Robust Motion Control - SMC Point of View

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***Abstract*— In this paper the robust motion control systems in the sliding mode framework are discussed. Due to the fact that a motion control system with n d.o.f may be mathematically formulated in a unique way as a system composed of n second order systems, design of such a system may be formulated in a unique way as a requirement that the generalized coordinates must satisfy certain algebraic constraint. Such a formulation leads naturally to sliding mode framework to be applied. In this approach constraint manifolds are selected to coincide with desired constraints on the generalized coordinates. It has been shown that the CMC can be interpreted as a realization of the acceleration controller thus possessing all robust properties of the acceleration controller framework. The possibility to treat both unconstrained motion (the motion without contact with environment) and constrained motion in the same way is shown.**

I. INTRODUCTION

The most salient feature of the Sliding Mode Control (SMC) is the possibility to constrain the system motion in selected manifold in the state space. Such motion results in a system performance that includes disturbance rejection and insensitivity to parameter variations [1]. The development of VSS has gone through oscillations with both very enthusiastic claims and the skepticism regarding the achieved results. In some cases researchers contributed to the confusion, especially in the case of so-called chattering phenomena, through incomplete analysis and design fixes, what provoked many analytical methods to be proposed to deal with chattering phenomenon [2,3,4,5]

The complexity and nonlinear dynamics of motion control systems along with high-performance operation require complex, often nonlinear control system design, to fully exploit system capabilities. Basic goal for motion control systems is to achieve smooth stable motion in the presence of unstructured environment with which plant in motion (robotic manipulator) can be in contact. The joint torques are treated as the control inputs. The torques or forces are on the other hand the outputs of actuators - often electrical machines, with their own complex nonlinear dynamics. In this paper we will demonstrate a generalized framework for sliding mode approach in motion control systems. It will be shown that sliding mode represents a method for acceleration control implementation and thus offers all advantages of the acceleration control framework like robustness to the parameters changes and the external disturbances. In order to illustrate the state observer design in the sliding mode framework we will discuss the sliding mode application for PZT actuator nolinearity and external force estimation.

II. SLIDING MODES IN MOTION CONTROL SYSTEMS

So-called sliding mode motion is represented by the state trajectories being forced to stay in the selected state space manifold (sliding mode manifold) with finite time convergence to sliding mode manifold. In the continuous time the control that may guaranty above properties happens to be discontinuous with high frequency switching, while in the discrete-time the control that guaranty the motion in sliding mode manifold may be continuous [6-11].

A. Mathematical Formulation of the Control Problem

For 'fully actuated' mechanical system (number of actuators equal to the number of the primary masses) mathematical model may be found from Euler-Lagrange formulation in the following form

$$\begin{aligned}&\dot{\mathbf{q}}_1 = \mathbf{q}_2\\ &\mathbf{M}(\mathbf{q}_1)\dot{\mathbf{q}}_2 + \mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) = \boldsymbol{\tau} - \boldsymbol{\tau}_{ext}\\ &\boldsymbol{\tau}_{ext} = \mathbf{J}_{aco}^T \mathbf{F}_{ext}\end{aligned} \tag{1}$$

where $\mathbf{q}_1 \in \Re^n$ stands for vector of generalized positions, $\dot{\mathbf{q}}_1 = \mathbf{q}_2$ stands for vector of generalized velocities $\mathbf{M}(\mathbf{q}_1) \in \Re^{nxn}$ is generalized positive definite inertia matrix with bounded parameters hence $M^- \le \|\mathbf{M}(\mathbf{q}_1)\| \le M^+$, $\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) \in \Re^{nx1}$ represent vector of coupling forces including gravity and friction and is bounded by $\|\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t)\| \le N^+$, $\boldsymbol{\tau} \in \Re^{nx1}$ with $\|\boldsymbol{\tau}\| \le \tau_0$ is vector of generalized input forces and $\boldsymbol{\tau}_{ext} \in \Re^{nx1}$ with $\|\tau_{ext}\| \le g_0$ is vector of generalized torques due to the presence of the external forces $\mathbf{F}_{ext} \in \Re^{nx1}$. $M^-, M^+, N^+, \tau_0, g_0$ are known scalars. In system (1) vectors $\boldsymbol{\tau}_{ext} \in \Re^{nx1}$ and $\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) \in \Re^{nx1}$ satisfy matching conditions [9], $\mathbf{J}_{aco}^T$ is Jacobian matrix.

Model (1) can be rewritten as n second order systems

$$\dot{q}_{i1} = q_{i2}, \quad m_{ii}\dot{q}_{i2} + n_i = \tau_i - \tau_{exti} - \sum_{j=1, j\neq i}^{n} m_{ij}\dot{q}_{j2}, \quad i = 1,...,n$$

where the elements of inertia matrix are bounded $m_{ij}^- \le |m_{ij}(t)| \le m_{ij}^+$, the elements of vector $\mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t)$ are bounded $n_i^- \le |n_i(t)| \le n_i^+$, the elements of the external torque vector are bounded by $g_{0i}^- \le |\tau_{exti}(t)| \le g_{0i}^+$ and the input generalized torques are bounded $\tau_{0i}^- \le |\tau_i(t)| \le \tau_{0i}^+$. The structure is simple each of the systems can be viewed as a second order system with generalized torque as input and $d_i = -n_i - \sum_{j=1, j\neq i}^{n} m_{ij}\dot{q}_{j2} - \Delta m_{ii}\ddot{q}_i$, as disturbance which satisfies matching conditions and external forces being result of the system contact with environment.

