Sliding Modes in Power Electronics and Motion Control

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Abstract - In the paper the general approach to motion control systems in the sliding mode framework is discussed in details. It has been shown that, due to the fact that a motion control system with n d.o.f may be mathematically formulated in a unique way as a system composed on n 2 d.o.f systems, design of such a system may be formulated in a unique way as a requirement that the generalized coordinates must satisfy certain algebraic constrain. Such a formulation leads naturally to sliding mode methods to be applied where sliding mode manifolds are selected to coincide with desired constraints on the generalized coordinates. In addition to the above problem the design of full observer for IM based drive is discussed.

I. INTRODUCTION

The complexity and nonlinear dynamics of motion control systems along with high-performance operation require complex, often nonlinear control system design, to fully exploit system capabilities. Basic goals for motion control systems include trajectory tracking, velocity control and control of the force exerted by the system to the environment with torque or force as the control input. The torques or forces are on the other hand the outputs of actuators, often electrical motors, with their own complex nonlinear dynamics. In most approaches to motion control systems the dynamics of torque or force is neglected and controllers are designed assuming perfect tracking in the torque or force control loop, which is not case in many systems and such a design procedure may create some difficulties in systems with high dynamical demands. In this paper, main problems in motion control systems like position tracking; force (torque) control along with control and state estimation in induction electrical machines will be discussed in the sliding mode control framework. In the first part a generalized approach to sliding mode control in motion control systems will be presented with some illustrative examples. After that we will discuss the control of induction machine as one example of systems, which include fast dynamics of electromagnetic system and control of mechanical coordinates. At the end we will present latest results in sliding mode application for induction machine state and parameters estimation.

II. SMC IN MOTION CONTROL SYSTEMS

For 'fully actuated' mechanical system (number of actuators equal to the number of the primary masses) mathematical model may be found from Euler-Lagrange formulation in the following form

$$\begin{aligned}&\dot{\mathbf{q}}_1 = \mathbf{q}_2\\&\mathbf{M}(\mathbf{q}_1)\dot{\mathbf{q}}_2 + \mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) = \boldsymbol{\tau} - \mathbf{G}_{ext}\end{aligned} \tag{1}$$

where $\mathbf{q}_1 \in \Re^n$ stands for vector of generalized positions, $\dot{\mathbf{q}}_1 = \mathbf{q}_2$ stands for vector of generalized velocities $\mathbf{M}(\mathbf{q}_1) \in \Re^{nxn}$ is generalized positive definite inertia matrix with bounded parameters hence $M^- \le \|\mathbf{M}(\mathbf{q}_1)\| \le M^+$, $\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) \in \Re^{nx1}$ represent vector of coupling forces including gravity and friction and is bounded by $\|\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t)\| \le N^+$, $\boldsymbol{\tau} \in \Re^{nx1}$ with $\|\boldsymbol{\tau}\| \le \tau_0$ is vector of generalized input forces and $\mathbf{G}_{ext} \in \Re^{nx1}$ with $\|\mathbf{G}_{ext}\| \le g_0$ is vector of generalized external forces. $M^-, M^+, N^+, \tau_0, g_0$ are known scalars. Note that many different norms may be employed but the most common one is 2-norm. Interested reader is referred to textbooks on robotics for a detailed treatment of derivations of equations (1). In system (1) vector $(\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) + \mathbf{G}_{ext})$, which contains most of unknown parameters of the system, can be treated as a disturbance vector satisfying matching conditions [1]. Model (1) may be rewritten as n second order systems of the form

$$\begin{aligned}&\dot{q}_1 = q_2\\&m_{ii}\dot{q}_{i2} + n_i = \tau_i - g_{exti} - \sum_{j=1, j\neq i}^{n} m_{ij}\dot{q}_{j2}, \quad i = 1,\dots,n\end{aligned} \tag{2}$$

where the elements of inertia matrix are bounded $m_{ij}^- \le |m_{ij}(t)| \le m_{ij}^+$, the elements of vector $\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t)$ are bounded $n_i^- \le |n_i(t)| \le n_i^+$ and the elements of the external force vector are bounded by $g_{0i}^- \le |g_{exti}(t)| \le g_{0i}^+$ and the input generalized torques are bounded $\tau_{0i}^- \le |\tau_i(t)| \le \tau_{0i}^+$.

A. *Control problem formulation*

Vectors of generalized positions and generalized velocities define configuration of a mechanical system. That allows motion control problems to be defined as a requirement to enforce certain dependence between generalized coordinates $\boldsymbol{\sigma}(\mathbf{q}_1, \mathbf{q}_2, t) = \mathbf{0}_{nx1}$. In general that dependence may be expressed by a nonlinear function. Without any loss of generality, in this paper we will assume

