A SURVEY ON MULTI TRIP VEHICLE ROUTING PROBLEM

Ahmet Şen[1] and Kerem Bülbül[2]

Abstract — *The vehicle routing problem (VRP) and its variants are well known and greatly explored in the transportation literature. The vehicle routing problem can be considered as the scheduling of vehicles (trucks) to a set of customers under various side constraints. In most studies, a fundamental assumption is that a vehicle dispatched for service finishes its duty in that scheduling period after it returns back to the depot. Clearly, in many cases this assumption may not hold. Thus, in the last decade some studies appeared in the literature where this basic assumption is relaxed, and it is allowed for a vehicle to make multiple trips per period. We consider this new variant of the VRP an important one with direct practical impact. In this survey, we define the vehicle routing problem with multiple trips, define the current state-of-the-art, and report existing results from the current literature.*

Keywords — *multi trip, vehicle routing problem*

VEHICLE ROUTING PROBLEM

In many transportation settings, the vehicle routing problem (VRP) is an important problem and many variants of VRP are well studied in the literature. In its basic form, there are m identical vehicles to serve n customers with known demands from a single depot. Let Q be the capacity of a single vehicle, and L be the time limit for a single trip of a vehicle. A trip is a sequence of customers starting from and finishing at the depot. In VRP, the basic assumptions are;

1. Each vehicle leaves the depot, visits at least one customer and returns to the depot.
2. Each customer must be served.
3. Each customer is served by only one vehicle.
4. For a vehicle, the total amount of demand of the customers on its trip can not exceed Q while the duration of the trip should be no more than L.

Then VRP can be described as finding m vehicle trips with the minimum total distance obeying the rules given above.

Many different types of mathematical programming formulations are given in the literature for VRP. For a thorough treatment of the problem formulations, variants, and exact and heuristic solutions, the reader is referred to (Toth and Vigo, 2001). The given definition of VRP is very theoretical and in real life practice many additional assumptions should be considered such that (1) there may be different types of vehicles, (2) while some customers ask for goods to be delivered (delivery or linehaul customers), some customers ask for goods to be picked from (pickup or backhaul customers), and some customers ask for both services, (3) some customers may have time windows for service, (4) there may be more than one depot, (5) a vehicle can serve only a subset of the customers due to time/space limitations, (6) the depot(s) may have time windows.

The assumption that a single vehicle can perform only one route in VRP is very limiting, since if it is possible a company would like to use its resources efficiently and as needed. Given the long history of transportation problems (see for example (Ulusoy, Bülbül and Şen, 2007)) in the literature, only in the last two decades some stress is given to the VRP with multiple use of vehicles, with much work being done more recently ((Taillard, Laporte and Gendrau, 1996), (Brandão and Mercer, 1998), (Petch and Salhi, 2004), (Alonso, Alvarez and Beasley, 2007), (Olivera and Viera, 2007), (Salhi and Petch, 2007)).

In many studies referenced in the previous paragraph, VRP with different assumptions are considered and a different name is given for the problem where vehicles are allowed to do more than one route over the planning horizon. “Multiple use of vehicles” (Taillard, Laporte and Gendrau, 1996), “multi—trip” (Brandão and Mercer, 1998), “multiple trips” (Petch and Salhi, 2004) and “multiple vehicle trips” (Salhi and Petch,

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[1] Ahmet Şen, Sabancı University, Faculty of Engineering and Natural Sciences, Tuzla, Istanbul, Turkiye, ahmetsen@sabanciuniv.edu
[2] Kerem Bülbül, Sabancı University, Faculty of Engineering and Natural Sciences, Tuzla, Istanbul, Turkiye, bulbul@sabanciuniv.edu

2007) are used for identifying these type of VRPs. In this study we will refer to them as VRP with multi—trips (VRPM) following (Taillard, Laporte and Gendrau, 1996).

In the following sections, we will first give a literature survey on VRPM, the solution methods used in the literature and the comparison of their results where available. In the final section we will provide future research opportunities that can be explored for VRPM.

