation we have put forward in the introduction; consumers choosing between a generic, low quality product and a newer, more advanced, higher quality product. Even though consumers are heterogenous with respect to their valuations of the two variants, they all value the higher quality product more. In this respect, there is a typical vertical differentiation between the two variants of the same product. However. unlike the standard model due to Shaked and Sutton (1982), our framework allows for elastic individual demands.

Let us label the two variants as H for "High Quality" and L for "Low Quality". Consumer heterogeneity in tastes for quality will come from the differences in the parameters s_H and s_L. We assume each consumer draws a bivariate vector of tastes for quality from the joint density function

$$f(s_H, s_L) = \begin{cases} \frac{(\lambda_H + \lambda_L)}{\lambda_H^2 \lambda_L} e^{-\frac{s_H}{\lambda_H}} e^{-\frac{s_L}{\lambda_L}} & s_H > s_L > 0, \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

with $0 \leq \lambda_H < 1$ and $0 \leq \lambda_L < 1$. The restrictions on λ_H and λ_L are required for the integrability of individual demands over the population that comes into play during the derivation of aggregate demands. Note that the draws of an individual are not independent. If a consumer draws a high s_L, her draw for s_H must be even higher. Alternatively, if the realization for the taste for high quality is low, the realization of the low quality taste must be even lower. Thus, each consumer's tastes for quality are positively correlated.[6] We assume however that the draws of different consumer are independent of one another. Of the two density parameters, λ_H plays an important role. Given the bivariate density function in (3), it turns out that $\lambda_H = E(s_H - s_L)$, i.e. it measures the average (expected) quality difference between the high and low quality variants.

Essentially when both qualities are sold at the same price, all consumers are going to prefer the high quality variant. In other words

$$V(p, s_H) > V(p, s_L) \quad \text{for all } p.$$

This implies that there will be positive sales for the lower quality variant only when the higher quality variant is priced higher. In order to calculate the proportion of consumers who choose the high quality variant, one needs to find the region on the domain of the bivariate random vector $[s_H, s_L]$ such that

$$V(p_H, s_H) \geq V(p_L, s_L).$$

Given the indirect utility function in (2), the set of consumers who choose to buy the high quality variant is given by the region

$$s_H \geq s_L + \beta(p_H - p_L).$$

And the set of consumer who choose to buy the low quality variant will then be given by

$$s_L \leq s_H < s_L + \beta(p_H - p_L).$$

The demand functions for the two quality variants can then be obtained by calculating the expected value of the demand in (1) over the corresponding regions of $[s_H, s_L]$. Formally,[7]

$$\begin{aligned} d_H^C(p_H, p_L) &= \int_{s_L=0}^{\infty} \int_{s_H=s_L+\beta(p_H-p_L)}^{\infty} d(p, s_H) f(s_H, s_L) ds_H ds_L \\ &= \frac{(\lambda_H + \lambda_L)}{(\lambda_H + \lambda_L - \lambda_H\lambda_L)(1-\lambda_H)} e^{\{\delta - \beta p_L - \frac{\beta(p_H - p_L)}{\lambda_H}\}} \\ &= e^{A^C - \alpha^C p_H}. \end{aligned}$$

[6]The correlation coefficient between the two elements of the bivariate vector is given by $\rho = \frac{\lambda_L}{\sqrt{(\lambda_H+\lambda_L)^2+\lambda_L^2}}$.

[7]The demand for the low quality product can be computed similarly as

$$\begin{aligned} d_L^C(p_H, p_L) &= \int_{s_L=0}^{\infty} \int_{s_H=s_L}^{s_L+\beta(p_H-p_L)} d(p, s_L) f(s_H, s_L) ds_H ds_L \\ &= \frac{(\lambda_H + \lambda_L)}{(\lambda_H + \lambda_L - \lambda_H\lambda_L)(1-\lambda_H)} e^{\delta - \beta p_L} (1 - e^{-\frac{\beta(p_H - p_L)}{\lambda_H}}). \end{aligned}$$

where $A^C = \delta - \beta p_L(1 - \frac{1}{\lambda_H}) + \ln(\frac{\lambda_H+\lambda_L}{(\lambda_H+\lambda_L-\lambda_H\lambda_L)(1-\lambda_H)})$, and $\alpha^C = \frac{\beta}{\lambda_H}$.

Now, consider the case where the low quality variant is removed from the market. This 'ban' is basically equivalent to setting the price of the low quality variant to infinity. In this case, all consumer purchase only the high quality product as long as its price is finite. The demand for the high quality variant becomes much easier to compute, as consumers do no longer have the option of choosing the low quality alternative. The demand function for high quality after the ban is given by

$$\begin{aligned} d_H^B(p_H) &= \int_{s_L=0}^{\infty}\int_{s_H=s_L}^{\infty} d(p,s_H)f(s_H,s_L)ds_H ds_L \\ &= \frac{(\lambda_H+\lambda_L)}{(\lambda_H+\lambda_L-\lambda_H\lambda_L)(1-\lambda_H)}e^{\{\delta-\beta p_H\}} \\ &= e^{A^B-\alpha^B p_H}. \end{aligned}$$

where $A^B = \delta + \ln(\frac{\lambda_H+\lambda_L}{(\lambda_H+\lambda_L-\lambda_H\lambda_L)(1-\lambda_H)})$, and $\alpha^B = \beta$.