$\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)=\mathbf{0}_{nx1}$ as linear with respect to generalized vectors as depicted in (3)

$$\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)=C\mathbf{q}_1+\mathbf{q}_2-\mathbf{f}(t)=\mathbf{0},\ \boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)\in\Re^{nx1},$$
$$\mathbf{C}>0.\ \boldsymbol{\sigma}=[\sigma_1,\sigma_2,....,\sigma_n]^T \tag{3}$$

where $\mathbf{f}(t)\in\Re^{nx1}$ is known continuous and bounded function of time $\|\mathbf{f}(t)\|\le f_0$ with continuous and bounded first time derivative. Requirement (3) is equivalent to enforcing sliding mode in manifold $S_q=\{\mathbf{q}_1,\mathbf{q}_2:\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)=\mathbf{0}\}$, elements of $\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)$ being $\sigma_i=c_iq_{i1}+q_{i2}-f_i(t)$, *i=1,2,..,n*. If sliding mode is established in manifold (3) then equivalent control [2], being solution of $\dot{\boldsymbol{\sigma}}\big|_{\tau=\tau_{eq}}=\mathbf{C}\dot{\mathbf{q}}_1+\dot{\mathbf{q}}_2-\dot{\mathbf{f}}(t)\big|_{\tau=\tau_{eq}}=\mathbf{0}$, is determined as

$$\boldsymbol{\tau}_{eq}=\mathbf{M}\left(\dot{\mathbf{f}}(t)-\mathbf{C}\mathbf{q}_2\right)+\mathbf{N}+\mathbf{G}_{ext}, \tag{4}$$

and equations of motion (1) with sliding mode in manifold (3) are reduced to $\mathbf{q}_2=\mathbf{f}(t)-\mathbf{C}\mathbf{q}_1$. Consequently, sliding mode control may be effectively applied in motion systems (1) to control problems that may be defined as depicted in (3). In robotics systems position tracking and force tracking are two basic control problems. Selecting reference trajectory as $\mathbf{q}_1^{ref}(t)$, the position tracking problem can be specified as a requirement that sliding mode is enforced in manifold (5)

$$S_{q_1}=\left\{\mathbf{q}_1,\mathbf{q}_2:\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)=\mathbf{C}\left(\mathbf{q}_1^{ref}-\mathbf{q}_1\right)+\left(\dot{\mathbf{q}}_1^{ref}-\mathbf{q}_2\right)=\mathbf{0},\mathbf{C}>0\right\}$$
$$S_{q_1}=\{\mathbf{q}_1,\mathbf{q}_2:\sigma(\mathbf{q}_1,\mathbf{q}_2,t)=\mathbf{f}(t)-(\mathbf{C}\mathbf{q}_1+\mathbf{q}_2)\},\mathbf{f}(t)=\left(\mathbf{C}\mathbf{q}_1^{ref}+\mathbf{q}_2^{ref}\right) \tag{5}$$

Assume that the contact force can be modeled as

$$\mathbf{F}=\mathbf{K}(\mathbf{q}_{e1}-\mathbf{q}_1)+(\dot{\mathbf{q}}_{e1}-\mathbf{q}_2) \tag{6}$$

where $\mathbf{q}_{e1}$ is the generalized coordinate of the contact point of the robot tip with environment, $\mathbf{K}>\mathbf{0}$ is spring coefficient matrix. The force control problem in which the contact force $\mathbf{F}$ should track its reference $\mathbf{F}^{ref}(t)$ can be specified as a requirement that sliding mode is enforced in the manifold (7)

$$\begin{gathered}S_f=\left\{\mathbf{q}_1,\mathbf{q}_2,t:\mathbf{F}^{ref}-\left(\mathbf{K}(\mathbf{q}_{e1}-\mathbf{q}_1)+\mathbf{q}_{e2}-\mathbf{q}_2\right)=\mathbf{0}\right\}\\ S_f=\{\mathbf{q}_1,\mathbf{q}_2:\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)=-\mathbf{f}(t)+(\mathbf{K}\mathbf{q}_1+\mathbf{q}_2)\},\\ \mathbf{f}(t)=-\left(\mathbf{F}^{ref}-\mathbf{K}\mathbf{q}_{e1}-\mathbf{q}_{e2}\right)\end{gathered} \tag{7}$$

Both, the trajectory tracking (5) and the force control (7) are mathematically defined is the same way as general motion control problem (3) thus both can be solved by enforcing sliding mode in selected manifolds. Moreover the combination of the two tasks is natural since it only requires change of the siding mode manifold.

II. SELECTION OF CONTROL INPUT

The design of control inputs for system (1), (2) with sliding mode in manifold (3) may follow a few different approaches. Here we will discuss some of the possibilities in order to demonstrate the richness of the sliding mode design approaches to motion control systems.

1) Discontinuous control: First we will demonstrate a straight forward sliding mode approach by selecting discontinuous control input [3]. In this framework control is selected in the following form

$$\boldsymbol{\tau}=-\tau_0 sign(\boldsymbol{\sigma})\Rightarrow\tau_i=-\tau_{0i}sign(\sigma_i),\ i=1,...,n \tag{8}$$

