Modeling Breach of Contract Risk through Bundled Options

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Abstract

In this paper, in order to model breach of contract risk, we design and value a bundled option that is composed of contract abandonment and price renegotiation. We numerically show that the bundled option is more valuable for the contract than either of the options, i.e., contract abandonment and price renegotiation, in isolation. This value increases monotonically as the spot price becomes more volatile. The value of the bundled option is less than the sum of the individual option values, hence showing the sub-additive property. We demonstrate that in the presence of high spot price volatility, the bundled option is more valuable when renegotiation date is selected to be closer to the half-life of the contract. We also show that early contract abandonment probability goes down in the presence of renegotiation option. We conclude that the commodity supplier should negotiate a supply chain contract with flexible options at the design stage with the buyer, obtaining contract abandonment and price renegotiation options –as a bundled option—in order to enhance the supply contract value and reduce the breach of contract risk.

Keywords: *Supply chain contracts, Procurement, Breach of contract risk, Abandonment and renegotiation options, Basel II -Clients, Products, Business Practices Category Risks-*

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1. Introduction

In different industries worldwide, executives state that designing flexible options in supply chain contracts creates value and mitigates operational risks for supply chain partners (IBM, 2009). It is imperative that the nature of this flexibility and risk needs to be carefully delineated. While managing supply chain contracts with suppliers worldwide, supply chain executives become increasingly concerned specifically about operational risk events such as breach of contract, supplier non-performance, improper business practices, and other supplier/vendor failures, which are classified by Basel II framework under the "Clients, Products, and Business Processes" as well as the "Execution, Delivery, and Process Management" categories. (Cruz (2002), Chernobai, Rachev, Fabozzi (2007), Hoffman (2002)). To hedge these operational risks, firms can use operational risk derivatives, insurance, and capital allocation methods (Cruz, 2002). Operational risk derivatives consist of different real and financial options as well as bonds and swaps. In this paper, we develop a method to value a particular type of flexible option, which is a *bundle* of *abandonment* and *renegotiation* options, designed into supply chain contracts. To that end, we shed light on the strategic use of renegotiation option in operational risk management. We show that not only the presence of this type of option, but also the timing of it, becomes important in supply chain contract valuation and operational risk mitigation.

This paper derives some motivation from the previous work by Haksöz and Seshadri (2007) where the abandonment option is valued for the seller of the commodity in which abandonment option allows the seller to breach the contract at any time and sell to the spot market. In our paper, we build on a similar model by introducing a formal renegotiation option, which we show to be quite useful in today's highly volatile markets. Nowadays, many buyers and sellers of different commodities, such as steel, iron ore, aluminum, zinc, and pulp prefer to revise and renegotiate the contract parameters via a negotiation process where the result is uncertain. Sometimes, these renegotiation processes create favorable results, yet sometimes they end in deadlocks and may lead to undesirable breach of contracts. A breach of contract is considered to be a type of

operational risk that is classified under the "Clients, Products, and Business Practices" category of the Basel II framework (Cruz (2002), Haksöz and Kadam (2009)). Hence, a method to model breach of contract risk will be highly desirable for procurement and supply chain executives.

Steel markets and its recent evolution is an interesting case in point. Until the summer of 2008, commodity prices at London Metal Exchange have appreciated and the spot markets were more beneficial for commodity traders. ArcelorMittal, the largest steelmaker in the world, claimed that it sold around 20% of its steel capacity via fixed price supply contracts, and the rest to the spot market according to Matthews (2008a). On the other hand, with increasing firm bankruptcies around the financial world and the real sector, commodity prices started to plummet. In this case, commodity players began to breach their supply contracts due to much lower demands for end products such as automotive, construction, hardware goods etc. As noted by Matthews (2008b) iron ore buyers turned towards the spot market instead of paying much higher negotiated contract prices to the sellers. One such example is the case of Australian iron-ore producer Mount Gibson Iron Ltd. stating that in November 2008, three of Mount Gibson Iron's customers had defaulted on their contracts to purchase iron ore in the spot market. Also, in our private communication with the Chief Risk Officer of Sabancı Group, the second largest industrial and financial conglomerate in Turkey, we learned that a firm in his industrial group was contemplating how to manage the default risk of a one year fixed price supply chain contract, as the supplier had asked for a %50 price increase in the contract as better price offers from competitors are available in the market (Saka, 2009).

It is clear from examples above that both parties in a supply contract may benefit from renegotiating the contract terms, especially the price, rather than being exposed to breach of contract event. Surely, a breach of contract may increase the reputational risk of that firm in its industry, which may eventually damage long-term relationships with other potential partners. Thus, in this paper, we propose a formal renegotiation option with an abandonment option in a bundle that will be designed into a supply chain contract at the beginning of the contract. To our knowledge, this particular type of bundled option does

not exist in real world contractual arrangements. Hence, in this paper, we aim to propose a novel approach in modeling the breach of contract risk through a bundled option. To that end, our work also sheds some light on the optimal supply chain contract design literature (See Cachon (2003) and references therein).

The definition of the renegotiation option in our setting is as follows: As time goes on while the procurement is made via a long term fixed price contract, spot market price may go up or down in the commodity market. The renegotiation option uses the spot price at the time of the renegotiation to determine the forward contract price. In the absence of the renegotiation option, both parties would go through an uncertain renegotiation process where the renegotiated contract price could not be known with certainty. Once the renegotiation option is exercised, if the renegotiated contract price is not accepted, the supplier may breach the contract, i.e., formally exercise the abandonment option, and sell to the spot market. If the renegotiated price is accepted, the contract continues with the revised price until the contract duration or until a contract abandonment occurs. In such a setting, we show that the buyer reduces the breach of contract risk of the supplier *ex post* by offering the contract price renegotiation option *ex ante*.

This paper is organized as follows. In Section 2, we present the related literature to our work. In Section 3, we provide our model of a bundled option. Next, we present our numerical solution approach for this bundled option in Section 4. In Section 5, we provide the model results and discuss managerial insights and practical significance. Finally, we conclude with a summary of main findings and future research directions in Section 6.