LITERATURE SURVEY ON VRPM

Main Studies Related with VRPM

Taillard, Laporte and Gendreau, 1996: In the literature, the first study including the multi trip idea for VRP is due to working paper (Fleischmann, 1990) which is referred to by (Taillard, Laporte and Gendreau, 1996). (Taillard, Laporte and Gendreau, 1996) work on the VRP with assumptions given in the first section plus the additional assumptions that (1) a vehicle can perform more than one trip in a planning period (2) it is allowed for vehicles to perform trips longer than given time limit, paying some penalty. Their solution algorithm is a population based algorithm using tabu search and bin packing approaches. In Table 1, the details of their algortihm are given. We will refer to their algorithm as TLG96 from here on.

TABLE 1
VRPM Algorithm of (Taillard, Laporte and Gendreau, 1996) (TLG96)

Step	Operations
1. Initial trip generation	Apply tabu search algorithm of (Taillard, 1993) to obtain *h* many VRP solutions to the problem with the basic assumptions and with an unspecified number of vehicles and insert all the individual vehicle trips to LIST.
2. Additional trip generation	Repeat *p* times the following steps: i. Randomly select solutions from the LIST with better trips having a higher probability of being selected. ii. Eliminate all the trips from LIST having common customers with the selected trips, if any trip is left go to i. iii. Using the trips selected in i, apply tabu search to obtain a new VRP solution, and all the separate trips to LIST while eliminating dominated trips from the LIST.
3. Generating VRP solutions	Select q (q>>m) of the trips generated in step 2. Generate a search tree based on the selected trips so that trips can be put together to produce all feasible VRP solutions. As a result, K feasible VRP solutions are kept.
4. Generating VRPM solutions	Solve bin packing problem using "best fit decreasing" approach for m bins (the items to be put into bins are single trips) and each bin having size (time capacity) of L (that is, the bin size is the duration of the planning period) .

Brandao and Mercer, 1998: This work indeed has a very interesting place in the literature. First in (Brandao and Mercer, 1997), the results for an empirical case based on the operations of a real life firm (Burton's Biscuit Ltd.) were given for VRPM with many more additional considerations. In (Brandao and Mercer, 1997) some of the considered assumptions are the following:

1. Multi—trips are allowed.
2. There exist delivery time windows.
3. Vehicles have heterogeneous capacities.
4. Access to customers are restricted by some rules.
5. There exists maximum legal driving time per day for drivers.
6. Unloading times of vehicles are considered.
7. Additional vehicles can be hired if the fleet of the firm is not enough and it is feasible to do so.
8. Real map data is used for the problem.
9. The objective function of the problem takes into account transportation costs which include fuel and maintenance, wage and fixed costs.
10. There is overtime charge.

The case study included 45—70 customers, 11 vans, 11 tractors and trailers, and concluded that the multi—trips were the best way for the company. (Brandao and Mercer, 1998), later, simplified the work previously done by them to have a comparable study with that of (Taillard, Laporte and Gendreau, 1996). As a result, the problem is defined exactly as in (Taillard, Laporte and Gendreau, 1996) in their later work. The study uses the same set of benchmark problems. In Table 2, the details of the algorithm from (Brandao and Mercer, 1998) are given. From here on, this algorithm will be referenced as BM98. The algorithm for (Brandao and Mercer, 1997) is more involved and includes all the aspects given above.

TABLE 2
VRPM Algorithm of (Brandao and Mercer, 1998) (BM98)

Step	Operations
1. Nearest neighbor and insertion procedure	Create a layer (a set of trips such that each trip belongs to a different vehicle). Apply insertion to include more customers to the trips of this layer. Insertion is applied for a customer if it is in some proximity (δ -neighborhood) of the customer it is being inserted before or after. After any layer is obtained, the procedures are reapplied in order to get another layer where the unserved customers are added to trips of the vehicles.
2. Tabu search algorithm	Insert move: Insert a customer into a trip if the neighborhood condition is satisfied and the capacity of a vehicle is not violated, or create a new trip. Swap move: A swap is made between two different trips.