The existence of sliding mode in manifold (3) can be proven by selecting, for each component σ_i of sliding mode function, Lyapunov function candidate as $v_i=\frac{1}{2}\sigma_i^2$ (*i=1,..,n*). Time derivatives $\dot{v}_i=\sigma_i\dot{\sigma}_i$ along the trajectories of the system (2) with control (8) are under assumption that the derivative of functions $f_i(t)$ as well as the elements of inertia matrix, the elements n_i (*i=1,2,…,n*) of vector $\mathbf{N}(\mathbf{q}_1,\mathbf{q}_2,t)$ and the elements of the external force vector are bounded becomes $\dot{v}_i\le-\mu|\sigma_i|,\mu>0$. Consequently the convergence to the intersection of the manifolds $\sigma_i=0$ is established. Each component of the control input undergoes discontinuity by taking values from the set $\{-\tau_{0i},\tau_{0i}\}$. Direct implementation of algorithm (8) may result in chattering [4] so it may not be suitable for direct application. An approach to reduce the effect of the discontinuous control is to implement (8) as $\tau_i=\hat{\tau}_{eq}^{est}-\tau_{0i}sign(\sigma_i)$ where $\hat{\tau}_{eq}^{est}$ is estimated control torque that may be calculated either from the system's model using available measurement and estimated parameters or from disturbance estimation. Asymptotic observers may be used as a bypass for high frequency component [5,6] to eliminate chattering.

2) Discrete-time sliding mode control: Opposite to continuous time SMC in discrete-time SMC motion in sliding mode manifold may occur if control is continuous [7,8,9]. The discrete-time implementation of the sliding mode control is essentially application of the equivalent control determined as a solution of $\sigma_{k+1}\big|_{u_k=u_k^{eq}}=0$. Such implementation requires information on parameters, system states and external disturbances and may not be easy to apply in some motion control systems. Another approach is based on enforcing certain structure of the time derivative for selected Lyapunov function candidate. For system (1) asymptotic stability of the solution $\boldsymbol{\sigma}(\mathbf{q}_1,\mathbf{q}_2,t)=C\mathbf{q}_1+\mathbf{q}_2-\mathbf{f}(t)=\mathbf{0}$ can be assured if a control input is selected as such that Lyapunov function candidate $v_l=\boldsymbol{\sigma}^T\boldsymbol{\sigma}/2$ has time derivative $\dot{v}_l=-\boldsymbol{\sigma}^T\mathbf{D}\boldsymbol{\sigma},\ \mathbf{D}>0$, [6] (for the simplicity in most of the cases $\mathbf{D}=diag\{d_{ii}\}$). After short algebra one can obtain $\dot{v}_l=-\boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}}=-\boldsymbol{\sigma}^T\mathbf{D}\boldsymbol{\sigma},\ \mathbf{D}>0,$ and $\boldsymbol{\sigma}^T(\dot{\boldsymbol{\sigma}}+\mathbf{D}\boldsymbol{\sigma})=\mathbf{0}$ which depends on control due to the presence of the term $\dot{\boldsymbol{\sigma}}$. Control can be selected to enforce $(\dot{\boldsymbol{\sigma}}+\mathbf{D}\boldsymbol{\sigma})\big|_{\sigma\ne0}=\mathbf{0}$. By applying sample and hold process with sampling interval T, the discrete-time control that satisfy given requirements can be determined as

$$\boldsymbol{\tau}(k)=\boldsymbol{\tau}(k-1)+T^{-1}\left[(1+\mathbf{D}T)\boldsymbol{\sigma}(k)-\boldsymbol{\sigma}(k-1)\right], \mathbf{D}>0 \quad (9)$$

Application of approximated control (9) to system (1), (3) leads to the

$$\boldsymbol{\sigma}^T(k)\boldsymbol{\sigma}(k-1)=\boldsymbol{\sigma}^T(k)(\mathbf{I}-T\mathbf{D})\boldsymbol{\sigma}(k) \quad (10)$$

If $\mathbf{D}$ is diagonal matrix, then for each of the components in (13), one can write $\sigma_i(k)\sigma_i(k-1)=\sigma_i^2(k)(1-Td_{ii})$ and d_{ii} may be selected so that $0<(1-Td_{ii})<1$, which ensures existence of quazi-sliding mode motion. This solution is similar with so-called β-equivalent control approach.

3) Sliding mode observers: Sliding mode methods can be applied to design disturbance observer and sliding mode controller. The disturbance observer design may be applied for system (1) by constructing model (11*)*

$$\dot{\hat{\mathbf{q}}}_2=\hat{\mathbf{M}}^{-1}(\boldsymbol{\tau}-\mathbf{u}) \quad (11)$$

where $\hat{\mathbf{M}}, \hat{\mathbf{q}}_2$ are estimates of inertia matrix and generalized velocity, $\mathbf{u}$ is model control input, which should be selected to enforce sliding mode in manifold $\boldsymbol{\sigma}_{q_2}=\mathbf{q}_2-\hat{\mathbf{q}}_2=0$. Equivalent control for observer (17) in manifold $\boldsymbol{\sigma}_{q_2}=\mathbf{0}$ can be calculated as $\mathbf{u}_{eq}=\mathbf{N}+\mathbf{G}_{ext}+\left(\mathbf{M}-\hat{\mathbf{M}}\right)^{-1}\dot{\mathbf{q}}_2$ - what represents total disturbance and parameter uncertainty in system (1). Following the same idea as in scalar case and selecting control input in (1) as $\boldsymbol{\tau}=\mathbf{u}_{eq}+\hat{\mathbf{M}}\mathbf{v}$ motion of the augmented system can be written as $\dot{\mathbf{q}}_1=\mathbf{q}_2$, $\dot{\mathbf{q}}_2=\mathbf{v}$. The equivalency with sliding mode control may be established in the same way as in the previous case. This leads to the simple realization of the acceleration controller that is very similar to the structure obtained in the disturbance rejection framework [10].