At this stage, we can make a few observations regarding the effects of a ban on the aggregate demand for the high quality alternative. The 'ban' clearly lowers the flexibility of consumers regarding their consumption choices, as they can no longer opt for the low quality alternative. First, consider the effect of the ban on the price elasticity of aggregate demand. The price elasticity of demand, in absolute value, before the ban is given by $\frac{\beta p_H}{\lambda_H}$, while after the ban it is given by βp_H. Since $\lambda_H < 1$ by assumption, demand becomes less elastic after the ban. This fact plays an important role in the pricing decisions of the firms that produce the high quality variant. Next compare the demand for the high quality product before and after the ban for the same price p_H. Namely,

$$d_H^B(p_H) - d_H^C(p_H,p_L) = d_H^B(p_H)(1 - e^{\{-\frac{\beta(1-\lambda_H)}{\lambda_H}(p_H-p_L)\}})$$

which is positive as long as $p_H > p_L$ and $\lambda_H < 1$. At prices that exceed the price of the low quality alternative, the demand for the high quality product after the ban is always higher. In summary, after the ban, the aggregate demand for the high quality product increases and becomes less elastic. Both changes are likely to increase the profitability of the high quality segment. This is exactly the reason why we expect a greater number of firms to be active after a ban.

2.2 Supply and Market Equilibrium

The two quality segments of the market are served by separate firms. We assume that the low quality variant is supplied by a competitive fringe at an equilibrium price of p_L. The firms that produce the high quality variant compete in quantities, 'a la Cournot'. They have identical production technologies with a constant marginal cost equal to k. In addition, high quality firms have to incur a one-time fixed entry cost of F. One can think of the additional fixed cost of producing the high quality variant as the cost of converting or upgrading to that technology of production.[8]

Given the similar shape of the aggregate demand functions for the high quality before and after the ban, we will derive equilibrium prices and quantities for a generic demand with parameters A and α, where $A \in \{A^C, A^B\}$ and $\alpha \in \{\alpha^C, \alpha^B\}$. Assuming that there are N active firms in the high quality segment, and all firms but firm i producing symmetric quantities of q, one can write the inverse demand as

$$p_H = \frac{A - \ln(q_i + (N-1)q)}{\alpha}.$$

Consequently, the profit function of firm i is then given by

$$\pi_i = \left[\frac{A - \ln(q_i + (N-1)q)}{\alpha} - k\right] q_i.$$

Solving for the symmetric Cournot equilibrium, the equilibrium quantities and the equilibrium price for the high quality variant become

$$q^*(A, \alpha, N) = \frac{e^{A-k\alpha-1/N}}{N}$$

and

$$p^*(A, \alpha, N) = k + \frac{1}{N\alpha}.$$

Note that the equilibrium price does not depend on A but it depends on α. As a consequence, while the equilibrium prices will be affected by the change in the demand elasticities before and after the ban, they are unaffected by the price of the low quality alternative. This is due to the stylized model of demand we adopt. On the other hand, the equilibrium production of the high quality product increases with the price of the low quality product before the ban. When the price of the low quality product is low, the high quality product is consumed by a small niche of consumers which has substantially higher valuations of its quality.

Subsequently, per firm equilibrium profits are given by

$$\pi^*(A, \alpha, N) = \frac{e^{A-k\alpha-1/N}}{N^2\alpha} - F$$

where naturally $\frac{\partial \pi^*(A,\alpha,N)}{\partial N} < 0$, i.e. profits decrease with the number of active firms with or without a ban.[9]

[8] For instance, converting conventional egg farms to free-range farms involves a one-time fixed cost. As stated in the introduction, empirical evidence also predicts higher variable or marginal costs of production for the higher quality product. Our model will capture this if $k > p_L$.

[9] $\frac{\partial \pi^*(A,\alpha,N)}{\partial N} = -\frac{(2N-1)\exp(A-k\alpha-1/N)}{N^4\alpha} < 0$ for $N \geq 1$.

Furthermore, when the number of active firms serving the high quality segment remains the same, those who operate in a market where the low quality product is banned earn higher profits.

Lemma 1 *Per firm profits after the ban are higher than per firm profits before the ban:* $\pi^*(A^B, \alpha^B, N) > \pi^*(A^C, \alpha^C, N)$.

Proof. See the Appendix. ■

Lemma 1 implies that in free entry equilibrium, we would expect to see more active firms in a market after the ban. A special case of interest occurs when there are only two active firms. There is a range of values for the fixed cost parameter F such that a second firm does not find it profitable to enter the high quality segment unless the low quality is removed from the market. Consequently, for a certain range of fixed costs we have $\pi^*(A^B, \alpha^B, 2) > 0 > \pi^*(A^C, \alpha^C, 2)$. For much of the following welfare analysis we will assume this to be the case.

Before starting our analysis of the effects of a ban on consumer surplus and welfare, we establish a simple result regarding the change in the price of the high quality variant as a result of the ban. Lemma 1 suggests that more firms will be active in the high quality segment after the institution of a ban. Let N denote the number of firms active before the ban and let $N + M$ firms be active after the ban.

Proposition 1 *The price of the high quality product is lower after a ban whenever* λ_H *is greater than the ratio of the pre- to after ban number of firms that supply high quality. That is,* $\Delta p_H = p_H^*(A^C, \alpha^C, N) - p_H^*(A^B, \alpha^B, N + M) > 0$ *whenever* $\lambda_H > \frac{N}{N+M}$*. For such values of* λ_H*, however,* Δp_H *decreases with the number of active firms prior to a ban* N.