Petch and Salhi, 2004: This study emphasizes the fact that by enabling multi—trips, companies may obtain savings in all transportation costs. (Petch and Salhi, 2004) also point out that VRPM can be essential both for tactical and strategic planning and aim at finding strategic planning insights as a result. (Petch and Salhi, 2004) gives a short literature review for the problem, and explain their method of tackling VRPM. Their solution approach is composed of a multi—phase construction heuristic which can be considered as the combination of the two solution approaches mentioned for (Brandao and Mercer, 1997) and (Taillard, Laporte and Gendreau, 1996). The main ideas of the solution methodology of (Petch and Salhi, 2004) are given in Table 3. Their algorithm, which we will refer to as PS04, is rather involved.

TABLE 3
VRPM Algorithm of (Petch and Salhi, 2004) (PS04)

Step	Operations	
Phase 1.	Construction: Yellow's savings algorithm (Yellow, 1970). Yellow's savings algorithm generates a VRP solution, if applied directly. In order to obtain a pool of VRP solutions, savings calculations are parameterized. Improvement: 2—opt and 3—opt exchange heuristics Refining: (Salhi and Rand, 1987) Elimination: Eliminate the repeated solutions.	
Phase 2.	Bin packing problem: Generation of VRPM solution from VRP solution obtained before. Here, items are the single trips and the bin size is the duration of the planning period. Improvement heuristics:	
	Meiosis	It is about having two new trips from a readily available trip.
	VRP Partition	2—opt , 3—opt for each trip. Combine trips if possible.
	Donate	Like an insert move, a customer is transferred to another trip.
	Exchange	A pair of customers is exchanged between two trips.
	Donate Exchange	2 trips transfer their customers to a third one.
Phase 3.	To obtain many trips, a tour partition approach is implemented. Available trips are partitioned into small feasible trips using a geographical "route codification". When the new solution population is obtained, search is redirected to Phase 2 in order to improve the available solutions.	

Salhi, S. and Petch, R.J., 2007 : This is the only work using a genetic algorithm approach dealing with VRPM. This work uses the same assumptions as in (Petch and Salhi, 2004). To apply a genetic algorithm heuristic, one needs the idea of a chromosome. In this study, the chromosomes are defined by a sector of a circle. The solution for customers in a sector composes the encoding. In their genetic algorithm, the authors use "chromosome injection and cloning" and "crossover and mutation" operators. Savings heuristic is used to solve smaller VRP sub-problems. To obtain a complete set of vehicle trips, bin packing heuristics are used.

The objective function (fitness value) is based on the overtime penalty and driver—time cost. We label their algorithm as SP07.

Alonso, Alvarez and Beasley, 2008: This work is based on a periodic VRP (PVRP). In contrast to VRP defined above, in PVRP the planning of trips are made such that customers can be served 1 to t times in a planning period of t time units. This study also incorporates the fact that not all type of vehicles can visit all customers, which leads to the site—dependent VRP (SDVRP). (Alonso, Alvarez and Beasley, 2008) combine the ideas of SDVRP and the main assumption of our study to work on SDVRP with multi—trips(SDVRPM). One difference of this study from the previous works is that it includes a mathematical model for the problem they are working on (SDVRPM), while the authors make no attempt to solve it. Like the previous works, tabu search is applied to solve the problem. The main points of their algorithm (AAB08) are given in Table 4. The AAB08 algorithm is developed from the tabu search algorithm of (Cordeau, Gendreau and Laporte, 1997) with some differences (such as neighborhood structure, objective function evaluation, initial solution).

TABLE 4
VRPM Algorithm of (Alanso, Alvarez and Beasley, 2008) (AAB08)

Step	Operations
Construction	Randomly assign to each customer a delivery day pattern. Use insertion heuristics to insert customers into vehicle schedules obeying time and capacity constraints. If inserting a customer into a current vehicle trip is not feasible, then, when possible, a new trip is initialized on that day for the vehicle. This results in multiple trips for vehicles.
Search	Objective function is composed of three parts which are put together after scaling: cost of routing, penalty for capacity violation, and penalty for overtime. Two search moves are used up to a specified number of times: 1. (Type—I): A customer is moved from a trip to another trip of the same vehicle or another vehicle on the same day. 2. (Type—II) Replace categories of delivery patterns of a customer.