2. Related Literature

Our work is mainly related to two streams of literature. One is the work in supply chain contracts and procurement in commodity markets. The second relevant literature is the financial and real options valuation for multiple options.

In supply chain contracts literature, value of flexibility in contract design and management has been under focus for some time. Provision of options such as flexible quantity, revenue/cost sharing, buyback, and contract abandonment (Cachon (2003), Haksöz and Seshadri (2007)) create value for supply chain partners and provide benefits for managing risks in supply chain contracts. In the presence of spot markets, buyers may use long-term contracts, option contracts and the spot market simultaneously for procurement. Seifert, Thonemann, Hausman (2004) study a single period model where a forward contract and spot market are used. Yet, in their model, no link exists between the forward contract and the spot market as one would expect in the commodity markets. Barnes-Schuster, Bassok, Anupindi (2002) show that backup agreements, quantity flexibility, pay-to-delay capacity reservation contracts are special cases of a general flexible options contract that enables buyers better manage demand risk. Martínez-de-Albéniz and Simchi-Levi (2005) develop a portfolio model of procurement with three alternatives such as fixed-price long term contracts, option contracts, and spot market trading in order to manage demand and price risks.

Haksöz and Seshadri (2007) review the extant literature on supply chain models in the presence of spot markets. Recently, in operational risk management, some research for contract default/breach risk has been conducted. In the presence of breach of contract risk by the suppliers, the buyers need to compute the supply risk exposure. For a portfolio of commodity suppliers, Haksöz and Kadam (2009) provide a risk assessment method based on CreditRisk+ model in the presence of a spot market.

On renegotiation, there is a large literature in economics (See for example Tirole (1986) and references therein). In supply chain literature, however, renegotiation has begun to appear rather recently. Plambeck and Taylor (2007) show that simple fixed quantity contracts with renegotiation design can coordinate investment in capacity and innovation for biopharmaceutical and semiconductor industries. In their work, renegotiation is modeled in a cooperative game theoretic setting that is different from our approach.

In financial and real options literature, there are a number of papers that provide some directions in valuation of interacting options, which are most relevant to our work. One of the pioneering articles in the real option valuation that shows the importance of option interactions is by Trigeorgis (1993). In his work, interactions among multiple real options such as deferral, abandonment, contraction, expansion, and switch use have been demonstrated numerically. One of the key results of his paper is that combined value of multiple options is different than the sum of each option's value in isolation. Option interactions may increase/decrease the combined value based on the nature and magnitude of different options. On the other hand, we should note that very few studies have been done to value the option interactions analytically. Studies by Agliardi (2006, 2007) demonstrate the value interactions analytically for only two (expansion or contraction) options. Agliardi (2007) shows that addressing more than two options is technically doable and may create more flexibility, yet increases the mathematical complexity by having to work with nested multinomial cumulative functions.

Last but not least, there is also a growing literature that studies a variety of exotic options that offers the flexibility to the option holder to change his strike price or maturity date (reset option), extend the length of the maturity (extendible option), choose a maximum of a call and put at fixed or flexible time points (chooser option). Our paper also relates to this stream of work. (See for example Detemple and Emmerling (2009), Dai and Kwok (2005), Gray and Whaley (1999)). We should point out that these flexible options are structurally different than compound options where the option holder gains another option by exercising his previous option (Geske, 1979).

3. The Model

Our model is developed for a fixed-price supply chain contract. The contract duration is given by $[0,T]$. At time 0, the contract is signed between a supplier and a buyer to procure a fixed amount of commodity, Q at unit price $F_{0,T}$ to be delivered at time T. This contract is essentially a commodity forward contract in our framework and the forward price is given by the following equation when the risk-free interest rate, r is non-stochastic.

$$F_{0,T} = S_0 e^{(r-\delta)T},$$

where S_0 is the spot market price at time 0 and δ is the convenience yield. As time moves on, the spot market price is assumed to evolve with a stochastic process assumed to be Geometric Brownian Motion, i.e.,

$$\frac{dS_t}{S_t} = \alpha_S dt + \sigma_S dW_S, \quad \text{t} \in [0, T].$$

α_S is the expected growth rate (drift) of the spot market price, σ_S is the instantaneous volatility of the spot market price, and dW_S is the standard Wiener process.

In this contract, the supplier is provided with two types of options in a bundle. We define these options below.

Abandonment Option: This option entitles the supplier to abandon the supply contract at any time before the end of the contract duration by paying a positive abandonment penalty fee denoted by A. This is an American call option. Note that this fee is A at time 0 and grows with given interest rate, i.e., Ae^{rt}, therefore a higher amount at the time of the option exercise. We consider the time value of money in these penalty fee computations. Otherwise, the option holder may delay the option exercise as having paid the fixed amount at the beginning of the contract.

Renegotiation Option: This option entitles the supplier to renegotiate the price of the contract by paying a positive renegotiation cost, R, at fixed time τ, $0 \le \tau \le T$, when the price of the contract is revised based on the spot price at τ, S_τ. Essentially, the contract price is revised to become $F_{\tau,T} = S_\tau e^{(r-\delta)(T-\tau)}$. This is a European call option. Again, the renegotiation cost is payable at the exercise of the option, therefore it is assumed to grow at risk-free interest rate.

We assume that the abandonment penalty fee, A is larger than the renegotiation cost R, $A>R$. This assumption is valid since the firms will be paying larger amounts of penalties if they default and abandon their contracts.

Merton (1973) shows that "an option on a portfolio is worth less than a portfolio of options", thus yields the sub-additive property. However, if one has sequential real options such as those defined in Agliardi (2006, 2007), then one can achieve super-additive (sub-additive) value if two real options are sequential expansion (contraction). Moreover, aforementioned, Trigeorgis (1993) shows numerically that multiple options in a real investment project cannot be valued as simple summation of their individual option values. The inherent interaction between these options has to be taken into account explicitly. Depending on their option types, whether an American or European and call/put, the combined value can be super or sub-additive.[1] Thus, we need to note that in this paper, we study neither an option on a portfolio nor multiple real options as defined by Trigeorgis (1993), but we have a much more complicated bundled option for which no analytical solution exists. However, as also noted by a referee, unlike many real options, this bundled option conforms much better to the assumptions required to apply standard option pricing theory since the underlying asset for the contract is actually a physical commodity. Hence, the arbitrage trade that option pricing formulae depend on can potentially be executed in the marketplace.