III. TIMING-BELT SERVOI SYSTEM

In the following section we will demonstrate application of the above results to a timing-belt driven servo system depicted in Fig. 1.

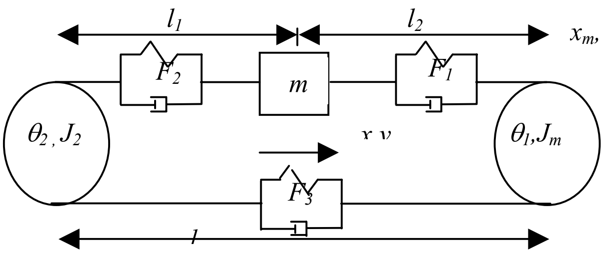

Fig. 1 Timing-belt servo system

Forces F_1, F_2 and F_3 acting on the load ddepend on the stretch of the belt and its derivative, thus depend on both motor and load position. The variables and parameters are: θ_1 angular position of the pulley driven by the servomotor; θ_2 angular position of the un-driven side pulley; $T=K_T i$ torque developed by the servomotor; $T_L(\theta,\omega)$ friction torque at the servomotor side; F_B belt elasticity force; F_D belt internal friction force; G gear ratio (if present in the system); $x_m=2\pi\theta/G$ and v_m longitudinal position and velocity of the belt on the periphery of pulley 1; x and v longitudinal position and velocity of the load; F_L friction force at the load side; m_{mot} equivalent mass on the motor side; m equivalent mass on the load side; r radius of the pulleys. By combining dynamics of the servomotor and the dynamics of the load side one can develop a state space description of the overall system (12), with total belt force given by (13) with its elasticity force $F_B(x_m,x)$of the equivalent spring defined in (14) and damping force $F_D(v_m,v)$due to the belt internal friction defined (15):

$$\begin{bmatrix}\dot{x}\\ \dot{v}\\ \dot{x}_m\\ \dot{v}_m\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K}{m} & 0 & \frac{K}{m} & 0\\ 0 & 0 & 0 & 1\\ \frac{K}{m_{mot}} & 0 & -\frac{K}{m_{mot}} & 0\end{bmatrix}\begin{bmatrix}x\\ v\\ x_m\\ v_m\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{m_{mot}}\end{bmatrix}\left[F_{mot}\right]+\begin{bmatrix}0 & 0\\ \frac{1}{m} & 0\\ 0 & 0\\ 0 & -\frac{1}{m_{mot}}\end{bmatrix}\begin{bmatrix}F_D-F_L\\ F_D+F_{Lmot}\end{bmatrix} \quad (12)$$

$$F_{tBelt}=F_B(\theta,x)+F_D(\omega,v),\ F_{mot}=GK_T i/r \quad (13)$$

$$F_B=K(x)(x_m-x); K(x)=\frac{1}{\frac{1}{C_0}+\frac{1}{K_1+K_2}}$$

$$K_1(x)=\frac{K}{l_{L0}-(x_0+x)};\quad K_2(x)=\frac{K}{x_0+x} \quad (14)$$

$$F_D=K_D(x)(v_{mot}-v);$$

$$K_D(x)=K_0\left|\sqrt{\frac{m_{mot}m}{m_{mot}+m}K(x)}\right|; K_0>0 \quad (15)$$

where C_0 stands for elasticity coefficient of the gear and coupling K_T stands for motor torque constant, I stands for motor current; $K_1(x)$ stands for elasticity coefficient of the un-driven side of the belt $K_2(x)$ stands for elasticity coefficient of the driven side of the belt l_{L0} the total length of the belt on the load side x_0 the length of the belt when $x=0$, $K_D(x)$ stands for damping coefficient. In the above model the dynamics of the actuator with current (torque) controller is disregarded.

Experimental verification is performed on timing-belt driven the linear drive DGEL25-1500-ZR-KF (FESTO) equipped by the electrical servomotor MTR-AC-70-3S-AA. Experimental set-up consists of the original motor driver attached to the dSPACE DS1103 module hosted in the PC. In all experiments sampling in controller loop is kept at T_l=1 millisecond. Position and velocity of motor are measured from an incremental encoder with 1024 ppr. Load position is measured by linear incremental encoder with resolution of $3*10^{-6}$ m per pulse. Selecting current $i^{ref}(k)=u_{eq}r/GK_T+sat\left(i^{ref}(k-1)+K_u((1+DT)\sigma(k)-\sigma(k-1))\right)$ sliding mode motion is guarantied in manifold $\sigma=C_F\left(x_m^{ref}-x_m\right)+\left(v_m^{ref}-v_m\right)$. The following parameters had been used $K_u=2\cdot10^{-5}$ $C_F=450$ $D=250$. In Fig. 2 transients for 1 cm motion with load of 26 kg. Experiments are showing very small overshoot and high positioning accuracy. The pulsation of the motor position error is visible while the load position is not changing.

