A Note on Export Subsidies and Exchange Rate

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Abstract

This note investigates effects of exchange rate uncertainty on optimal trade policies and market prices within a standard export subsidy model. As exchange rate changes, relative efficiencies of firms in different countries change. In accordance with the conventional result, we show that changes in expected exchange rate effects optimal subsidies through relative costs. In particular, increase in expected depreciation of own currency increases subsidy levels when marginal cost is constant. Introducing import dependency, however, violates this uniform relation, and subsidy levels may decrease with increasing depreciation. Subsidy levels always decrease in import dependency when depreciation is expected. We also show that market price is less sensitive to exchange rates, compared to the free trade case (no subsidies).

JEL Classifications: F12

Key Words: Subsidies, exchange rate uncertainty, cost asymmetry, demand asymmetry.

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1. Introduction

This note introduces exchange rate uncertainty into a standard export subsidy model. While optimal trade intervention policies have been analyzed under various market structures in the existing international trade literature, effects of exchange rates on optimal policies and resulting market structures have not been considered.

Starting from leading studies of Brander and Spencer (1985) and Eaton and Grossman (1986) in the area, many studies focused on the models analyzing trade and trade policies in countries with oligopolistic markets. The main result of this line of literature is that the optimal trade policies heavily depend on the market structure. Focusing on export subsidies, Meza (1986) and Neary (1994) showed that when subsidies are the optimal policies they should be higher for the more efficient firms. In other words, governments should subsidize the winners more. On the other hand, Bandyopadhyay (1997) found that demand elasticity is also a major determinant of the subsidy level and this result can be reversed for the inelastic demand case. Later, Bandyopadhyay et al. (2004) determined that the optimal subsidy level is higher with higher level of cost heterogeneity. There are various other papers looking at the trade policy issues under uncertainty on either demand or cost sides\(^1\). The results again vary depending on the environment modeled.

In this paper, we aim to fill a gap by investigating effects of exchange rate uncertainty on optimal subsidy policies and relative price levels with cost and demand asymmetries. Consider a two-country environment, referred to as domestic and foreign, where one firm is located in each country and these firms compete in both countries. In our model, governments set subsidy policies before observing the actual exchange rate, but taking into account its expected value, $\bar{e}$. Firms compete after exchange rate is realized.

With constant marginal costs, we find that the optimal subsidy level of domestic country increases and foreign developed country decreases with $\bar{e}$, where exchange is measured in domestic currency per foreign currency. This result is in accordance with conventional results of the existing literature which states that higher cost firms gets less subsidy, as higher $\bar{e}$ corresponds to a lower relative cost for the domestic

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country’s firm. However, when import dependency for the domestic firm is considered, where the marginal cost may partially depend on exchange rate, subsidy level could go either way as expected exchange rate increases. Subsidy level is always decreasing in the import dependency parameter if currency value depreciation is expected.

We show that higher cost firm may receive a higher subsidy in equilibrium, which is reminiscent of Bandyopadhyay (1994)’s result. We also show that domestic market price may be decreasing in exchange rate, if intensity of subsidy by the foreign government is high. Otherwise, sensitivity of market price to exchange rate is less compared to the free trade (no subsidy) regime.

2. The Model

Consider a two-country world, where there is one firm in each country. Following the literature, one of these countries is a developing country and the other is a developed one. The developing country is referred to as domestic country ($d$) and the other as foreign country ($f$). Both domestic and foreign firms produce the same good, which is sold in both countries, however, domestic firm has higher costs due to inferior technology.

We consider a two stage game. In the first stage, governments choose subsidy levels, facing exchange rate uncertainty. In the second stage, exchange rate is realized and firms enter a Cournot competition in both domestic and foreign markets.

Let $\tilde{e}$ denote the exchange rate in the second stage of the game, representing value of unit foreign currency in domestic currency. In the first stage, probability distribution of $\tilde{e}$ is known, and expected exchange rate is denoted by $\bar{e}$. For discussion purposes, assume that actual exchange rate in the first stage is 1; hence expected devaluation of domestic currency is represented by $\bar{e} > 1$.

