Discrete-time Sliding Mode Control of High Precision Linear Drive using Frictional Model

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Abstract- **The paper deals with high precision motion control of linear drive system. The accuracy and behavior of the linear drive system are highly affected by the non-linear frictional component compromising of stiction, viscous and stribeck effect present in the system especially in the vicinity of zero velocity. In order to achieve the high accuracy and motion it is mandatory to drive our system with low velocity resulting in many non linear phenomena like tracking error, limit cycles and undesired stick-slip motion etc. This paper discuss the design and implementation of discrete time sliding mode control along with the implementation of dynamic frictional model in order to estimate and compensate the disturbance arising due to frictional component. Experimental results are presented to illustrate the effectiveness and achievable control performance of the proposed scheme.**

I. INTRODUCTION

In most of the high precision control application it becomes mandatory to control the shaft of the linear drive to a varying reference angle with high accuracy in spite of the high disturbance present in the system mostly dominated by the highly disturbance component present in the system. In case of the multidimensional motion of a linear drive it becomes inevitable to compensate the effect of friction in order to accurately place the end effectors of the system in the desired location as well as desired dynamics.

The most dominant disturbance which hinders the precise positioning and the dynamics is due to frictional part which is highly non-linear in nature. Thus there is immense need to remove the frictional part present in the system and to come out with a robust controller in order to achieve a high accuracy and desired dynamics. In the literature many methods are proposed in order to compensate for the disturbance, Yang and Tomizuka [1] compensated the friction component by adaptively changing the frequency of the control input, Tung et al. [2] compensated the friction with a repetitive control scheme. Rao et al [3] have proposed a friction observer and disturbance observer. Pan [4] used a PID controller using a defined model [5] for angular positioning. Apart from these, many recent approaches fuzzy control [6] and learning control [7] can be found on the literature.

In all the approaches the frictional model produced destabilizing effects in the low velocities. The above approaches did not take the hysteretic behavior when studying friction for non stationary velocities nor variation in the break-away force with the experimental conditions nor small displacement that occur at the contact interface during stiction.

In this paper a dynamic friction model [8] is implemented which describes the stiction behavior along with coulombs friction, viscous friction, and stribeck effect [9]. This paper also describes the implementation of variable structure control for continuous system with discrete-time implementation of the control algorithm by maintaining sliding mode. A considerable amount of work has been done analyzing discrete-time sliding modes [10-17]. Most of the proposed control strategies use, in one or another way, the calculation or estimation of the discrete-time equivalent control explicitly, which requires the transformation of the plant model into a discrete-time form.

II. MODELING AND SYSTEM DESCRIPTION

In the ball screw drive mechanism the friction is the major dominant disturbance exists in the system which is highly non-linear in nature. Thus our system can be modeled as

$$\frac{dx}{dt} = f(x,t) + B(x,t)u(x,t)\ ,\ x \in R^n, u \in R^m \quad (2.1)$$

where all elements of vector f(x,*t*) and matrix B(x,*t*) are continuous and bounded, and having continuous and bounded first order time derivatives; rank(B(x,*t*)=*m*, $\forall \mathbf{x}, t > 0$; all components of the control input u(x,*t*) are bounded by known functions $u(x,t)_{imax}$ and $u(x,t)_{imin}$, i=1,2,...,m.The design goal is to stabilize the system motion the smooth manifold

$$S = \left\{ x : \sigma(x,t) = 0; G(x,t) = \frac{\partial \sigma}{\partial x} \right\} \quad (2.2)$$

Where, all elements of vector ∂σ/∂t and matrix G(x,*t*) are, by assumption, continuous and bounded and rank(G(x,*t*))=*m* >0.

In the experimental setup, a linear drive system equipped with DC servo from PI Physik Instrumente model M-415.DG which uses closed loop DC motor with shaft mounted position encoders and backlash-free gear heads. Two such linear drive systems were used for the x-y stage motion as shown in the fig. 1:

TABLE I
PARAMETERS OF THE LINEAR DRIVE

Motor Voltage	12 V	J (Moment of Inertia)	2.1992e-7 Nm/A
Motor Power	3 W	Kt (Torque constant)	0.0133908 Nm/A
Encoder resolution	2000 counts/rev	B (Friction constant)	0.00012002 Nm
Gear head ratio	29.641975309:1	Travel Range	150 mm
Lead-screw Pitch	0.5 mm/r	Design resolution	0.0085 μm

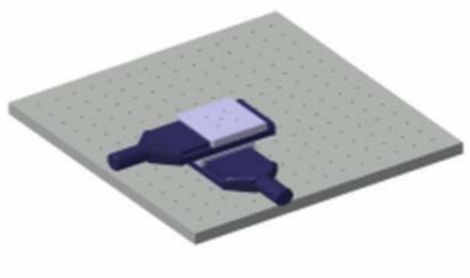

Fig. 1 shows the X-Y stages of linear drive attached with one another for trajectory following.