Proof. See the Appendix. ■

The implication of Proposition 1 is that whenever there are sufficiently many active firms serving the high quality segment prior to a ban, the likely impact of a ban of the low quality product would be to reduce prices, albeit not drastically. Thus, if we wish to find situations where a ban would improve consumer surplus—a prerequisite for improving welfare—, those instances where a relatively small number of firms serve the high quality segment prior to a ban would be good candidates. Henceforth, we will restrict our attention to a situation where only a monopoly firm is active prior to a ban, and two firms are active after the ban.

3 Welfare Impact of the Ban

3.1 Consumer Welfare

It is clear that, in the event of the removal of the low quality variant from the market, any increase in consumer surplus may only result from a significant lowering of the price of the high quality variant. Consumers with draws of s_H very close to s_L, would be worse off due to a ban as long as the price of the high quality product is above the price offered by the competitive fringe for the low quality product. In order to offset this loss in surplus, those consumers with higher realizations of s_H must become significantly better off as a result of the ban. Since the demand for the high quality becomes less elastic after the ban, this is only possible if the supply of high quality increases through the entry of new firms.[10]

We will first derive the consumer surplus for a generic number of firms in the two subcases: with and without the low quality variant in the market. Obviously, a decrease in prices as demonstrated in proposition 1 is a necessary but not sufficient condition for consumer surplus to increase. When consumers have the option to buy either variant, their surplus becomes the sum of indirect utilities from consuming either quality. Namely, consumer surplus before the ban is

$$\begin{aligned} CS^C(p_H, p_L) &= \int_{s_L=0}^{\infty}\int_{s_H=s_L+\beta(p_H-p_L)}^{\infty} V(p_H, s_H) f(s_H, s_L) ds_H ds_L \\ &\quad + \int_{s_L=0}^{\infty}\int_{s_H=s_L}^{s_L+\beta(p_H-p_L)} V(p_L, s_L) f(s_H, s_L) ds_H ds_L \\ &= \frac{(\lambda_H+\lambda_L)\left(e^{\delta-\beta p_L}(1-\lambda_H+\lambda_H e^{\frac{\beta p_L-\beta p_H}{\lambda_H}})\right)}{(\lambda_H+\lambda_L-\lambda_H\lambda_L)(1-\lambda_H)\beta} \end{aligned}$$

whereas after the ban surplus consists of only the sum of indirect utilities from the consumption of the high quality variant, and depends only on the price of the high quality

$$\begin{aligned} CS^B(p_H) &= \int_{s_L=0}^{\infty}\int_{s_H=s_L}^{\infty} V(p_H, s_H) f(s_H, s_L) ds_H ds_L \\ &= \frac{(\lambda_H+\lambda_L)e^{\delta-\beta p_H}}{(\lambda_H+\lambda_L-\lambda_H\lambda_L)(1-\lambda_H)\beta}. \end{aligned}$$

The removal of the low quality variant changes the market outcome in two different ways. On the one hand, as the demand for the high quality variant becomes less elastic, there is an upward pressure on its price. Moreover, the removal of a substitute product raises the demand for the high quality product. Both of these effects increase the profits of an active firm in the high

[10] When the equilibrium number of firms remains the same after the ban, the price of the of the high quality variant changes by $\frac{1-\lambda_H}{N\beta} > 0$.

quality segment. As a result, new firms will be induced to enter the market, and this will put a downward pressure on the equilibrium price. The net effect on the equilibrium price depends on the magnitudes of these two effects. As stated earlier, one also needs to take into account the welfare loss of the consumers who after the ban have to switch to their less preferred product. The highly parametric nature of the demand model makes it difficult to establish concrete results on the net change in consumer surplus. Besides, as we have already shown in proposition 1, when sufficiently many firms are active prior to a ban, the likelihood of an increase in consumer surplus and welfare following the ban decreases. To this effect, we concentrate on a pre-ban monopolistic high quality segment. We will then identify the parameter regions where without a ban additional entry does not occur, but where the ban leads to 1) the entry of at least one additional firm into the high quality segment, and 2) an improvement in consumer and aggregate welfare.

Substituting in the equilibrium price of the high quality product where $N = 1$ firms are active before the ban and $N = 2$ firms are active after the ban, the expressions for consumer surplus become

$$CS^C(1, p_L) = \frac{(\lambda_H + \lambda_L)\left(e^{\delta - \beta p_L}\left(1 - \lambda_H + \lambda_H e^{\frac{\beta p_L - \beta k}{\lambda_H} - 1}\right)\right)}{(\lambda_H \lambda_L - \lambda_H - \lambda_L)(\lambda_H - 1)\beta}$$

and

$$CS^B(2) = \frac{(\lambda_H + \lambda_L)e^{\delta - \beta k - \frac{1}{2}}}{(\lambda_H \lambda_L - \lambda_H - \lambda_L)(\lambda_H - 1)\beta}$$

Under the scenario in consideration, the ratio of consumer surpluses is equal to

$$\frac{CS^C(1)}{CS^B(2)} = e^{\beta(k - p_L) + \frac{1}{2}}\left(1 - \lambda_H + \lambda_H e^{-1 - \frac{\beta(k - p_L)}{\lambda_H}}\right).$$

Note that the ratio of consumer surpluses depends only on four parameters: λ_H, β, k, and p_L. Furthermore, the last three of those parameters enter the ratio in a single term: $\beta(k - p_L)$.This suggests that we can conduct our welfare analysis on a two-dimensional parameter space. Denote $z \equiv \beta(k - p_L)$.[11] Then we have

$$\frac{CS^C(1)}{CS^B(2)} = e^{z + \frac{1}{2}}\left(1 - \lambda_H + \lambda_H e^{-1 - \frac{z}{\lambda_H}}\right).$$

We show in the appendix that, this ratio of consumer surpluses is monotonically decreasing in λ_H for $\lambda_H \in [0, 1]$. Furthermore, for all values of z, $\frac{CS^C(1)}{CS^B(2)}$ is greater than 1 at $\lambda_H = 0$ and it is less than 1 at $\lambda_H = 1$. These findings are brought together in the following proposition.