Linear demand functions are assumed for both developing and developed countries as follows,

$$ p_j = a_j - b_j Q_j, \quad Q_j = q_{j1} + q_{j2}, \quad j = d, f $$

(1)

where $p_d$ and $p_f$ are domestic and foreign market prices in each country’s own currency. We refer to the firm residing in the domestic country as firm 1, and the other as firm 2. Thus, $q_{d1}$ and $q_{f1}$ denote quantities sold by the domestic firm in
domestic and foreign markets, respectively. We assume constant marginal costs for both firms,

\[ C_i(q_i) = c_i q_i, \quad q_i = q_{d_i} + q_{f_i}, \quad i = 1, 2. \]  

(2)

Following the developing country theme, we will assume that domestic firm remains inefficient even with expected devaluation, or \( c_1 > c_2 \bar{e} \).

Under these assumptions for both supply and demand sides, the profit functions of domestic and foreign firms can be written as follows;

\[ \pi_1 = p_d q_{d1} + e p_f q_{f1} - c_1 (q_{d1} + q_{f1}) + s_d q_{f1} \]

\[ \pi_2 = \frac{p_d q_{d2}}{e} + p_f q_{f2} - c_2 (q_{d2} + q_{f2}) + s_f q_{d2} \]  

(3)

where \( s_d \) and \( s_f \) are the subsidies given by the domestic and foreign countries respectively, and \( e \) is the realization of the exchange rate in stage 2. Note that both profit functions are given in own currencies.

Social welfare in each country is given by the sum of consumer surplus and the profit of its firm net of subsidy transfers,

\[ SW_d = CS_d + \pi_1 - s_d q_{f1} \]

\[ SW_f = CS_f + \pi_2 - s_f q_{d2} \]  

(4)

In the first stage, governments set the subsidy level that maximizes expected social welfare.

3. Exchange Rate and Subsidy Levels

Solving the last stage of the game, we obtain Cournot-Nash equilibrium quantity levels as follows:

\[ \bar{q}_{d1} = \frac{a_d - 2c_1 + (c_2 - s_f)\bar{e}}{3b_d}, \quad \bar{q}_{f1} = \frac{(a_f + c_2)\bar{e} - 2c_1 + 2s_d}{3b_f\bar{e}} \]

\[ \bar{q}_{d2} = \frac{a_d + c_1 - 2(c_2 - s_f)\bar{e}}{3b_d}, \quad \bar{q}_{f2} = \frac{(a_f - 2c_2)\bar{e} + c_1 - s_d}{3b_f\bar{e}} \]  

(5)

In the first stage, governments maximize expected social welfare. We note that derivative of social welfare functions with respect to subsidy levels are as follows,
\[
\frac{d \bar{SW}_{d}}{ds_{d}} = \frac{a_{f} + c_{2} - (2c_{1} + 4s_{f})\bar{e}_{f}}{9b_{f}}, \text{ and}
\]
\[
\frac{d \bar{SW}_{f}}{ds_{f}} = \frac{a_{d} + c_{1} - (2c_{2} + 4s_{f})\bar{e}}{9b_{d}},
\]
where \(\bar{e}_{f} = 1/\bar{e}\) is the exchange rate expressed in foreign currency per domestic currency. Therefore, the first order conditions \(\frac{d}{ds_{f}} E\bar{W}_{j} = 0\) yield,
\[
\bar{s}_{d} = \frac{(a_{f} + c_{2})\bar{e} - 2c_{1}}{4} \text{ and } \bar{s}_{f} = \frac{a_{d} + c_{1} - 2c_{2}\bar{e}}{4\bar{e}}.
\]
as the optimal subsidy levels. In order obtain (7), one needs to assume congruence of expected values of the two versions of the exchange rate, i.e., \(\bar{e}_{f} = 1/\bar{e}\), which is certainly suggested by common sense.