Specifically, we have an American call option for the duration $[0,T]$ and we do have a European call option at time τ. This is structurally similar to an American-style reset call option where at time τ the strike price is reset from $F_{0,T}$ and $F_{\tau,T}$. Gray and Whaley (1999) proposes a numerical procedure for valuing American-style reset put options. In our solution, we develop and use a method inspired by their approach.

[1] Sub-additivity property states that evaluating the function for the sum of two elements in a particular domain always yields values less than or equal to the sum of the function's values at each element. Super-additive function is the negative of sub-additive function.

4. Numerical Solution Approach

In this section, we develop a numerical solution approach for valuing the bundled option (abandonment and renegotiation) that is written in the supply chain contract. We apply a modified version of the binomial tree method developed by Cox, Ross, and Rubinstein (1979). This modification was first proposed by Gray and Whaley (1999) for valuing American-style reset put options. A reset put (call) option is similar to a standard put (call) option except that the strike price is reset equal to the asset price on the pre-specified reset date if this asset price is above (below) the original strike price. If this option is American, it can be exercised early before or after this pre-specified reset date.

As explained earlier, the bundled option in this paper is structurally equivalent to an American-style reset-strike call option. It is an American-style call option since we have the right to abandon a short forward position by getting into a long forward contract at the same forward price at any time. Furthermore, it is a reset-strike option since the renegotiation helps change the forward price (i.e., the strike price) to a favorable one on a pre-specified date. However, there are subtle differences arising from the nature of these supply chain contracts. First, in the options-embedded supply chain contracts, penalty costs must be paid only when these options are exercised; whereas in standard options, premiums are paid up front. To adjust for this difference and to reduce a potential bias towards early exercise due to having constant penalty costs over time, we compound these penalty costs at the risk-free rate as we go further in time. This modeling feature is also aligned with industrial practice. The penalties paid for a later abandonment are much higher than early ones. Second, as these are short forward contracts, the strike (i.e., forward price) is reset when forward prices at the renegotiation date sufficiently goes above (not below) the original forward price.

In our base case, we consider an 8-year supply chain contract[2] where the supplier has an option to abandon this contract at the end of any period during the life of the contract. We

[2] 8-year supply chain contract can be also considered as a generic 8-period contract without loss of generality, where periods could be considered as one month to a quarter depending on the specific supply chain in focus.

assume that when the supplier abandons the contract, she will immediately sell the product in the spot market and receive the proceeds. Furthermore, the supplier also has an option to renegotiate the forward price at the end of the year four (i.e., half-way through). As a result, if the forward price at the end of the 4th year is favorable (i.e., higher enough to cover the cost of renegotiation), this becomes the new contract price. It should be noted that if abandonment takes place before the renegotiation date (end of year 4), the renegotiation cannot take place as the product is already sold in the spot market (i.e., the bundled option dies just like an abandonment option). However, in case a renegotiation takes place at the end of year four, abandonment is possible afterwards. That is to say, the abandonment component of the bundled option is still alive even if the renegotiation right is exercised. This bundled option therefore creates flexibility for both parties in revising the contract price in the middle of the contract duration. Price changes in the spot market could be reflected formally in the contract price.

The binomial tree for spot prices is calibrated according to Cox, Ross, and Rubinstein (1979). Valuation of abandonment and renegotiation options as single options is straightforward within this framework. For bundled option, however, the possibility of renegotiation creates an implicit path-dependency that complicates the valuation procedure. To remedy this, we use the following modifications similar to those proposed in Gray and Whaley (1999).

The renegotiation date (τ) breaks the binomial tree into two parts. The first part is an American abandonment option with a terminal date τ. For sub-trees emanating from each tree node on this date, we value two American abandonment options: *with* and *without renegotiated prices*. The terminal values of these options are given respectively as follows:

$$V_T^R = \max\left(QS_T - Ae^{rT}, QF_{\tau,T}\right) \text{ and } V_T = \max\left(QS_T - Ae^{rT}, QF_{0,T}\right).$$

For either option, at each node on the sub-tree prior to these terminal nodes, standard American option valuation routine is used, implying a value function in the form

$V_t = \max\left(QS_t - Ae^{rt}, e^{-r\Delta t}E\left[V_{t+\Delta t}\right]\right)$ for a node at time t. The only exception is the root node for the sub-tree with renegotiated prices, where we also subtract the cost of renegotiation, $R \times e^{r\tau}$, from this max function. The value of the bundled option on each node at time τ is the greater of the value of these two options. This comparison also gives whether it is optimal to renegotiate or not for this specific node.

It should be noted that, regardless of the cost of renegotiation, it will never be optimal to renegotiate if $F_{\tau,T} \leq F_{0,T}$. Therefore, for nodes where $S_\tau \leq S_0 e^{(r-\delta)\tau}$, we do not analyze the sub-tree with renegotiated prices. Finally, having found the value of the bundled option on each of the nodes on the renegotiation date, we use the value function above to proceed backwards in time until t = 0, at which point we have the value of the supply chain contract with the bundled option.

Figure 1 illustrates the implementation of the procedure for our base case assuming a further simplification of 1-year step-size. In the case of reset strike options, resetting the strike is a trivial procedure based on comparing the spot price and the strike price, therefore the comparison explained above is not needed. In the case of our bundled option, renegotiation penalty cost complicates the situation and necessitates a comparison, therefore our approach deviates slightly from that of Gray and Whaley (1999).