The Basic block diagram is shown in Fig. 2.

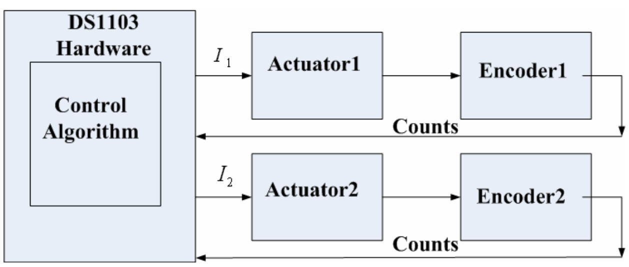

Fig. 2 Block diagram of the system with two linear axes driven by dSpace1103.

According to the motor specification the design resolution 0.0085 μm which means theoretical minimum movement that can be made based on the selection of the mechanical drive components (drive screw, gear ratio, angular motor position etc). Design resolution is much higher than the practical position resolution or minimum incremental motion. Thus using a robust discrete time SMC controller and frictional compensation a high accurate system along with the desired dynamics is achieved in the vicinity of the design resolution without any stick slip motion which arises from the disturbance due to friction.

III. DESIGN OF THE DISCRETE SMC

For system (2.1) asymptotic stability of the solution $\sigma(x,t)=0$ can be assured if one can find a control input such that the Lyapunov stability criteria are satisfied. Natural selection of the candidate Lyapunov function is a quadratic form $v=\sigma^T\sigma/2$. The design procedure can be started from the requirement that the time derivative of the Lyapunov function should have the following form $dv/dt=-\sigma^T\mathbf{D}\sigma,\ \ \mathbf{D}>0$. Then $v(x,t)>0$, $dv(x,t)/dt<0$ $\forall x\notin S$, $v(x^*,t)=0$, $dv(x^*,t)/dt=0$, $x^*\in S$, $\forall t>0$ and solution $\sigma(x,t)=0$ is asymptotically stable on the trajectories of the system (2.1). A control input that satisfies given requirements can be calculated from

$\sigma T d\sigma/dt=-\sigma T D\sigma \Rightarrow \sigma T(d\sigma/dt+D\sigma)=0 \Rightarrow d\sigma/dt+D\sigma=0,\ \forall\ \sigma(x,t)\neq 0$.

By solving this equation for an unknown control u, it is easy to obtain

$$u(x,t)=-(GB)^{-1}(\frac{\partial\sigma(x,t)}{\partial t}+Gf)-(GB)^{-1}D\sigma \tag{3.1}$$

To have a unique solution for the control input, matrix GB must be regular. Selected control input guaranties that the motion of system (2.1) satisfies the dynamical constrain $d\sigma/dt+D\sigma=0$

That means all distances from the manifold S exponentially tend to zero and the system motion will remain in ε-vicinity of the manifold S after reaching it. Strictly speaking, control (3.1), being continuous, does not provide the sliding mode motion on manifold (2.2) because that manifold could be reached only in infinite time. Taking into account that the equivalent control, required to keep the motion in the manifold S if initial state belongs to this manifold, equation (5.1) can be rewritten as

$$u(x,t)=u_{eq}(x,t)-(GB)^{-1}D\sigma \tag{3.2}$$

In (3.2) the resulting control action is continuous (the equivalent control is continuous and function σ(x,t) is continuous by assumption) and $u(x,t)=u_{eq}(x,t)$ for $\sigma(x,t)=0$.

From $d\sigma/dt=GB\ (u-u_{eq})$ equivalent control can be substituted into (5.2) to obtain

$$u(x,t)\equiv u(x,t)-(GB)^{-1}\left(D\sigma(x,t)+\frac{d\sigma(x,t)}{dt}\right) \tag{3.3}$$

Equation (3.3) can be easily modified for the use in the discrete time systems with *no computational delay* to obtain

$$u(kT^{+}) = \mathbf{sat}(u(kT^{-}) - (GB)^{-1}(D\sigma(kT^{-}) + \frac{d\sigma(kT^{-})}{dt})) \quad (3.4)$$

$$u_{\min} \le \mathbf{sat}(\bullet) \le u_{\max}$$

Since the control input is assumed to be limited by a saturation function in (3.4).Thus the control (3.4) is used in order to achieve a robust control system.