(Alonso, Alvarez and Beasley, 2008) provide computational results for SDVRP, VRPM and VRP test problems. Since SDVRPM is first introduced in their work, they generated some test instances specifically for this problem.

Olivera and Viera, 2007: This work is a study directly for VRPM. Their solution methodology is based on the adaptive memory procedure of (Rochat and Taillard, 1995). They also present a mathematical programming model based on a set covering formulation for VRPM. (Olivera and Viera, 2007) give results for the same set of benchmark problems as in the previous references. We will denote their algorithm as OV07, and the details of the algorithm is given in Table 5.

TABLE 5
VRPM Algorithm of (Olivera and Viera, 2007) (OV07)

Step	Operations
Construction	Generate solutions using the sweep algorithm, each time starting from a random customer. Put solutions into memory ***M***. While stopping criteria not met: { Construct a new solution s from M. Apply local search to s to obtain s*. Update M using s*. }
Improvement	Like in (Alonso, Alvarez and Beasley, 2008), a three part objective function is used. Two type of moves are employed: 1. Swap: Customers in two separate trips are swapped if certain criteria are met. 2. Move: A customer is moved from one trip to another.

Other Works in the Literature Including Multi—Trips

(Golden, Laporte and Taillard, 1996) presented a work on some variants of VRP as well as on VRPM where it is assumed that vehicles have infinite capacity while the objective is to minimize the length of the maximum distance covered by any vehicle at the end of the day. They report computational results on the same benchmark problems as (Taillard, Laporte, and Gendreau, 1996).

(Fagerholt, 1999) works on the liner shipping problem which deals with a feeder system at the North Sea and covers a week of planning time. In (Fagerholt, 1999), the feeder system problem is considered to be similar to VRPM. Their solution approach resembles that of (Taillard, Laporte and Gendreau, 1996)) and generates single ship trips, combines single trips into multiple trips, and solves a set partitioning problem. The test problems are composed of 20, 30, and 40 customers. In the real world problem the study provides, there exist 15 customers.

The problem of providing high service levels for postal services is dealt in (Grünert, Sebastian and Thärigen, 1999) for the reorganization of Deutsche Post, Germany. One of the main goals of this study is to satisfy all demand at a minimum cost. One of the "scenarios" in their routing problem is allowing multi—trips. This paper does not further elaborate on the multi—trip problem.

(Cheung and Hang, 2003) focus on the VRP with time windows and backhauls with the additional assumptions that vehicle capacities and working plans are heterogeneous, multi—trips are allowed, and linehaul and backhaul customers can be served in any order. (In many VRP with backhaul problems, all linehaul customers are served before any backhaul customers on a single trip.) If a vehicle arrives at a customer earlier than the associated time window it incurs a cost and there is reward for delivery. Their solution approach is rather involved. In the computational study, results from three different problem sets are incorporated: R1—type instances from Solomon problems (Solomon, 1987), randomly generated instances and real world instances. The real life problem is from Hong Kong air cargo forwarders including 122 customers. In their results, no emphasis is given for multi—trip solutions.

A real life example for multi—trip VRP is given in (Gribkovkaia et al., 2006) for optimizing the collection of livestock. A mixed integer mathematical programming model is constructed for the problem, but no numerical study is conducted.

Computational Results

The main results for VRPM are given for benchmark problems well known in the literature. Seven test problems (CMT problems) from (Christofides, N., Mingozzi, A., and Toth,P., 1979) and two test problems (F problems) from (Fisher, 1994) are reported in the references cited before. Since a vehicle can do more than one trip in the planning period in VRPM, a total time limit is imposed on the total workload of a single vehicle. In many of the problem instances, the algorithms TLG96, BM98, PS04, SP07, AAB08 and OV07 found feasible solutions. When no feasible solution is obtained for some instances, then the results are reported based on the ratio of the longest tour length to the allowed time limit a single trip (see next paragraph) in all the corresponding references. The total working time limits for the vehicles are M_1=$(1.05z^*/m)$ and M_2=$(1.10z^*/m)$, where z^* is the optimal solution times obtained by (Rochat and Taillard, 1995) and m is the number of vehicles. The properties of the problem instances as well as the CPU times for a number of algorithms are given in Table 6. These results are based on the instances for which feasible solutions are identified. All CPU times are in minutes. The computation times are scaled in order to account for different computing platforms based on the work of (Salhi and Petch, 2007).