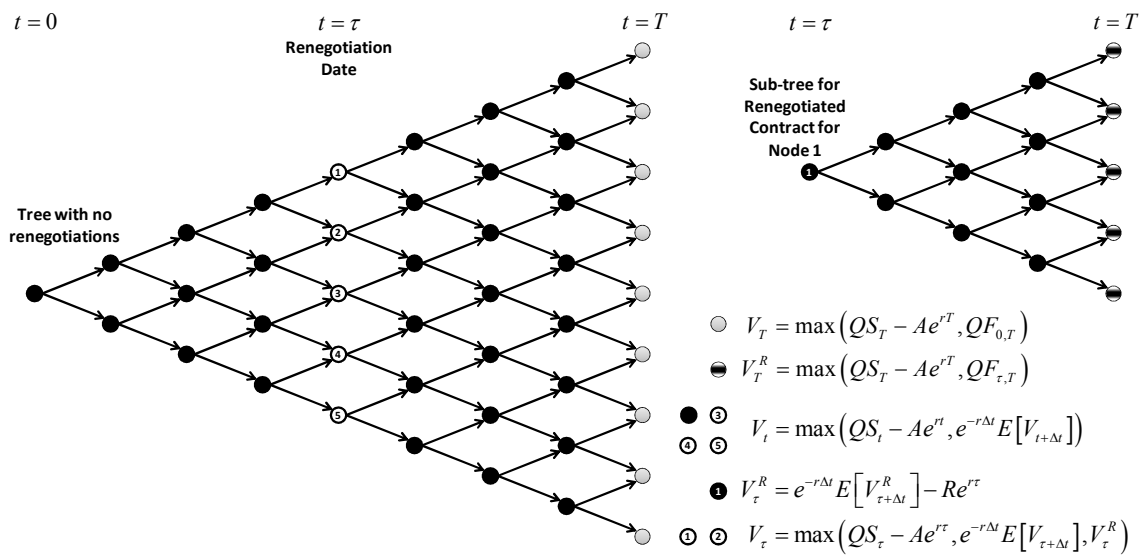

Figure 1. ***Binomial tree illustrating the procedure for bundled option valuation. The legend on the lower right shows the value functions for respective nodes. The sub-tree on the upper right represents one such sub-tree assuming the renegotiation option is exercised on a given node.***

Table 1 summarizes the parameter values for the binomial tree analysis. We remind that abandonment and renegotiation costs given below are present values of actual payments discounted at the risk-free rate. For example, if abandonment takes place at the end of period t, the payment will be $A \times e^{rt}$. This adjustment is done to prevent a bias that constant costs might create for late abandonment and also to create a similarity between option premiums and these penalty costs.

Parameter	Value
Initial spot market price (S_0)	\$100/ton
Contract quantity (Q)	1,000 tons
Contract term (T)	8 years
Renegotiation date (τ)	4 years
Spot price volatility (σ)	20%
Abandonment cost (A)	\$40,000
Renegotiation cost (R)	\$20,000
Annual risk-free rate (r)	5%
Annual average convenience yield (δ)	2.5%
Binomial tree step-size (Δt)	0.01 years

Table 1. ***Parameter values for the binomial tree model in the base case***

Although the base case parameter values are selected to represent a typical supply chain contract, we provide further justification for some of the parameters. The binomial tree

step-size, Δt, is chosen based on the results of a convergence test. Figure 2 provides evidence and shows how the total contract value changes as number of tree steps increases (i.e., step-size decreases). Moreover, a step-size of 0.01 years corresponds to 2-4 days, which appears to be a sufficient frequency to monitor spot prices for such long-term contracts in practice.[3]

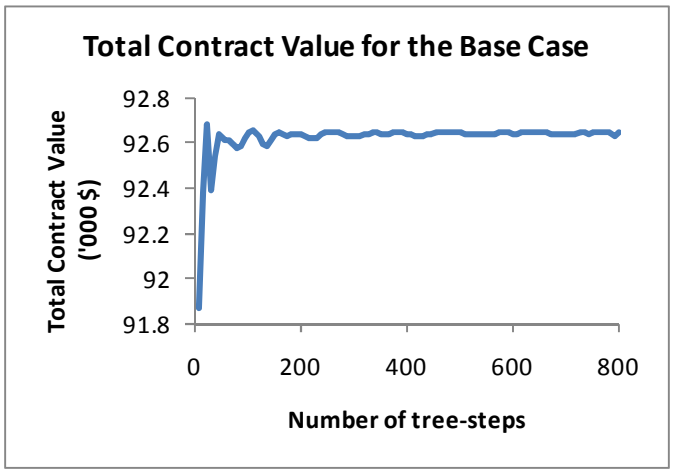

Figure 2. ***Convergence of total contract value in the presence of bundled option as number of tree-steps increases (i.e., the step-size decreases)***

Abandonment and renegotiation costs, A and R, are chosen to be compatible with other parameter values. For this purpose, we first assume that these costs are zero and we compute the incremental value of having these options mutually exclusively. For the base case, in which the value of the option-free contract is $81,873, we find that a free abandonment option increases the contract value by 28.04%, whereas a free renegotiation option adds 15.86%. This implies that the abandonment option is roughly twice as valuable as the renegotiation option. We keep this 2-to-1 ratio for the penalty costs but set them higher than their incremental values since the penalties are paid only when the options are exercised.

[3] In our discussions with procurement executives, it is stated that frequency of spot price information monitoring intimately depends on the type of commodity purchased, market features, and supplier characteristics. In practice, 2-4 days monitoring frequency may reduce to daily checks or in some cases increase to bi-weekly examination.

5. Results and Discussion

In this section, we present various results and discuss their managerial implications.

5.1 Value of the Bundled Option

Base case results suggest that bundled option adds value to either single option and this added value increases with volatility. However, the value of the bundled option is less than the sum of the values of the single options, i.e., aforementioned sub-additive property (Agliardi 2006, 2007). This finding is displayed in Figure 3 and emphasized by the following values. For the base volatility level of 20%, the incremental value (above the forward contract value of $81,873) of the abandonment option is $9,135 and that of the renegotiation option is $6,638. We find that the bundled option has an incremental value of $10,771, which implies that $5,002 = ($9,135 +$6,638) – $10,771 is lost due to option interaction. Figure 4 shows that the loss is also a non-decreasing function of volatility.