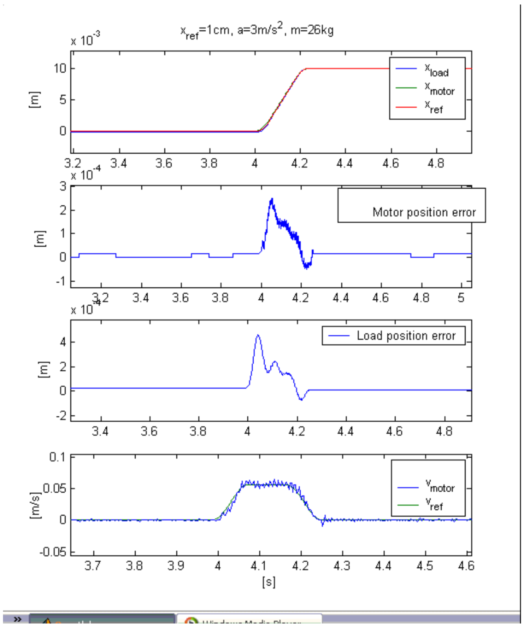

Fig. 2. Transients in the motor position change for 1 cm with m=26 kg and acceleration 1m/s[2]

IV. CONTROL OF INDUCTION MACHINE

A. *Control of Induction Machine*

Control of induction machine (IM) is still a challenging problem due to its nonlinear dynamics, limited possibility to measure or estimate necessary variables and presence of the switching converter with its own nonlinearity as a power modulator in control loop. The dynamics of IM consists of the mechanical motion (16), the dynamics of the stator electromagnetic system (17) and the dynamics of the rotor electromagnetic system (18).

$$\frac{d\omega}{dt} = \frac{1}{J}\left(\tau - T_L\right); \quad \tau = \frac{3L_m}{2L_r}\left(i_{s\beta}\phi_{r\alpha} - i_{s\alpha}\phi_{r\beta}\right) \tag{16}$$

$$\begin{aligned} \frac{di_\alpha}{dt} &= \beta\eta\phi_\alpha + \beta\omega\phi_\beta - \gamma i_\alpha + \frac{1}{\sigma L_s}u_\alpha; \quad \beta = \frac{L_m}{\sigma L_s L_r} \\ \frac{di_\beta}{dt} &= \beta\eta\phi_\beta - \beta\omega\phi_\alpha - \gamma i_\beta + \frac{1}{\sigma L_s}u_\beta; \quad \sigma = 1 - \frac{L_m^2}{L_s L_r} \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{d\phi_\alpha}{dt} &= -\eta\phi_\alpha - \omega\phi_\beta + \eta L_m i_\alpha \\ \frac{d\phi_\beta}{dt} &= -\eta\phi_\beta + \omega\phi_\alpha + \eta L_m i_\beta; \quad \eta = \frac{R_r}{L_r} \end{aligned} \tag{18}$$

where ω is the rotor angular velocity, $\Phi^T_{\alpha\beta} = \left[\phi_\alpha \quad \phi_\beta\right]$, $\mathbf{i}^T_{\alpha\beta} = \left[i_\alpha \quad i_\beta\right]$ and $\mathbf{u}^T_{\alpha\beta} = \left[u_\alpha \quad u_\beta\right]$ are rotor flux, stator current and stator voltage vectors; τ is torque developed by IM and T_L is external load, L_m, L_s and L_r are mutual, stator and rotor inductances respectively. Model (16-18) is written in stationary frame of references (α,β). For power modulation in IM control system a switching power converter is employed with possibility to connect each stator winding of a machine either to + or – bar of a DC power source. The converter switches may take eight distinct configurations $S_i, i = 1,2,\ldots,8$ thus defining eight distinct values $\mathbf{u}(S_i)$. Converter's output voltages u_1, u_2, u_3 are taking values from the discrete set $\{0, V_0\}$. With motor stator windings in star connection the relationship between machine phase voltages u_a, u_b, u_c, stator voltage vectro $u^T_{\alpha\beta} = \left[u_\alpha \quad u_\beta\right]$ and converter output voltages u_1, u_2, u_3 are given as in (19)

$$\mathbf{u}_{\alpha\beta} = \mathbf{T}^{\alpha\beta}_{abc}\mathbf{T}^{abc}_{123}\mathbf{u}_{123} \tag{19}$$

where $\mathbf{T}^{\alpha\beta}_{abc}$ stands for transformation matrix from $(a,b,c) \rightarrow (\alpha,\beta)$ frame of references; $\mathbf{T}^{abc}_{123}$ stands for transformation matrix from $(1,2,3) \rightarrow (a,b,c)$ frame of references. For mechanical motion control system design model (26-28) is usually rewritten in so-called field oriented frame of references (d,q) in which d-axis is collinear with, and q-axis is orthogonal to, vector of rotor flux. Matrix $\mathbf{T}^{dq}_{\alpha\beta}$ describes transformation from (α,β) to (d,q) frame of references and matrix $\mathbf{T}^{dq}_{\alpha\beta}\mathbf{T}^{\alpha\beta}_{abc}\mathbf{T}^{abc}_{123}$ describes transformation from converter output voltages $\mathbf{u}_{123}$ to $\mathbf{u}_{dq}$ voltages (20)