TABLE 6
Benchmark Problems and CPU Times

Problem	# of cust.	z^*	TLG96	BM98	PS04	OV07	SP07
CMT-1	50	524.61	5	2.5	1.8	0.16	0.26
CMT-2	75	835.26	7	5	5.5	0.33	0.50
CMT-3	100	826.14	24	10	13.8	0.40	1.17
CMT-4	150	1028.42	51	25	16.4	0.88	3.44
CMT-5	199	1291.44	66	62.5	40.9	1.68	8.06
CMT-11	120	1042.11	45	25	40.5	0.37	18.86
CMT-12	100	819.56	23	10	2	0.37	0.75
F-11	71	241.97	26	2.5	4.3	0.13	1.55
F-12	134	1162.96	75	80	13.5	0.50	9.73

When a feasible solution cannot be computed for a given instance, it may be plausible to have a solution slightly violating some of the constraints. All the works for VRPM incorporated this idea. If there is no feasible solution with respect to M_1 or M_2, then LTR (the ratio of the longest trip of a vehicle to M_1 or M_2) is reported. Table 7 summarizes the solutions which were infeasible regarding M_1. An empty cell in the table means that the corresponding algorithm obtained a feasible solution. In Table 7, the total length of the VRPM solution is not given for some algorithms since it is not reported in the corresponding references. Note that in Table 7, results for problem F-12 are not given since the problem is solved to optimality by all the algorithms.

TABLE 7
Comparison of Infeasible Solutions to Benchmark Problems with respect to M_1

Problem	m	TLG96 Length	TLG96 LTR	BM98 Length	BM98 LTR	PS04 LTR	AAB07 Length	AAB07 LTR	OV07 Length	OV07 LTR	SP07 LTR
CMT-1	3	533.00	1.115	556.34	1.041	1.026	554.37	1.041	558.82	1.024	1.003
	4	546.29	1.027	547.10	1.027	1.085	547.10	1.027	547.10	1.027	1.056
CMT-2	6	841.60	1.032	858.04	1.031	1.019	851.59	1.052			1.068
	7	843.60	1.073	870.07	1.088	1.064	855.59	1.050	873.40	1.009	1.102
CMT-3	5	829.50	1.062			1.052					1.056
	6	842.85	1.032	837.82	1.003	1.061	839.58	1.002			1.050
CMT-4	7	1042.39	1.033	1063.95	1.071	1.072	1053.39	1.041	1072.82	1.002	1.09
	8	1049.02	1.075	1069.54	1.031	1.058	1062.53	1.036			1.1
CMT-5	9			1329.28	1.056	1.024	1327.63	1.041			1.047
	10	1316	1.024	1336.92	1.051	1.064	1350.52	1.045			1.076
CMT-11	4	1042.11	1.020	1069.24	1.011	1.052	1097.05	1.038			1.052
CMT-12	4			819.56	1.012						
	5	819.56	1.050	828.94	1.036		825.65	1.035			1.015
	6	819.56	1.064	819.56	1.072	1.029	819.99	1.113	852.19	1.014	1.024
F-11	2	241.97	1.031	255.12	1.011	1.020					1.020
	3	244.60	1.075	254.40	1.011	1.020			256.85	1.025	1.020

The results with respect to M_2 are not given here, since most of the algorithms solve VRPM to feasibility under M_2, except for TLG96. Tables 6-7 indicate that the algorithm OV07 seems to be the most promising algorithm cited in this survey based on both computation times and LTR values.

CONCLUSIONS

VRPM is an important extension of the well known vehicle routing problem where vehicles may be dispatched several times in a scheduling period. As it is also stated in (Petch and Salhi, 2004), VRPM can be used in strategic decision making. The real life applications may require more assumptions than considered for VRPM, as considered in (Brandao and Mercer, 1997). These assumptions may very well include other assumptions such as backhauls, more than one depot, pickup and delivery, simultaneous pickup and delivery. Currently we are working on VRPM with some of the real life assumptions taken into account.

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