It should be noted that in reality the abandonment option can only be exercised at certain pre-determined dates (e.g., at the end of each period/year). This characteristic would make the option a Bermudan-type one, which is a special case for American options. On the following charts, namely Figures 3, 6, and 7, we also present the results for the Bermudan abandonment option case in which the contract can only be abandoned at the end of each year. Clearly, the contract value is smaller than the comparable American option; however, the difference is too small to change our qualitative findings.

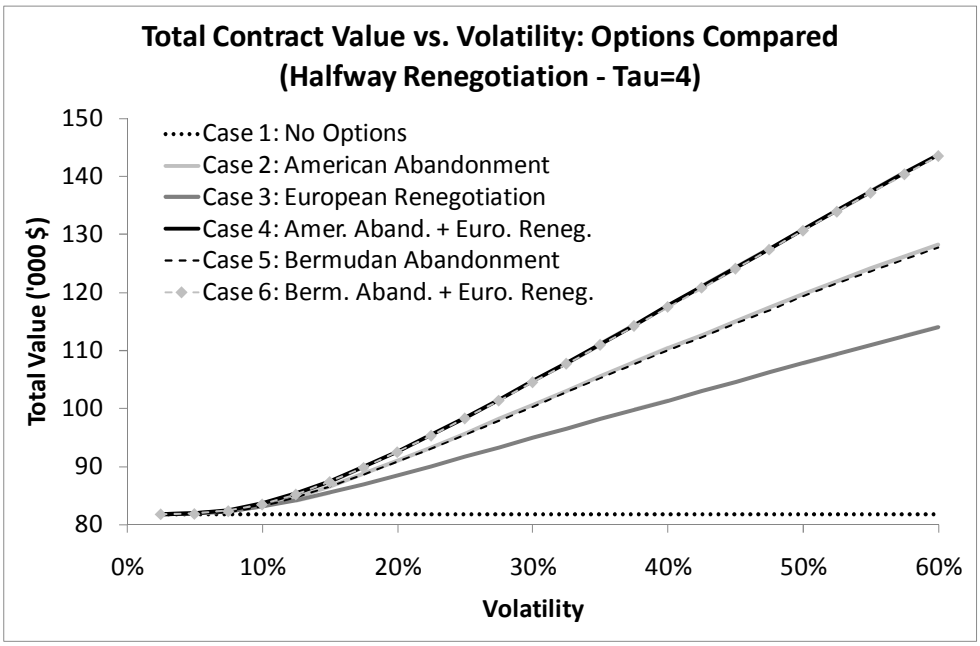

Figure 3. ***Total contract value in the presence of different type of abandonment and/or renegotiation options and its relationship with volatility of spot price***

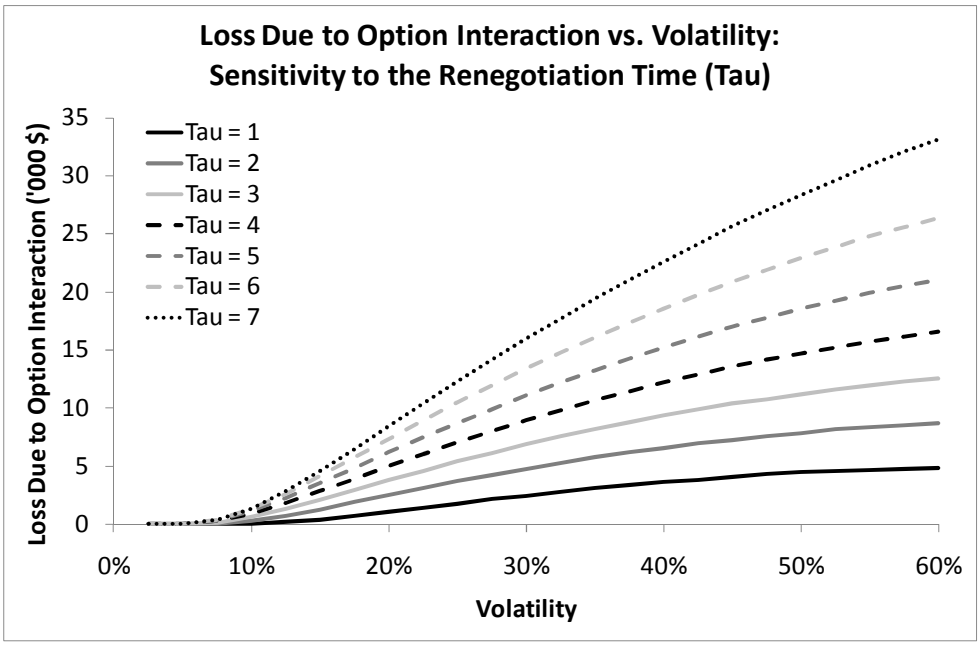

Figure 4. ***Loss due to interaction between abandonment and renegotiation options in the bundle for different dates of renegotiation and its relationship with volatility of spot price***

5.2 Sensitivity on the Renegotiation Date

We conduct sensitivity analysis for different renegotiation dates under different volatility assumptions and see if an early/late renegotiation date results in a higher value for the bundled option. In the absence of the abandonment option, regardless of the volatility level, extending the renegotiation date creates more value as shown in Figure 5. This is an expected result for European options on forward contracts as values of such options increase with time-to-maturity. Intuitively, if the supplier only has one opportunity to renegotiate, in the absence of contract abandonment option, she should select a date as close as possible to the contract maturity. However, as pointed out by a referee, should the supply contract were a swap, the option should reach its maximum value with a renegotiation date somewhere in the middle of the contract duration even in the absence of the abandonment option. Figure 5 also displays that for a given renegotiation date option value increases with volatility.