IV. Frictional Compensation

Till now we have been discussing that the disturbance is resulting from smooth disturbance but for friction the smoothness of the disturbance d_k is lost in the vicinity of zero velocity hence the tracking error may be large due to $|d_k - d_{k-1}|$ may be large in this region e.g. zero-velocity crossing, d_k and d_{k-1} have opposite signs. Clearly, the amplitude of the tracking error depends on the difference in friction amplitude between two sampling values. It is a well established fact that friction can not be described by a pure discontinuity at zero velocity instead friction is a continuous function of time with complicated and fast dynamics around the zero velocity. Therefore the difference in friction values between two successive sampling instances is made smaller by selecting a smaller period, which reduces the friction estimated error and the following tracking error during zero-velocity crossing is also reduced. The obvious solution is to decrease the sampling period as the friction becomes more "discontinuous". However, as sampling time T $\rightarrow$ 0, the control signal $\pm u(sat)$ but due to the saturation the u_k will actually chatter with amplitude of $u_{\min} \le \mathbf{sat}(\bullet) \le u_{\max}$, which is unacceptable as it may excite the high frequency modes in the system.

In order to cope up with the above written problem, the dynamic friction model proposed by Canudas de Wit[8] is used as a friction acting on the system. The advantage for selecting this model is because inherits most of the frictional phenomena that gives rise to control problem such as stick-slip behavior as explained above. The model is expressed as

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$

$$\frac{dz}{dt} = v - |v|\, z / g(v)$$

$$where, \sigma_0 g(v) = F_c + (F_s - F_c)e^{-(v/v_s)^2} \quad (4.1)$$

F_s Denotes the static friction level, F_c is the level of Coulomb force and v_s is the stribeck velocity.

σ_0, σ_1 and σ_2 represents stiffness, damping co-efficient and viscous friction co-efficient respectively.

In the same way the observer compensate the frictional effects as enforce by the above model and remove the stick-slip motion in the steady state stage. The observer model can be expressed as:

$$\hat{F} = \sigma_0 \hat{z} + \sigma_1 \frac{d\hat{z}}{dt} + \sigma_2 v$$

$$\frac{d\hat{z}}{dt} = v - |v|\, \hat{z} / g(v) - ke \quad (4.2)$$

$$where, \sigma_0 g(v) = F_c + (F_s - F_c)e^{-(v/v_s)^2}$$

e Denotes the position error and the term ke in the observer is a correction term from the position error. The closed loop system block diagram in below figure 3

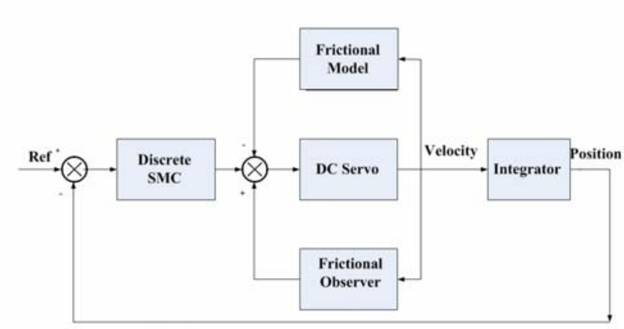

Figure 3 shows the block diagram of the closed loop system with frictional model and observer.

V. Simulation and Experimental Description

The above shown figure 3 is simulated with the following parameters shown in Table II and the same parameters are used for the experiments to control the linear drives.

TABLE II
PARAMETERS OF THE FRICTIONAL MODEL AND OBSERVER

σ_0	100000 N/m	v_s	0.001 m/s
σ_1	316.2277 Ns/m	T(Sample Time)	0.0001 s
σ_2	0.4 Ns/m	k	0.0001
F_c	1 N	F_s	1.5 N

In the experiments setup the control (3.3) was applied along with the friction model (4.1) and the frictional observer (4.2) to the system as shown in figure 1.

The two linear drive systems equipped with DC servo Model M-415.DG from PI (Physik Instrumente) as shown in figure 1 are driven in current controller mode through dSpace DS1103 PPC controller board equipped with Real-Time Interface (RTI) Implementation software. It facilitates to capture and view the time varying data through its GUI based software Control Desk. The programming can be done through

MATLAB/SIMULINK or C-code with built in Libraries and functions. The DS1103 provides a great selection of interfaces, including 50 bit-I/O channels as well as 36 A/D channels and 8 D/A channels. For additional I/O tasks, a digital signal processor can be used.

As an input to the linear drive a smooth step is applied as its more realistic solution in practical application.