$$\underbrace{\begin{bmatrix} x_d \\ x_q \end{bmatrix}}_{x_{dq}} = \underbrace{\begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix}}_{T^{dq}_{\alpha\beta}} \underbrace{\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}}_{x_{\alpha\beta}}, \quad \mathbf{u}_{dq} = \underbrace{\mathbf{T}^{dq}_{\alpha\beta}\mathbf{T}^{\alpha\beta}_{abc}\mathbf{T}^{abc}_{123}}_{T^{dq}_{123}}\mathbf{u}_{123} \tag{20}$$

$$\begin{aligned} \frac{di_d}{dt} &= \beta\eta\phi_d + \omega i_q - \gamma i_d + \frac{1}{\sigma L_s}u_d \\ \frac{di_\beta}{dt} &= -\omega\left(\beta\phi_d + i_d\right) - \gamma i_q + \frac{1}{\sigma L_s}u_q \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{d\phi_d}{dt} &= -\eta\phi_d + \eta L_m i_d \\ \frac{d\rho}{dt} &= \omega + \frac{\eta L_m}{\phi_d}i_q \end{aligned} \tag{22}$$

$$\dot{\theta} = \omega, \qquad J\dot{\omega} = \tau - T_L = \frac{3L_m}{2L_r}\phi_d i_q - T_L \tag{23}$$

Design of IM motion control can be performed in two steps. In the first step components of the current vector $\mathbf{i}^T_{dq} = \left[i_d \quad i_q\right]$ should be selected to provide reference tracking in the rotor flux control loop i_d, and in mechanical motion loop i_q. In the second step the voltages u_d, u_q should be determined to ensure reference current tracking and then from (30) converter voltages $\mathbf{u}_{123}$ should be selected. This procedure is the same as used for sliding mode control of systems in regular form [11]. The rotor flux dynamics is one of first order system with scalar control and reference tracking can be achieved if sliding mode is guarantied in $S_d = \left\{\phi_d, i_d : \sigma_d = \phi_d^{ref} - \phi_d = 0\right\}$. The mechanical motion is of the same form as system (2) and reference position tracking requires establishment of sliding mode motion in $S_q = \{\theta, \omega, i_q : \sigma_q = c(\theta^{ref} - \theta) + (\dot{\theta}^{ref} - \theta) = \sigma_q = 0\}$ [12]. Selection of discontinuous control is not suitable here due to the fact that thus determined components of the current vector will be set as references in the current control loop. One of possible solutions for enforcing quazi-sliding mode in manifolds S_d and S_q is selection of $i_d(k) = i_d(k-1) + K_d((1 - Td_d)\sigma_d(k) - \sigma_d(k-1))$ and $i_q(k) = i_q(k-1) + K_q((1 - Td_q)\sigma_q(k) - \sigma_q(k-1))$ which results in sliding mode motion $d_d(\phi_d^{ref} - \phi_d) + (\dot{\phi}_d^{ref} - \dot{\phi}_d) = 0$ and $cd_q(\theta^{ref} - \theta) + (c + d_q)(\dot{\theta}^{ref} - \theta) + (\ddot{\theta}^{ref} - \ddot{\theta}) = 0$ respectively. Thus determined values of the stator current shoul be treated as references $i_d^{ref} = i_d(k)$ and $i_q^{ref} = i_q(k)$ respectively, sliding mode in the intersection of manifolds $i_d^{ref} - i_d = \sigma_{di} = 0$ and $i_q^{ref} - i_q = \sigma_{qi} = 0$ can be enforced by selecting $u_d = U_0 sign(\sigma_{di})$ and $u_q = U_0 sign(\sigma_{qi})$ with $U_0 > \max\left(\sup_t |u_{deq}|, \sup_t |u_{qeq}|\right)$, thus guarantying that components i_d, i_q of stator current track their references.

As result of such a design procedure the stator voltage vector in the (d,q) frame of references is determined. Each of the control vector components is taking values from the set $\{-U_0, +U_0\}$. In order to complete control system design switching sequence of the converter switches defining outputs u_1, u_2, u_3 should be determined. To determine which one of the eight configurations $S_i, i = 1,2,...,8$ should be applied one should map vector $\mathbf{u}_{dq}$ to vector $\mathbf{u}(S_i)$. Matrix $\mathbf{T}_{123}^{dq}$ is 2x3 matrix, thus different algorithms for mapping $\mathbf{u}_{dq}$ to $\mathbf{u}_{123}(S_i)$ can be used offering a room for deriving different PWM strategies for the selection of the switching sequences. Indeed, many diiferent solutions can be found in litereture [6,13,14]. Above algorithms can be applied for three phase voltage source converters or for other types of three phase electrical machines without any change.