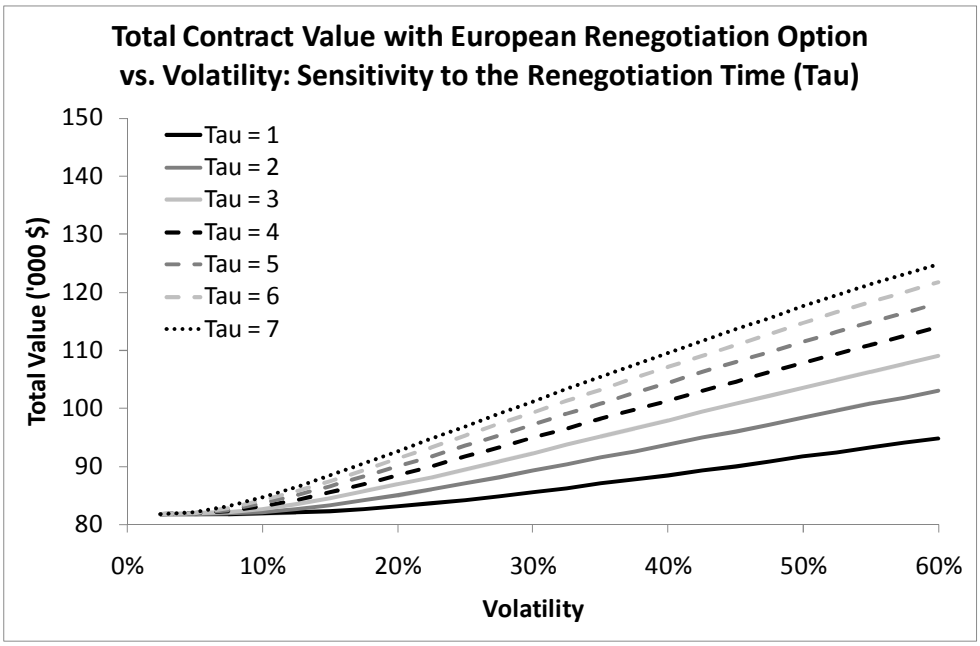

Figure 5. ***Total contract value in the presence of European renegotiation option with different maturities and its relationship with volatility of spot price***

On the other hand, for the bundled option we can only reach a similar conclusion in the presence of high spot price volatility. That is, as the volatility increases (beyond the base

case of 20%), the optimal renegotiation date that creates the most value shifts to the middle of the contract horizon (Table 2). This result can be best understood by analyzing the incremental value of the renegotiation option on top of the abandonment option. Under low volatility, extremely high prices are not too likely and a late and inexpensive renegotiation might be preferred to an early and expensive abandonment. On the other hand, when volatility is high, it is more likely to have very high prices early on and therefore the possibility of early abandonment is higher. This implies that having a too late renegotiation date might really be too late and add little value. When we look at the difference between the bundled option value and abandonment option value for different value of volatility and renegotiation dates, we find that for volatility values less than 22.5%, $\tau = 7$ seems to be the best date for renegotiation. For volatility values greater than 40%, the optimal renegotiation date moves to $\tau = 4$.

	Total Contract Value	**Renegotiation Date (0 means no renegotiation)**							
		0	1	2	3	4	5	6	7
Spot Price Volatility	2.5%	81.87	81.87	81.87	81.87	81.87	81.87	81.87	**81.87**
	5.0%	81.89	81.89	81.89	81.90	81.92	81.97	82.04	**82.12**
	7.5%	82.24	82.24	82.25	82.30	82.43	82.61	82.84	**83.11**
	10.0%	83.24	83.24	83.27	83.39	83.60	83.87	84.22	**84.64**
	12.5%	84.77	84.77	84.87	85.06	85.33	85.65	86.03	**86.49**
	15.0%	86.65	86.68	86.88	87.17	87.49	87.81	88.15	**88.59**
	17.5%	88.76	88.85	89.20	89.59	89.95	90.26	90.55	**90.86**
	20.0%	91.01	91.21	91.75	92.25	92.64	92.93	93.13	**93.30**
	22.5%	93.36	93.73	94.46	95.07	95.51	95.77	**95.88**	95.85
	25.0%	95.75	96.36	97.30	98.02	98.49	98.73	**98.74**	98.51
	27.5%	98.19	99.06	100.25	101.07	101.57	**101.78**	101.68	101.23
	30.0%	100.64	101.86	103.26	104.18	104.73	**104.89**	104.69	104.03
	32.5%	103.09	104.68	106.33	107.37	107.92	**108.07**	107.76	106.86
	35.0%	105.54	107.58	109.43	110.59	111.18	**111.27**	110.86	109.73
	37.5%	107.96	110.46	112.57	113.81	114.44	**114.50**	113.96	112.60
	40.0%	110.37	113.37	115.72	117.09	117.73	**117.75**	117.10	115.51
	42.5%	112.73	116.30	118.85	120.34	**121.02**	120.99	120.21	118.39
	45.0%	115.09	119.23	122.04	123.60	**124.32**	124.26	123.35	121.27
	47.5%	117.39	122.12	125.20	126.87	**127.59**	127.49	126.47	124.15
	50.0%	119.64	124.98	128.32	130.12	**130.86**	130.70	129.56	126.99
	52.5%	121.87	127.89	131.43	133.36	**134.14**	133.91	132.64	129.82
	55.0%	124.05	130.76	134.52	136.56	**137.38**	137.11	135.68	132.61
	57.5%	126.16	133.59	137.61	139.72	**140.58**	140.27	138.71	135.36
	60.0%	128.22	136.38	140.66	142.88	**143.73**	143.38	141.70	138.09

Table 2. ***Total contract value in the presence of bundled option with different renegotiation dates and its relationship with volatility of spot price. For each volatility level, the maximum contract value corresponding to the optimal renegotiation date is in bold text.***

When volatility is low, date of the renegotiation option determines which option is dominant in the bundled option. If renegotiation can take place after the half-life of the contract, then the renegotiation option explains a significant portion of the value of the bundled option as displayed on the left part of Figure 6. Otherwise, the renegotiation option adds little value, as can be seen on the left part of Figure 7. Both figures show that for high volatility, having the bundled option really makes a difference above either single option (abandonment and renegotiation option) regardless of the renegotiation date.