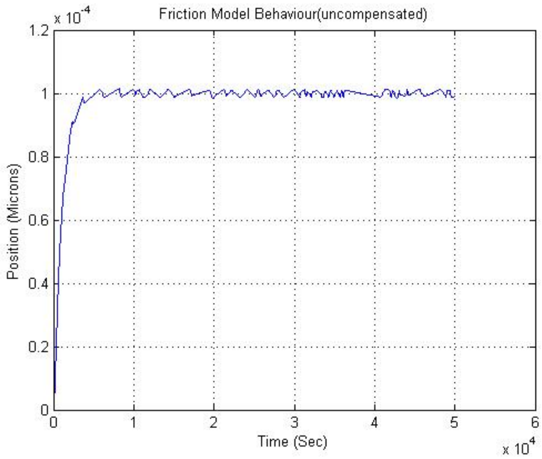

Figure 4 shows the simulation result of unit step response of the closed loop system by applying frictional model and without the frictional observer.

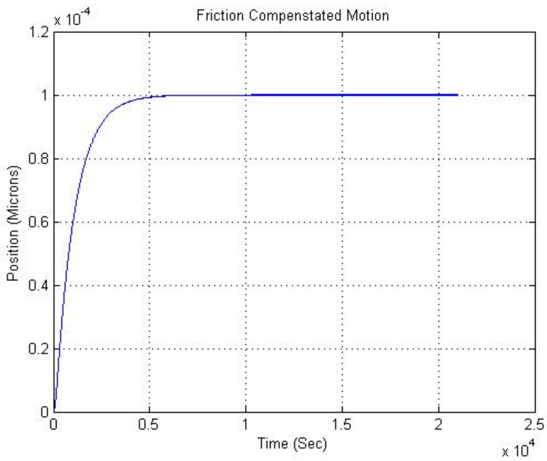

Figure 5 shows the simulation result of unit step response of the closed loop system by applying the frictional model and compensating with the frictional observer.

The figure 4 represents the simulated response of the system by the implementation of frictional model (without frictional observer) for a step input of 10 micrometer. It clearly indicates stick-slip behavior in the steady state due to the various frictional components by the frictional model. The figure 5 shows the simulated effect of the frictional observer that the stick-slip behavior is totally removed and the steady state error is very close to zero.

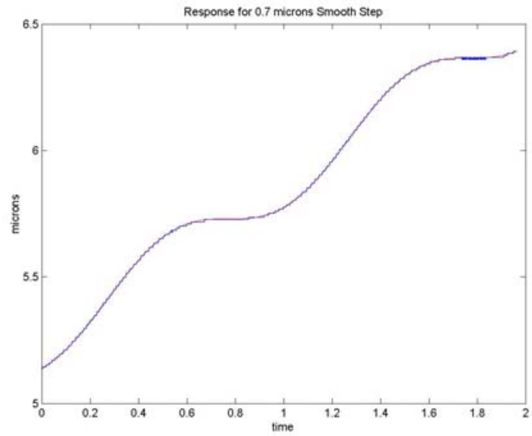

Figure 6 shows experimental results of response smooth step of less than 1 micron for a single axis.

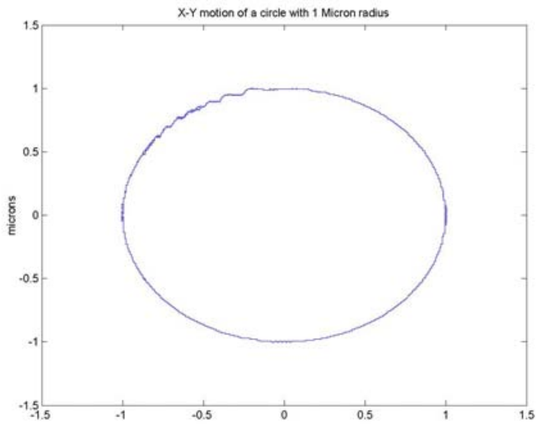

Figure 7 shows the experimental results for circular motion of two axes of radius 1 micron.

Figure 6 represents the experimental results of smooth response of less than 1 micron with the implementation of the Frictional observer and it is seen that it tracks the reference position and the dynamics very accurately. Figure 7 represents the experimental result of circular path of radius of 1 micron followed by the X and Y axis of the linear drive and the result indicates that it follows the trajectory very accurately.

VI. CONCLUSION

In this paper the design of a Discrete-Time Sliding Mode Controller based on Lyapunov stability criteria is presented. It is also shown that the implementation of Frictional Model and Frictional observer compensation along with the discrete time sliding-mode controller allows to reach a very high accuracy of the closed loop system and the stick-slip behavior is eliminated at the steady state. It has been demonstrated that chattering in the control system is avoided. In spite of the disturbance, the controller maintains the system on the sliding surface with a high accuracy and stick -slip behavior in the steady state is not observed. Also system does not exhibits any undershoot or overshoot which is a mandatory requirement in micro-manipulation applications.

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