B. *Induction Machine Flux and Velocity Observer*

Design of observer that will give good estimate of the rotor flux is key to motor control. In so-called sensorless drives estimation of rotor flux and rotor angular velocity is a key to successful design. In this section we explore the IM estimation issues in the framework of sliding mode control. In [14] - the first ideas on IM identification in sliding mode framework - rotor time constant η and angular velocity ω are treated as control in stator current model. That solution is further used in a closed loop torque control system [15]. In general SMC based IM observers use stator current dynamics and selection of the additional control input in such a way that estimated current tracks real currents. A stator current observer may be generalized in the following form

$$\frac{d\hat{i}_\alpha}{dt} = E_\alpha + \frac{1}{\sigma L_s} u_\alpha + V_\alpha \qquad \frac{d\hat{i}_\beta}{dt} = E_\beta + \frac{1}{\sigma L_s} u_\beta + V_\beta \tag{24}$$

where V_α and V_β are components of the observer's control vector. Then estimation error dynamics becomes:

$$\frac{d\varepsilon_{i\alpha}}{dt} = \beta\eta\phi_\alpha + \beta\omega\phi_\beta - \gamma i_\alpha - E_\alpha - V_\alpha \qquad \frac{d\varepsilon_{i\beta}}{dt} = \beta\eta\phi_\beta - \beta\omega\phi_\alpha - \gamma i_\beta - E_\beta - V_\beta \tag{25}$$

If components V_α and V_β of control vector are selected such that sliding mode exists in $\varepsilon_{i\alpha} = 0, \varepsilon_{i\beta} = 0$ then the following is true:

$$V_{\alpha eq} = \beta\eta\phi_\alpha + \beta\omega\phi_\beta - \gamma i_\alpha - E_\alpha = f_\alpha(\phi, i, \omega, \eta, \beta, \gamma) \qquad V_{\beta eq} = \beta\eta\phi_\beta - \beta\omega\phi_\alpha - \gamma i_\beta - E_\beta = f_\beta(\phi, i, \omega, \eta, \beta, \gamma) \tag{26}$$

By selecting different structures of vector $\mathbf{E}^T = \begin{bmatrix} E_\alpha & E_\beta \end{bmatrix}$ equivalent control $\mathbf{V}_{eq}^T = \begin{bmatrix} V_{\alpha eq} & V_{\beta eq} \end{bmatrix}$ will have different values. This offers a range of possibilities in determining $f_\alpha(\phi, i, \omega, \eta, \beta, \gamma), f_\beta(\phi, i, \omega, \eta, \beta, \gamma)$ as functions of selected variables (rotor flux, rotor angular velocity, currents) and some of the machine parameters. By proper selection of functions (26) one is able to determine at least two of the unknown variables or parameters or combination of variables and parameters of machine. This leads to variety of structures that may be derived from this approach. Selection of observer control vector $\mathbf{V}^T = \begin{bmatrix} V_\alpha & V_\beta \end{bmatrix}$, to enforce sliding mode in $\varepsilon_{i\alpha} = 0, \varepsilon_{i\beta} = 0$, may follow different procedures of sliding mode control. In the discontinuous control framework selection of $V_\alpha = V_0 sign(\varepsilon_{i\alpha})$ and $V_\beta = V_0 sign(\varepsilon_{i\beta})$ with $V_0 > \max\left(\sup_t |f_\alpha|, \sup_t |f_\beta|\right)$ then sliding mode in $\varepsilon_\alpha = 0, \varepsilon_\beta = 0$ is guarantied and observer outputs are equal to the motor currents. With such selection equivalent control $V_{\alpha eq}, V_{\beta eq}$ can be determined using simple first order filters. Discrete-time design may be also used in determining the structure of the controller in the motor current tracking loop. After determining the equivalent control and knowing the structure of f_α, f_β from (26) one can determine two unknowns – being variables or parameters of machine. In [16,17] relation (26) was used to determine rotor flux vector assuming that parameters of the machine and the angular velocity are known. If $\mathbf{E}^T = \begin{bmatrix} -\gamma i_\alpha & -\gamma i_\beta \end{bmatrix}$ rotor flux can be determined as

$$\begin{bmatrix} \phi_\alpha \\ \phi_\beta \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \eta & \omega \\ -\omega & \eta \end{bmatrix}^{-1} \begin{bmatrix} V_{\alpha eq} \\ V_{\beta eq} \end{bmatrix} \tag{27}$$

In the same works approach that allows for angular velocity estimation is described. The idea uses the fact that in addition to the stator circuit observer (24) a rotor flux observer may be derived by substituting $V_{\alpha eq} = \beta\eta\phi_\alpha + \beta\omega\phi_\beta$ and $V_{\beta eq} = \beta\eta\phi_\beta - \beta\omega\phi_\alpha$ into (18) to obtain

$$\frac{d\hat{\phi}_\alpha}{dt} = -\frac{1}{\beta}V_{\alpha eq} + \eta L_m i_\alpha \qquad (28)$$
$$\frac{d\hat{\phi}_\beta}{dt} = -\frac{1}{\beta}V_{\beta eq} + \eta L_m i_\beta$$

From (28) rotor flux can be estimated thus providing additional information that can be used to determine rotor angular velocity and rotor time constant from (29)

$$\begin{bmatrix}\hat{\phi}_\alpha \\ \hat{\phi}_\beta\end{bmatrix} = \frac{1}{\beta}\begin{bmatrix}\hat{\eta} & \hat{\omega} \\ -\hat{\omega} & \hat{\eta}\end{bmatrix}^{-1}\begin{bmatrix}V_{\alpha eq} \\ V_{\beta eq}\end{bmatrix} \qquad (29)$$