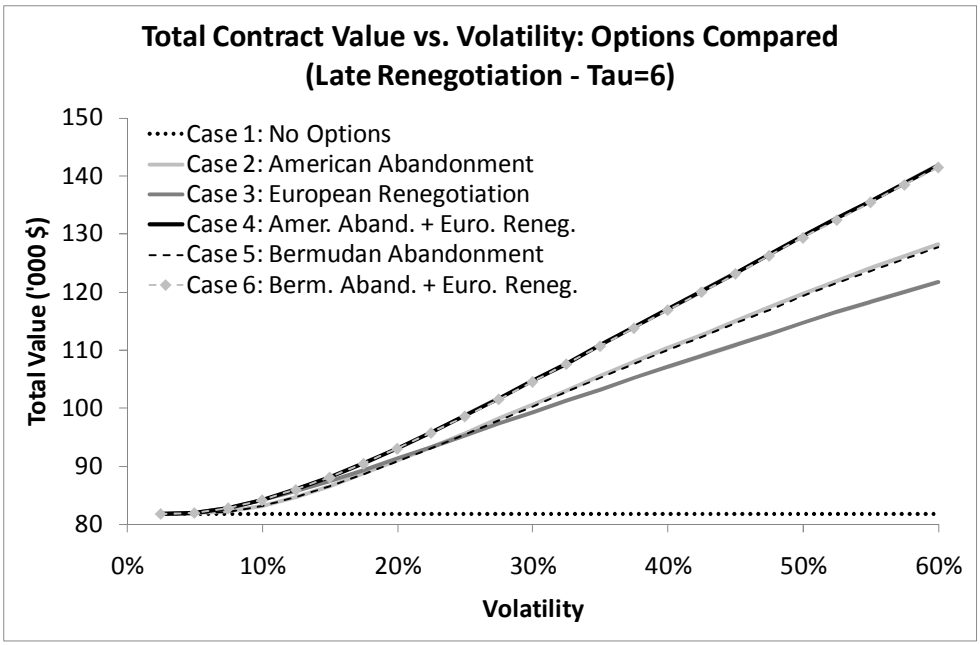

Figure 6. ***Total contract value in the presence of different type of abandonment and/or renegotiation options for a late renegotiation date and its relationship with volatility of spot price***

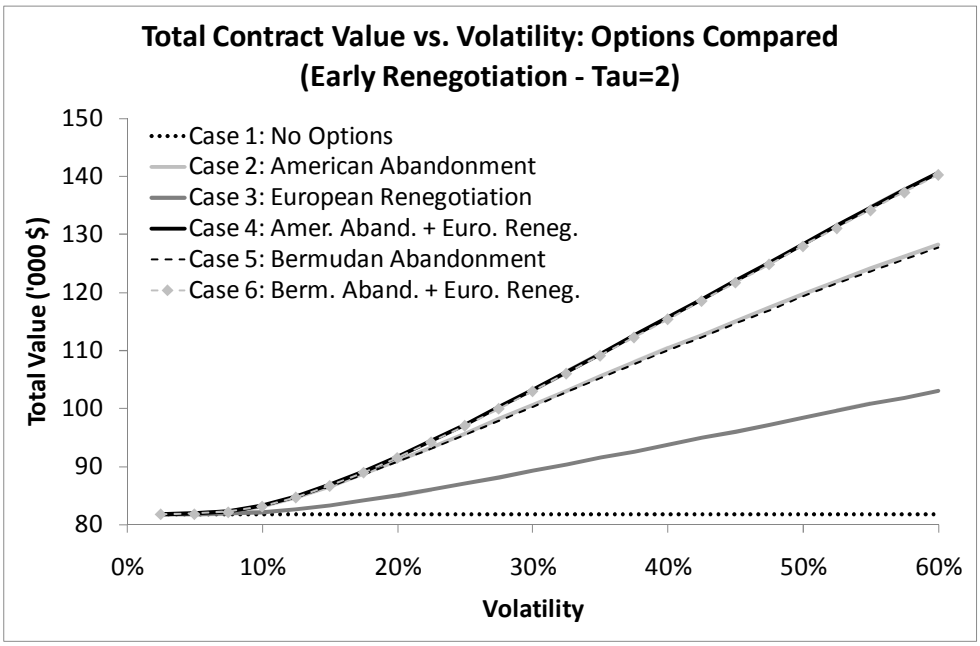

Figure 7. ***Total contract value in the presence of different type of abandonment and/or renegotiation options for an early renegotiation date and its relationship with volatility of spot price***

5.3 Optimal Abandonment Strategy

We should note that the following analysis is conducted for only American type abandonment in the bundled option, disregarding the Bermudan type abandonment as their values are quite similar. Although the bundled option value is sub-additive and a large amount is lost due to the interaction between two options, the renegotiation option does change the optimal behavior of the supplier to a great extent. In Figure 8, the optimal early abandonment boundary price is displayed as a function of time for three cases: i) No renegotiation, ii) Renegotiation at the end of 6th year, and iii) Renegotiation at the end of 4th year. The plots display the spot prices above which it would be optimal to abandon the contract before a possible renegotiation (i.e., to abandon "early"). There are two important findings. First and more significant observation is that the minimum spot price that triggers an early abandonment is considerably higher in the presence of a renegotiation option. Furthermore, not even the highest possible price on the binomial tree can make it optimal to early abandon within about 2.5 years before the renegotiation date. The supplier finds it optimal to wait and renegotiate instead of abandoning the contract. Second observation is that the supplier slightly postpones the earliest optimal

early abandonment time in the presence of a renegotiation option as displayed by the right-shift in the left-end of the curves. Both of these findings point to an obvious result, that the early abandonment is much less likely in the presence of a renegotiation option.