[11]It turns out that z is a convenient function of these three parameters. Changes in z have a very clear interpretation in terms of the changes in these parameters. The value of z decreases as the marginal cost of producing the high quality variant approaches the price of the low quality variant and/or as the price elasticity of the high quality decreases.

Proposition 2 *There is a value $\tilde{\lambda}_H(z) \in (0,1)$, such that for all $\lambda_H > \tilde{\lambda}_H(z)$, we have $\frac{CS^C(1)}{CS^B(2)} < 1$. Thus, as a result of the ban, consumer surplus will rise if λ_H is sufficiently large.*

Proof. See the above discussion for the intuition behind the result and the appendix for the derivations. ■

It is interesting to further investigate how the critical value of the average quality difference, $\tilde{\lambda}_H(z)$, depends on the value of z—and consequently on the remaining parameters of the model. For example, at $z = 0$, which occurs when $p_L = k$, the critical value becomes $\tilde{\lambda}_H = 0.62$. This implies that for all $\lambda_H \in (0.62, 1]$, the ratio of consumer surpluses is going to be less than 1 and the ban improves consumer welfare. For values of $z > 0$, the behavior of $\tilde{\lambda}_H(z)$ is not immediately clear. Hence, we first identify the link between z and $\tilde{\lambda}_H(z)$ by taking the total derivative of $\frac{CS^C(1)}{CS^B(2)} = 1$ to compute $\frac{d\lambda_H}{dz}$ as:

$$\frac{d\tilde{\lambda}_H(z)}{dz} = \frac{\lambda_H(z)(1-\lambda_H(z))\left(1 - e^{-\frac{z}{\lambda_H(z)}-1}\right)}{\lambda_H - (\lambda_H(z)+z)e^{-\frac{z}{\lambda_H(Z)}-1}}$$

which is positive for all z and for all λ_H.[12] The implication is that the critical value $\tilde{\lambda}_H(z)$ increases in z. Since z simply equals $\beta(k - p_L)$, we have the obvious result that an increase in the marginal cost of production of high quality suppliers will restrict the parameter region where consumer surplus rises after the ban. The next proposition summarizes the effects of the parameters β, p_L, and k on the ratio of consumer surpluses.

Proposition 3 *An increase in k raises, and an increase in p_L lowers the critical value $\tilde{\lambda}_H$, where for all $\lambda_H > \tilde{\lambda}_H(z)$ consumer surplus rises after the ban. The effect of an increase in β on $\tilde{\lambda}_H(z)$ is positive whenever $k > p_L$.*

Proof. See the Appendix. ■

Proposition 3 describes the mechanism underlying the change in consumer surplus after the ban. As the production of the high quality variant becomes more costly, as the demand for high quality becomes more inelastic when the price of the high quality variant exceeds that of the low quality, and as the price of the low quality variant is lower, the possibility of the ban leading to an increase in consumer surplus diminishes. In other words, as the price differential between the two qualities is increasingly in favor of the low quality variant, the ban becomes more costly to the consumers.

[12] See the Appendix for the proof.

In sum, there is a range of the parameters of the model where the removal of the low quality variant from the market leads to a strict increase in consumer welfare as measured by the aggregate consumer surplus. Figure 1 presents $\tilde{\lambda}_H(z)$ as a function of $z > 0$. In the diagonally shaded and crosshatched regions, aggregate consumer surplus increases. As can be seen the most likely cases where a ban results in an improvement in aggregate consumer surplus involves higher values of the average quality difference, lower values of price sensitivity of individual demand and/or a small difference between the marginal cost of producing the high quality product and the price of the fringe.

It should also be noted that the ban does not lead to a Pareto improvement of consumer welfare. In other words, the distribution of consumer surplus changes as some consumers become better-off and some become worse-off. At this point we are only stating results regarding aggregate consumer welfare. Especially those consumers who do not view the two alternatives as being too different in terms of quality will suffer as a result of the ban.

Next, we will state our results regarding aggregate welfare, which also includes firm profits.

3.2 Aggregate Welfare

Let us consider the aggregate welfare effects of the ban on low quality. Our definition of aggregate welfare is a simple sum of consumer surplus and firm profits. Equilibrium firm profits, $\pi^*(A, \alpha, N)$, for a generic number of firms was given in Section 2.2. Again, we have to consider the before and after ban cases separately. Before the ban, the profit of a monopolist is given by

$$\pi^*(A^C, \alpha^C, 1) = \frac{(\lambda_H + \lambda_L)\lambda_H e^{\delta - \beta k - 1}}{(\lambda_H + \lambda_L - \lambda_H \lambda_L)(1 - \lambda_H)\beta} - F$$

and the after ban profit of a duopolist is simply

$$\pi^*(A^B, \alpha^B, 2) = \frac{1}{4} \frac{(\lambda_H + \lambda_L)e^{\delta - \beta k - \frac{1}{2}}}{(\lambda_H + \lambda_L - \lambda_H \lambda_L)(1 - \lambda_H)\beta} - F.$$

For simplicity of the calculations let us equate the fixed cost of production to the maximum value that will sustain two firms after the ban. Namely, set $F = F^{\max}$ such that $\pi^*(A^B, \alpha^B, 2) = 0$. Note that this is the most 'conservative' scenario. For all smaller values of$F$, the duopolists will be making positive profits which would then be added to the after ban measure of welfare.[13] We are in essence providing a lower boundary on the welfare increase following the ban.