The estimated motor angular velocity and time constants can be found as

$$\begin{bmatrix}\hat{\eta} \\ \hat{\omega}\end{bmatrix} = \frac{1}{\sqrt{\hat{\phi}_\alpha^2 + \hat{\phi}_\beta^2}}\begin{bmatrix}\hat{\phi}_\alpha & \hat{\phi}_\beta \\ \hat{\phi}_\beta & -\hat{\phi}_\alpha\end{bmatrix}\begin{bmatrix}V_{\alpha eq} \\ V_{\beta eq}\end{bmatrix} \qquad (30)$$

Further improvement of the above approach is presented in [15]. An observer that allows estimation of rotor flux, angular velocity and rotor time constant is discussed. In this solution vector **E** in (24) is selected as $\mathbf{E}^T = \begin{bmatrix}-\vartheta i_\alpha & -\vartheta i_\beta\end{bmatrix}; \vartheta = R_s/\sigma L_s$, and then the components of equivalent control in (26) are determined as $V_{\alpha eq} = \beta\eta\phi_\alpha + \beta\omega\phi_\beta - \beta L_m \eta i_\alpha$ and $V_{\beta eq} = \beta\eta\phi_\beta - \beta\omega\phi_\alpha - \beta L_m \eta i_\beta$. Under assumption that rate of change of angular velocity ω and the rate of change of rotor time constant η are small $\dot{\omega} = 0, \dot{\eta} = 0$ one can design an observer of components of vector V_{eq}^T in the following form

$$\begin{bmatrix}\dot{\hat{L}}_\alpha \\ \dot{\hat{L}}_\beta\end{bmatrix} = -\begin{bmatrix}\hat{\eta} & \hat{\omega} \\ -\hat{\omega} & \hat{\eta}\end{bmatrix}\begin{bmatrix}L_\alpha \\ L_\beta\end{bmatrix} - \beta L_m \hat{\eta}\begin{bmatrix}i_\alpha \\ i_\beta\end{bmatrix} - K\begin{bmatrix}\varepsilon_{L\alpha} \\ \varepsilon_{L\beta}\end{bmatrix}; \begin{bmatrix}\varepsilon_{L\alpha} \\ \varepsilon_{L\beta}\end{bmatrix} = \begin{bmatrix}V_{\alpha eq} - \hat{L}_\alpha \\ V_{\beta eq} - \hat{L}_\beta\end{bmatrix} \qquad (31)$$

where adaptation of rotor time constant and speed is governed by (32)

$$\begin{bmatrix}\dot{\hat{\eta}} \\ \dot{\hat{\omega}}\end{bmatrix} = \begin{bmatrix}V_{\alpha eq} + \beta L_m i_\alpha & V_{\beta eq} + \beta L_m i_\beta \\ V_{\beta eq} & -V_{\alpha eq}\end{bmatrix}\begin{bmatrix}V_{\alpha eq} - \hat{L}_\alpha \\ V_{\beta eq} - \hat{L}_\beta\end{bmatrix}, \qquad (32)$$

$$\begin{bmatrix}\phi_\alpha \\ \phi_\beta\end{bmatrix} = \frac{1}{\beta}\begin{bmatrix}\eta & \omega \\ -\omega & \eta\end{bmatrix}^{-1}\begin{bmatrix}V_{\alpha eq} + \beta L_m \eta i_\alpha \\ V_{\beta eq} + \beta L_m \eta i_\beta\end{bmatrix} \qquad (33)$$

Convergence is assured since derivative of Lyapunov function $v_l = \frac{1}{2}\left[\varepsilon_{L\alpha}^2 + \varepsilon_{L\beta}^2 + \varepsilon_\omega^2 + \varepsilon_\eta^2\right]$ where $\varepsilon_\omega = \omega - \hat{\omega}$ and $\varepsilon_\eta = \eta - \hat{\eta}$ can be expressed as $v_l = -k\left[\varepsilon_{L\alpha}^2 + \varepsilon_{L\beta}^2\right] \le 0$. This solution shows applicability of SMC approach for design of nonlinear observers, it represents very good background for the sensorless drive design. Limitation due to the assumption that angular velocity is slow changing variable seems acceptable in most of the operational modes of the drive. Presented solution for observer seems the most complete until now. Further work should be directed towards elimination of the assumption of constant angular speed what could be done only if mechanical motion and load torque of the drive are estimated.

V. CONCLUSIONS

In this papaer the sliding mode design methods and their applications motion control systems are discussed. In this framework the dynamics of the subsystem that generate generalised force is neglected and the force control system is assumed ideal in a sense that it perfectly tracks reference value. The realisation of the control input in continuous time and discrete-time framework is discussed. IM induction machine motion control and state estimation is discussed with an aim to show validity of the SMC approach in the cases when dynamics of the torque/force generation is taken into account. It was shown that the same motion dynamics as attained in previous case could be achieved here too. The design of the IM rotor flux and velocity observer is discussed in last part of the chapter. The usefulness of the SMC approach is demonstrated in this case too.

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