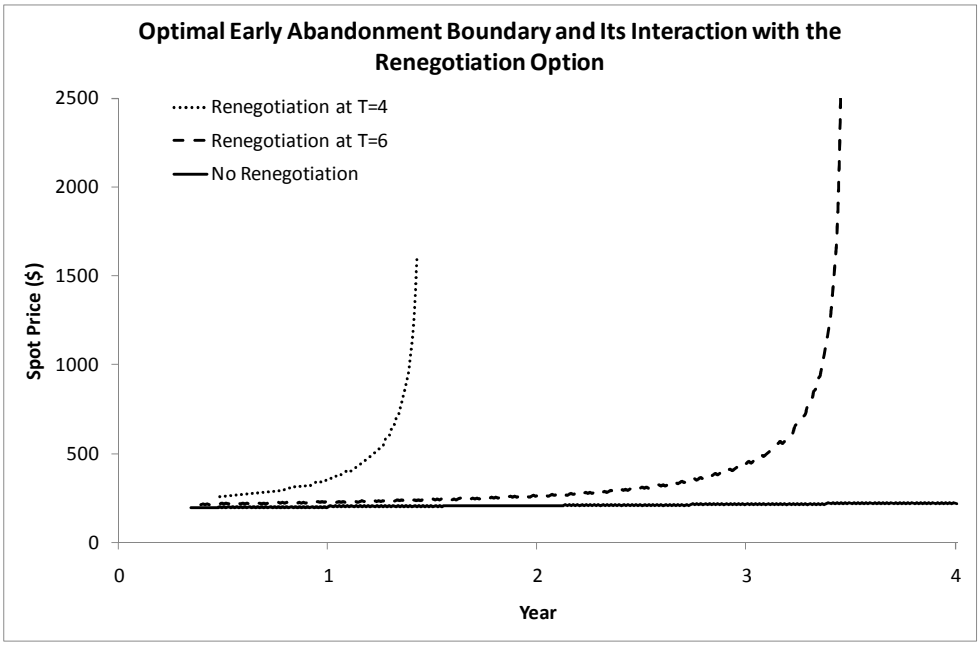

Figure 8. ***Spot price boundaries which show the level above which it is optimal to abandon the contract prior to a possible future renegotiation for three different cases***

To emphasize this result, we feel obliged to calculate the early abandonment probabilities explicitly.[4] In order to do this, for each tree-step, we first compute the number of nodes that are unreachable as a result of early abandonment using the boundary prices in Figure 8. Next, we calculate the single-path probabilities for each node on the early abandonment boundary. Due to the re-combining structure of the binomial lattice, many paths may lead to the same node. Therefore, we determine the number of paths that lead to each early abandonment node without going through an earlier abandonment. This is not a trivial task since early exercise causes path-dependency. For each early abandonment node, we compute the paths that will arrive from earlier abandonment

[4] As pointed out by a referee, these risk-neutral probabilities are not necessarily very close to true probabilities.

nodes using a recursive algorithm[5] and subtract this from the total number of paths leading to that node. Finally, we multiply the single-path probability of each early abandonment node with the calculated number of paths and sum them over all such nodes to arrive at the probability of early abandonment. Our findings are even more striking than those from Figure 8. When there is no renegotiation option, the contract is optimally abandoned with a probability of 19.89%. When there is a renegotiation option at $\tau = 6$, the probability that the contract will be abandoned prior to this date is a mere 0.14%. When the renegotiation date is pulled to $\tau = 4$, this probability shrinks to $5*10^{-10}$. Hence, the presence of a renegotiation option creates an incentive for the supplier to postpone the contract abandonment. Moreover, the closer the renegotiation dates to the beginning of the contract, the lower the probability of contract abandonment.

6. Conclusion

In this paper, for a supply chain contract, we designed and valued a bundled option that is composed of contract abandonment (American type) and price renegotiation (European type). This bundled option was used to model breach of contract risk --being one of the critical operational risks in practice, which is classified by Basel II framework under "Clients, Products, and Business Processes" category. We numerically solved this bundled option valuation model in a Binomial setting and obtained the following managerial insights.

First, the bundled option is more valuable for the contract than either of the options, i.e., contract abandonment and price renegotiation, in isolation. This value increases monotonically as the spot price becomes more volatile. However, the value of the bundled option is less than the sum of the individual option values, hence showing the sub-additive property, aligned with the previous literature. Second, as spot price volatility increases, one should prefer early renegotiation date. In the presence of low spot price volatility, if the renegotiation option can take place after the half-life of the contract, the bundled option value is significantly determined by the renegotiation option value. If the

[5] The algorithm and its MATLAB code are available with the authors upon request.

renegotiation date is earlier than the half-life of the contract, then the renegotiation option adds little value. Therefore, in markets with low spot price risk, obtaining renegotiation options that are exercisable after the half-life of the contracts are more valuable in a bundled option.

Third, having the renegotiation option changes the optimal strategy of the supplier (option holder). Early contract abandonment probability goes down in the presence of renegotiation option. Thus, offering a price renegotiation option to a supplier in a bundle with contract abandonment option is beneficial for the buyers such that favorable market conditions will not as much encourage the supplier for early abandonment. In sum, a price renegotiation option bundled with abandonment option may somewhat reduce the breach of contract risk. This managerial insight is also corroborated by Tirole (1986) for a different setting, in which renegotiation is shown to be valuable in the presence of breach of contract due to impossibility of initial contract enforcement.

We provide a number of future research directions. Our model values a bundled option that is composed of two options only (contract abandonment and price renegotiation). In practice, one can potentially design multiple flexible options in a supply chain contract at the same time such as renegotiation of contract quantity, duration, payment terms etc. These added option valuations will naturally complicate the solution procedure. Yet, one can use a similar setting as ours for multiple interacting options. Furthermore, in this paper, we disregard any gaming possibility in the supplier-buyer relationship. A game theoretic model in which multiple options are available to both players will be another interesting extension. Last but not least, quantifying the impact of such a bundled option on operational risk exposure will be a fruitful study. We hope that our paper motivates more research in designing and valuing flexible options for supply chain contracts such that various operational risks can be better managed.

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