[13]When F is lower than this maximum value, the pre-ban monopolist's profit would also increase. However, since there are two active firms after the ban, their total gain in profits will be the double of that of the monopolist.

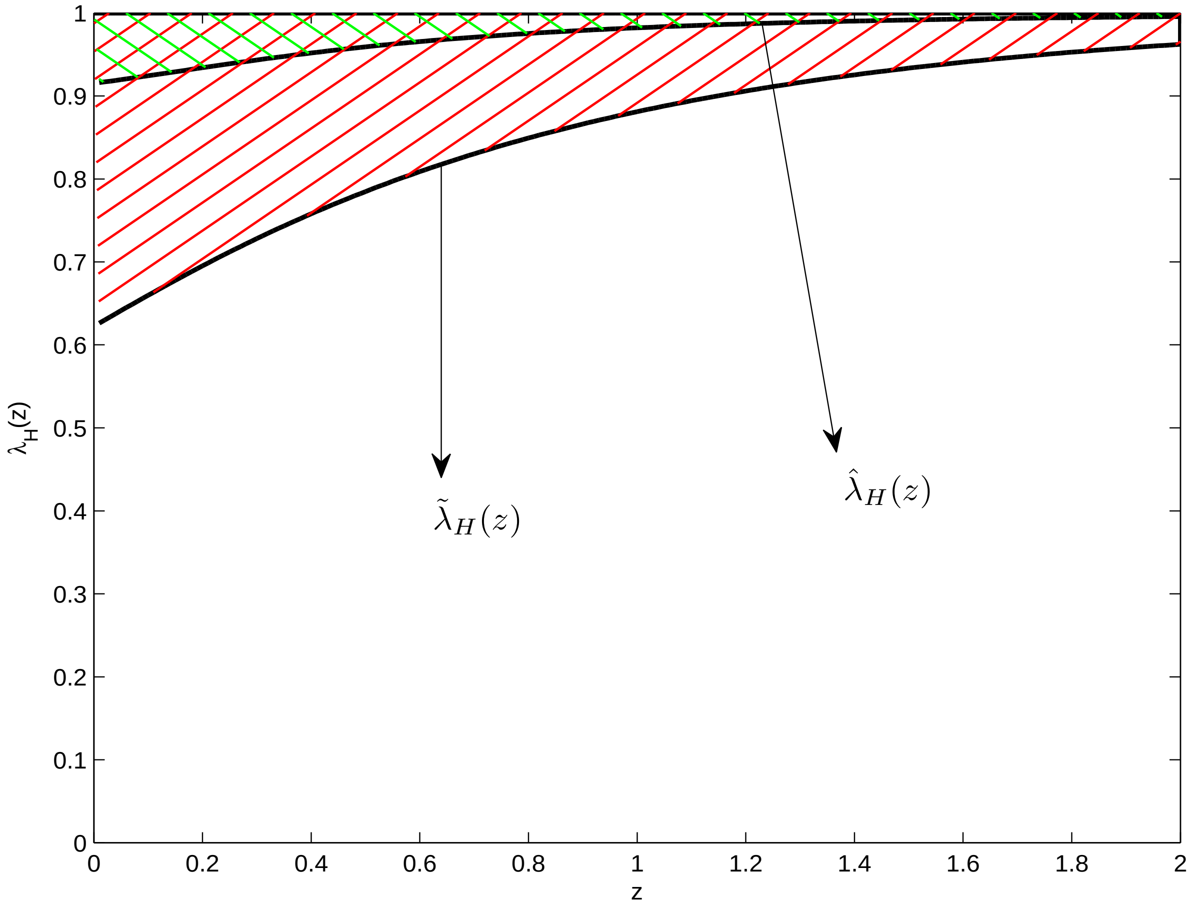

Figure 1: Regions where consumer surplus and welfare improve

Adding the profit terms to the consumer surplus expressions we have calculated earlier, we obtain our measure of aggregate welfare. Our interest essentially lies in the change in this measure which is given by

$$\Delta W = CS^B(2) - CS^C(1, p_L) - \pi^*(A^C, \alpha^C, 1)|_{F=F^{\max}}.$$

It turns out that the sign of ΔW depends on the same set of parameters that determine the consumer surplus ratio: λ_H, β, k, and p_L. Furthermore, once again, the last three of those parameters enter the welfare difference as a single term: $z \equiv \beta(k - p_L)$. The following proposition proves the existence of a welfare improving ban.

Proposition 4 *There is a value $\hat{\lambda}_H(z) \in (0, 1)$, such that for all $\lambda_H > \hat{\lambda}_H(z)$ and $z > 0$ we have $\Delta W > 0$. Therefore, following a ban of the low quality alternative, aggregate welfare will rise if λ_H is sufficiently large.*

Proof. See the Appendix. ■

The idea behind Proposition 4 is very similar to that behind Proposition 2. Welfare after the ban as well as the welfare difference, ΔW, increase in the average quality difference λ_H. There is a critical level of average quality difference, $\hat{\lambda}_H(z)$, such that when λ_H exceeds this threshold, consumers on average value the high quality so much more than the low quality alternative. this in turn lowers the price of the high quality product after additional entry so substantially that the increase in aggregate consumer surplus offsets the loss of monopoly profits before ban. And, in turn, aggregate welfare increases.