We now turn to the effects of exchange rate expectations on subsidy levels. The following proposition immediately follows from (7).

**Proposition 1.** The subsidy level of the domestic country increases and that of the foreign country decreases as expected exchange rate, \(\bar{e}\), increases.

Intuitively, export subsidies increase welfare through grabbing welfare from the foreign country. Potential welfare to be grabbed increases as the foreign market gets larger, or own firm becomes more efficient. Thus, subsidy levels intensify as the size of the foreign market increases and marginal cost of own firm decreases. The latter is the conventional result in this literature that the higher cost firm gets lower subsidy. From the perspective of the domestic country, devaluation of domestic currency makes both the foreign market relatively larger (\(\bar{e}_{a_{f}}\)), and the domestic firm relatively more efficient as its cost is given by \(c_{i}/\bar{e}\) in local currency at the foreign market. Consequently, subsidy level of the domestic country increases as expected exchange rate increases. The argument for the foreign country is symmetric.

It is, however, unrealistic to assume that marginal cost of the domestic firm remains constant while domestic currency depreciates. We will therefore extend the model to include possible dependency of the marginal cost to exchange rate sensitivity. In order to avoid confusion, let us re-label domestic firm’s marginal cost as \(c_{d}\), and assume that

\[
\frac{d \bar{SW}_{d}}{ds_{d}} = \frac{a_{f} + c_{2} - (2c_{1} + 4s_{f})\bar{e}_{f}}{9b_{f}}, \text{ and}
\]
\[
\frac{d \bar{SW}_{f}}{ds_{f}} = \frac{a_{d} + c_{1} - (2c_{2} + 4s_{f})\bar{e}}{9b_{d}},
\]
\[ c_d = \alpha \tilde{c}_1 + (1 - \alpha)c_1. \] (8)

Here, \( \alpha \in [0, 1] \) represents the fraction of the cost that is procured from world markets, and can be thought as import dependency of the domestic firm. Remainder of the cost \((1 - \alpha)\) does not depend exchange rates hence represents local inputs such as labor. Setting \( \alpha = 0 \) obtains the original model. Also, let \( \bar{c}_d = \alpha \tilde{c}_1 + (1 - \alpha)c_1 \) be the expected value of \( c_d \).

With this assumption, as exchange rate increases, marginal cost of the domestic firm increases in local currency, and decreases in foreign currency. The countervailing effect may invalidate Proposition 1 as stated in the next proposition. Note that, the new assumption does not affect maximization problems. In particular, (7) still holds with \( \bar{c}_d \) substituted for \( c_1 \).

**Proposition 2.** If \( 2c_1 > a_f + c_2 \) and \( \alpha > \frac{a_f + c_2}{2c_1} \), then domestic country’s subsidy level \( s_d \) is decreasing in \( \bar{c} \). Otherwise, \( s_d \) is increasing in \( \bar{c} \) but its sensitivity is less compared to the case with \( \alpha = 0 \).

**Proof.** Note that \( \frac{ds_d}{d \bar{c}} = (a_f + c_2 - 2ac_1)/4 \), from which the proposition follows.

If depreciation of domestic currency is expected, increasing \( \alpha \) makes domestic firm less efficient in either currency. Therefore, it is expected that subsidy level decreases in \( \alpha \).

**Proposition 3.** Domestic country’s subsidy level is decreasing (increasing) in \( \alpha \) if domestic currency is expected to depreciate (appreciate).

**Proof.** Follows from \( \frac{ds_d}{d \alpha} = c_1(1 - \bar{c})/2 \).

It is clear that subsidy levels are decreasing in respective firms’ marginal costs. However, high cost (domestic) firm may end up receiving a higher subsidy is foreign market sufficiently large, or expected devaluation is sufficiently high.