Next, let us consider how this critical level, $\hat{\lambda}_H(z)$, depends on the value of z. From the previous discussion on the change in consumer surplus, it is clear that as the marginal cost of producing high quality rises or as the price of the price of the low quality falls, it becomes less likely that the ban improves welfare. Since both of these changes imply an increase in z, we would expect the critical level $\hat{\lambda}_H(z)$ to increase in z. Formally, one needs to take the total derivative of $\Delta W = 0$ and solve it for $\frac{d\lambda_H(z)}{dz}$. Evaluating this expression at $\lambda_H = \hat{\lambda}_H$ gives us the desired result which is stated in the next proposition.

Proposition 5 *The critical level of the average quality difference parameter increases in z: $\frac{d\hat{\lambda}_H(z)}{dz} > 0$. Furthermore, an increase in k raises, and an increase in p_L lowers the critical value $\hat{\lambda}_H$, where for all $\lambda_H > \hat{\lambda}_H(z)$ aggregate welfare rises after the ban. The effect of an increase in β on $\hat{\lambda}_H(z)$ is positive if $k > p_L$.*

Proof. See the above discussion and the Appendix. ■

The effects of all four parameters on the welfare difference function have clear interpretations. An increase in the marginal cost of production of high quality suppliers will restrict the parameter region where aggregate welfare rises after the ban. This makes perfect sense: the higher the costs of producing high quality, the higher the equilibrium price of it. An increase in the price of the low quality substitute, on the other hand, makes the adverse impact of its removal from the market less pronounced. Finally, as β increases both pre and after ban prices fall, however their difference also decreases. This is due to the fact that an increase in β raises the price sensitivity of aggregate demand faster before the ban. It follows that an increase in β lowers the possibility of a welfare improvement.

In figure 1, the crosshatched area on the (λ_H, z) space corresponds to the region where an increase in welfare is realized. It is clear that for all values of z, we have $\hat{\lambda}_H(z) > \tilde{\lambda}_H(z)$. The region that corresponds to a welfare increase is unambiguously smaller than which corresponds to an increase in consumer surplus. This is something to be expected. As stated earlier, we set the fixed cost of production such that our duopolists make zero profits. As the lower quality product is banned, the increase in consumer surplus is in part canceled by the loss of the pre-ban monopoly profit. Nevertheless, the main result we would like to stress out is that there is a non-negligible range of parameters over which the removal of the low quality variant can be welfare improving. Additionally, one should note that the region in figure 1 is based on a conservative benchmark. There are fixed costs of entry below F^{max}, for which entry into the high quality segment by two firms without the removal of the low quality alternative is not profitable. On all these cases, while one duopolist's profit is offset by the loss of monopoly profit when the ban is not implemented, the other duopolist's profit enters as a positive contributitor to the aggregate welfare calculation.

4 Conclusion

Recent regulations in agricultural and animal products markets that disallow a certain subgroup of products resulted in a heated public policy debate. With the help of a theoretical model, we elaborate on this discussion by considering the welfare effects of such a 'ban'. More concretely, we ask if the removal of the low quality product could increase welfare in a vertically differentiated market as it encourages entry into the high quality segment. The mechanism through which a welfare improvement might arise is simple: Once the low quality low cost alternative is banned, entry into the high quality segment becomes more likely. Entry puts a downward pressure on the price of the high quality product. If this decline in the price is sufficiently large, it will yield an increase in consumer surplus which might offset the additional entry costs the new firms need to incur. As a consequence, the resulting change in aggregate welfare can be positive.

In order the address this question, we develop a model of vertical differentiation. Consumers have heterogenous tastes for high and low quality. We first derive an aggregate demand function for the high quality product when it competes with a low quality product that is supplied by a competitive fringe. We then implement the ban by setting the price of the low quality product to infinity, and derive the corresponding aggregate demand for the high quality product after a ban. The effect of a ban on the aggregate demand for high quality is two-fold. First, demand elasticity decreases as a result of the ban. Second, provided that the price of the high quality product exceeds the price of the low quality alternative, the demand function for the high quality product shifts outwards. Both of these effects are likely to increase the profitability of the high quality product.

We then move on to the analysis of competition between firms in the high quality segment. We model the supply of high quality a la Cournot. We assume that the high quality variant can be produced with a constant returns to scale technology; however, a firm who wishes to produce needs to incur a fixed cost of entry. Our results indicate that as long as the number of active firms that produce the high quality product increases, and the average difference of the valuations of quality is sufficiently high, the price of the high quality product decreases as a result of the ban.

In order to obtain relatively simple analytical results regarding the change in consumer and aggregate welfare, we restrict our attention to a specific scenario. We consider a situation where the high quality segment is served by a monopolist before the ban, and by a pair of duopolists afterwards. We first derive the conditions under which the ban results in an increase in consumer surplus and find that the difference of quality valuations has to be sufficiently high, but this condition alone is not sufficient. Additionally, for the ban to lead to an increase in consumer surplus, either the inherent price sensitivity of consumers has to be sufficiently low and/or the marginal cost of producing the high quality should not be too different from the price of the low quality alternative. The region of parameters where the ban yields an increase in aggregate welfare is smaller. The main reason for this is the requirement that consumer surplus increase substantially in order to offset the loss of monopoly profits earned by the monopolist prior to a ban. Overall, we find that there exists a nonnegligible region in the permissible parameter space where banning a low quality alternative results in an increase in aggregate welfare.

In principle, this phenomenon could be relevant in any type of industry with vertical product differentiation or with a steady rate of technological innovation. It is of great economic importance to identify the economic factors that lead to an inefficient adoption of a higher quality (more generally, a higher valued) product. Even though we are unable to provide exact analytical results that pertain to the general case with any number of firms before the ban, we demonstrate the possibility of a persistent market failure under vertical product differentiation. Another step forward would be to generalize the production technologies of the low quality and high quality firms to allow for non-constant returns to scale. When there are diminishing returns to scale in the production of the low quality firm, there can be an added factor of efficiency gains from banning the low quality.

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Appendix

A–1 Proof of Lemma 1

The difference between pre and after ban profit functions for an equal number of firms is given by

$$\begin{aligned}\pi^*(A^B,\alpha^B,N)-\pi^*(A^C,\alpha^C,N) &= \frac{e^{A^B-k\alpha^B-1/N}}{N^2\alpha^B}-\frac{e^{A^C-k\alpha^C-1/N}}{N^2\alpha^C}\\ &= \frac{e^{\delta}(\lambda_H+\lambda_L)\left(e^{\beta k\left(\frac{1-\lambda_H}{\lambda_H}\right)}-\lambda_H e^{\beta p_L\left(\frac{1-\lambda_H}{\lambda_H}\right)}\right)}{N^2\beta e^{\frac{\beta k}{\lambda_H}+N}(\lambda_H\lambda_L-\lambda_H-\lambda_L)(\lambda_H-1)}\end{aligned}$$

The denominator and the first two terms on the numerator are clearly positive. For $k > p_L$ the term inside the parentheses is also positive since $\lambda_H < 1$.

A–2 Proof of Proposition 1

Denoting the number of firms before the ban by N and that after the ban by $N + M$, the price change can be computed as

$$\Delta p_H(N,M)=p_H^*(A^C,\alpha^C,N)-p_H^*(A^B,\alpha^B,N+M)=\frac{\lambda_H(N+M)-N}{\beta N(N+M)}.$$

The sign of this term is positive when $N < \lambda_H(N + M)$.

Next, consider change in the price differential as the number of active firms prior to the ban increases. Namely,

$$\frac{\partial\Delta p_H(N,M)}{\partial N}=-\frac{\lambda_H(N+M)^2-N^2}{\beta N^2(N+M)^2}$$

which clearly is negative whenever $\frac{N}{N+M} < \sqrt{\lambda_H}$. Thus, as N increases the price differential before and after the ban becomes smaller. ∎

A–3 Proof of Proposition 2

Let $\gamma(z, \lambda_H)$ denote the ratio of $CS^C(1, p_L)$ and $CS^B(2)$, which is given by

$$\gamma(z,\lambda_H)=\frac{CS^C(1)}{CS^B(2)}=e^{z+\frac{1}{2}}\left(1-\lambda_H+\lambda_H e^{-1-\frac{z}{\lambda_H}}\right)>0.$$

It is easy to verify that the derivative of $\gamma(z, \lambda_H)$ with respect to λ_H is given by

$$\frac{\partial}{\partial\lambda_H}\gamma(z,\lambda_H)=-e^{z+\frac{1}{2}}\left[1-\psi e^{-\psi}\right]$$

with $\psi = 1 + \frac{z}{\lambda_H}$. It is also easy to verify that $\psi e^{-\psi}$ is log-concave, and hence has a unique maximum at $\psi = 1$. Thus, the smallest value of the term in the brackets is obtained at $\psi = 1$

and is given by 0.632. Since $e^{z+\frac{1}{2}} > 0$ for all z, we have that $\frac{\partial}{\partial \lambda_H}\gamma(z, \lambda_H) < 0$. Thus, $\gamma(z, \lambda_H)$ is decreasing in $\lambda_H \in [0, 1)$.

Note that $\gamma(z, 0) = e(z + \frac{1}{2}) > 1$ and $\gamma(z, 1) = e^{-\frac{1}{2}} < 1$. There exists a $\tilde{\lambda}_H(z)$ which solves $\gamma(z, \tilde{\lambda}_H(z)) = 1$. Furthermore, for all $\lambda_H > \tilde{\lambda}_H(z)$, we have $\gamma(z, \lambda_H) < 1$ and hence $CS^B(2) > CS^C(1, p_L)$. ■

A–4 Proof of Proposition 3

We can compute $\frac{\partial}{\partial z}\tilde{\lambda}(z)$ by differentiating $\gamma(z, \tilde{\lambda}(z)) = 1$ with respect to z and solving for $\frac{\partial}{\partial z}\tilde{\lambda}(z)$ which is given by

$$\frac{\partial}{\partial z}\tilde{\lambda}(z) = \frac{(1 - \lambda_H)(1 - e^{-\psi})}{1 - \psi e^{-\psi}}$$

with $\psi = 1 +' \frac{z}{\lambda_H}$. We have shown earlier that $1 - \psi e^{-\psi} > 0$ for all ψ. The numerator has the same sign as $1 - e^{-\psi}$ which is positive whenever $\psi > 0$, or equivalently as long as $z > -\lambda_H$. Thus, $\tilde{\lambda}_H(z)$ increases with z as long as $0 \leq \lambda_H < 1$ and $z > -\lambda_H$.