**Proposition 4.** \( s_d > s_f \) iff \( \bar{c}(a_f + 3c_2) > a_d + 3c_d \)

\(^2\) Note that first derivatives of social welfare functions given in (6) are still linear in exchange rate.
It is interesting to note that optimal subsidy level may exceed marginal cost even in the standard model if the marginal cost is sufficiently low. In this model, over- (under-) subsidy occurs if the exchange rate realization is lower (higher) than the expected rate. Thus, low realizations of exchange may also cause subsidy level to exceed marginal cost. Algebraic manipulation of $s_d - c_d$ yields the following result.

**Proposition 5.** Domestic country’s subsidy exceeds marginal cost of domestic firm if

(i) $\alpha = 0$ and $c_1 < \frac{e(a_f + c_f)}{6}$, or

(ii) $\alpha > 0$ and $e \frac{a_f + c_f}{4c_1\alpha} - \frac{3(1-\alpha)}{2} \frac{1}{\alpha} - \frac{1}{2}$

Proof is omitted.

4. **Effects of Subsidies and Exchange Rates on Relative Prices**

In this section, we investigate comparative statics of market prices in both countries. Equilibrium prices are as follows\(^3\),

\[
\begin{align*}
 p_d &= \frac{a_d + c_d + e(c_2 - s_f)}{3} \\
 p_f &= \frac{e(a_f + c_f) + (c_1 - s_d)}{3}
\end{align*}
\] (9)

where $s_d$ and $s_f$ are given in (7), and $e$ is again the realized exchange rate. Let $\hat{p}_d$ and $\hat{p}_f$ denote free-trade prices, which can be obtained by setting both subsidies to zero above.

As exchange rate increases, marginal cost of foreign firm in the domestic market increases. Therefore, under free trade, domestic market price increases with exchange rate. This relationship can reverse with intense subsidies as shown in the next proposition.

**Proposition 6.** Domestic market price is decreasing in $e$ iff $s_f > c_2 + \alpha c_1$. When domestic price is increasing, its sensitivity to the exchange rate is less compared to the free trade case.

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\(^3\) Both prices are stated in domestic currency unit.
Proof. Since $s_f$ is independent of the realized exchange rate, we have

$$\frac{d p_d}{d e} = \frac{1}{3}(\alpha c_1 + c_2 - s_f) .$$

Under free trade, we have

$$\frac{d \hat{p}_d}{d e} = \frac{1}{3}(\alpha c_1 + c_2) .$$

Since $s_f > 0$, we have

$$\frac{d p_d}{d e} < \frac{d \hat{p}_d}{d e} .$$

When foreign subsidy level exceeds $c_2$, foreign firms effective marginal cost is negative. Hence, increasing exchange rate decreases effective marginal cost, resulting in a decrease of the market price. With $\alpha > 0$, an increase in exchange rate also increases domestic firm’s marginal cost, forcing prices upwards. A large enough foreign subsidy overcomes this additional effect.

For completeness, we state the corresponding result for foreign market price.

**Proposition 7.** Foreign market price, measured in domestic currency, is always increasing in $e$. Furthermore, its sensitivity to the exchange rate is same under both free trade and subsidy regime.

The asymmetry between domestic and foreign prices is purely an artifact of currency translation. When translated to domestic currency, subsidy of the domestic firm becomes independent of exchange rate, hence derivatives are identical whether there is subsidy or not.

5. **Concluding Remarks**

Introducing exchange rate uncertainty in an export subsidy model, this note essentially recasts some of the earlier results. Consider conventional finding that optimal subsidy levels depend on relative costs and foreign market size. Since movements in exchange rate changes relative costs of a firm in the foreign market as well as relative size of the foreign market, both of these relationships can be translated to a relationship with exchange rates. In this sense, comparative statics of subsidy levels and exchange rates are not surprising. Introduction of exchange rates leads to some new results, such as inverse relationship of subsidy levels with import dependency of own firm and reduced exchange rate sensitivity of market prices under subsidy regime. Hopefully, this modeling innovation will open research possibilities.
References