The rest of the claims in the proposition follows by straightforward differentiation. We know that $\frac{d\tilde{\lambda}_H}{dz} > 0$. Since by definition $\frac{dz}{dk} > 0$, and $\frac{dz}{dp_L} < 0$, by the chain rule we have $\frac{d\tilde{\lambda}_H}{dk} > 0$, and $\frac{d\tilde{\lambda}_H}{dp_L} < 0$. > 0. Finally, $\frac{dz}{d\beta} = k - p_L$. Therefore, $\frac{d\tilde{\lambda}_H}{d\beta} > 0$ for $k > p_L$, $\frac{d\tilde{\lambda}_H}{d\beta} < 0$ for $k < p_L$ and $\frac{d\tilde{\lambda}_H}{d\beta} = 0$ otherwise. ■

A–5 Proof of Proposition 4

Comparison of before and after ban welfare is a complicated issue. Given that what we want to do is to highlight the possibility of having a duopoly after the ban, and a monopoly before the ban, we will set the fixed entry cost so that the total industry profit is zero after the ban. This is the least likely case that would result in a welfare gain.

Note that the after ban equilibrium profit when two firms enter is given by

$$\pi(A^B, \alpha^B, 2) = \frac{1}{4\beta}\frac{\lambda_H(\lambda_H + \lambda_L)}{(1 - \lambda_H)(\lambda_H + \lambda_L - \lambda_H\lambda_L)}e^{\delta - \beta k - \frac{1}{2}} - F$$

Thus, the critical fixed cost which allows two firms to enter is simply given by the zero profit condition as

$$F^{\max} = \frac{1}{4\beta}\frac{\lambda_H(\lambda_H + \lambda_L)}{(1 - \lambda_H)(\lambda_H + \lambda_L - \lambda_H\lambda_L)}e^{\delta - \beta k - \frac{1}{2}}.$$

In this case, the welfare after ban is simply equal to the consumer surplus. The welfare difference before and after the ban is then

$$\Delta W = CS^B(2) - CS^C(1, p_L) - \pi^*(A^C, \alpha^C, 1)|_{F = F^{\max}}.$$

It is straightforward though cumbersome to verify that $\Delta W = w_0 w_1(z, \lambda_H)$ where

$$w_0 = \frac{1}{4\beta} \frac{(\lambda_H + \lambda_L)}{(1-\lambda_H)(\lambda_H + \lambda_L - \lambda_H \lambda_L)} e^{\delta - \beta p_L}$$

and

$$w_1(z, \lambda_H) = 5e^{-z-\frac{1}{2}} - 4(1-\lambda_H) - 8\lambda_H e^{-\frac{z}{\lambda_H}-1}$$

with $z = \beta(k - p_L)$. Given that $w_0 > 0$ for all relevant parameter values, the sign of ΔW is the same as the sign of $w_1(z, \lambda_H)$.

First note that

$$\frac{\partial w_1(z, \lambda_H)}{\partial \lambda_H} = 4(1 - 2\psi e^{-\psi}).$$

with $\psi = 1 + \frac{z}{\lambda_H}$ as defined above. Once again, since $\psi e^{-\psi}$ is logconcave, it is uniquely maximized at $\psi = 1$. Therefore, $\frac{\partial w_1(z,\lambda_H)}{\partial \lambda_H}$ is minimized at $\psi = 1$, and $\min[4(1 - 2\psi e^{-\psi})] \approx 1.057 > 0$. Consequently, $\frac{\partial w_1(z,\lambda_H)}{\partial \lambda_H} >$, for all λ_H and z.

We have $w_1(z, 0) = 5e^{-z-\frac{1}{2}} - 4 < 0$ for all $z > 0$. On the other hand, we have $w_1(z, 1) \approx 0.090e^{-z} > 0$. Therefore, there exists a $\hat{\lambda}_H(z)$, such that $w_1(z, \hat{\lambda}_H(z)) = 0$. For $\lambda_H > \hat{\lambda}_H(z)$, we have $w_1(z, \lambda_H) > 0$ and therefore $\Delta W > 0$ whenever $\lambda_H > \hat{\lambda}_H(z)$. ■

A–6 Proof of Proposition 5

Totally differentiating $w_1(z, \lambda_H) = 0$ we obtain

$$\frac{d\hat{\lambda}_H(z)}{dz} = -\frac{\frac{\partial w_1(z,\lambda_H)}{\partial z}}{\frac{\partial w_1(z,\lambda_H)}{\partial \lambda_H}}|_{\lambda_H = \hat{\lambda}_H(z)}.$$

Since we have shown earlier that $\frac{\partial w_1(z,\lambda_H)}{\partial \lambda_H} > 0$,

$$\text{sign}(\frac{d\hat{\lambda}_H(z)}{dz}) = \text{sign}(-\frac{\partial w_1(z, \lambda_H)}{\partial z})|_{\lambda_H = \hat{\lambda}_H(z)}$$

Straightforward differentiation yields

$$\begin{aligned}
\frac{\partial w_1(z, \lambda_H)}{\partial z}|_{\lambda_H = \hat{\lambda}_H(z)} &= 5e^{-z-\frac{1}{2}} - 8e^{-(1+\frac{z}{\hat{\lambda}_H(z)})} \\
&= \underbrace{-w_1(z, \hat{\lambda}_H(z))}_{=0} + 5e^{-z-\frac{1}{2}} - 8e^{-(1+\frac{z}{\hat{\lambda}_H(z)})} \\
&= 4(1 - \hat{\lambda}_H(z))(1 - 2e^{-1}e^{-\frac{z}{\hat{\lambda}_H(z)}}) \\
&> 4(1 - \hat{\lambda}_H(z))(1 - 0.735e^{-\frac{z}{\hat{\lambda}_H(z)}}) > 0
\end{aligned}$$

whenever $z > 0$. Therefore, we have $\frac{d\hat{\lambda}_H(z)}{dz} > 0$.

The remaining claims follow by a straightforward application of the chain rule. ■