In the following sections we will discuss the motion control design assuming the generalized torque as a control input i.e. the motion dynamics (1).

B. Control Problem Formulation

Vector of generalized positions and generalized velocities defines configuration of a mechanical system [12-13]. The motion control tasks such as position tracking, force control and impedance control can be presented as requirements to maintain certain specified configuration of the mechanical system expressed in the form vector equations $\boldsymbol{\sigma}(\mathbf{q}_1, \mathbf{q}_2, t) = \mathbf{0}_{nx1}$ as depicted in (2)

$$\begin{aligned} &\boldsymbol{\sigma}(\mathbf{q}_1, \mathbf{q}_2, t) = \mathbf{C}\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{f}(t) = \mathbf{0}, \\ &\boldsymbol{\sigma}(\mathbf{q}_1, \mathbf{q}_2, t) \in \Re^{nx1}, \mathbf{C} > 0, \boldsymbol{\sigma} = [\sigma_1, \sigma_2, ..., \sigma_n]^T \end{aligned} \qquad (2)$$

where $\mathbf{f}(t) \in \Re^{nx1}$, is known bounded continuous function with bounded elements and their first order time derivatives. In the sliding mode framework requirement (2) is equivalent to enforcing sliding mode in manifold

$$S_q = \{\mathbf{q}_1, \mathbf{q}_2 : \mathbf{C}\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{f}(t) = \mathbf{0}\}, \qquad (3)$$

Matrix **C** is generally selected as diagonal so elements of vector function $\boldsymbol{\sigma} = \mathbf{0}_{nx1}$ are $\sigma_i = c_i q_{i1} + q_{i2} - f_i(t)$, *i=1,2,..,n*. If sliding mode is enforced in manifold (3) then equivalent control, being solution of $\dot{\boldsymbol{\sigma}}\big|_{\tau=\tau_{eq}} = \mathbf{C}\dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_2 - \dot{\mathbf{f}}(t)\big|_{\tau=\tau_{eq}} = \mathbf{0}$, is determined as $\boldsymbol{\tau}_{eq} = \mathbf{M}(\dot{\mathbf{f}}(t) - \mathbf{C}\mathbf{q}_2) + \mathbf{N} + \tau_{ext}$ and the equations of motion for system (1) with sliding mode in manifold (3) can be derived in the form

$$\mathbf{M}\dot{\mathbf{q}}_2 = \mathbf{M}(\dot{\mathbf{f}}(t) - \mathbf{C}\dot{\mathbf{q}}_1) = \mathbf{M}\dot{\mathbf{q}}_2^{des}, \dot{\mathbf{q}}_2^{des} = (\dot{\mathbf{f}}(t) - \mathbf{C}\dot{\mathbf{q}}_1) \qquad (4)$$

Which, after some manipulations may be written as

$$\begin{aligned} &\dot{\mathbf{q}}_1 = \mathbf{q}_2 \\ &\dot{\mathbf{q}}_2 = \dot{\mathbf{q}}_2^{des}, \ \dot{\mathbf{q}}_2^{des} = \frac{d}{dt}(\mathbf{f}(t) - \mathbf{C}\mathbf{q}_1) \end{aligned} \qquad (5)$$

From (5) it follows that in the sliding mode on manifold (3) the motion of the general fully actuated motion control system (1) depends only on the selection of the manifold (matrix **C**) and the desired configuration of the system $\mathbf{f}(t) \in \Re^{nx1}$. The motion in sliding mode is robust against the plant parameter changes and the generalized disturbance vector expressed as $\mathbf{d} = \mathbf{N}(\mathbf{q}_1, \mathbf{q}_2, t) + \boldsymbol{\tau}_{ext}$. The structure of the sliding mode control system is depicted in Fig. 1.

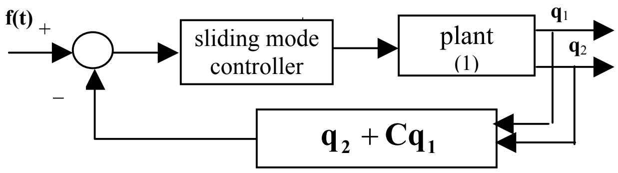

Fig. 1 The structure of the SMC motion control system

The sliding mode control (4) realizes the acceleration controller [12] with desired acceleration being $\frac{d}{dt}(\mathbf{f}(t) - \mathbf{C}\mathbf{q}_1)$. This indicates that results attained in acceleration control framework may be realized in sliding mode framework as well.

In the sliding mode framework desired configuration of the system can be realized while all advantages of the acceleration control framework can be retained. The fact that the closed loop system motion (5) is the same as the equation of sliding mode manifold is due to the specifics of the system's dynamics (1).

1) Position tracking in robotic systems - By selecting reference trajectory as $\mathbf{q}_1^{ref}(t)$, the position tracking problem can be specified as a requirement that sliding mode be enforced in manifold (6)

$$S_{q_1} = \{\mathbf{q}_1, \mathbf{q}_2 : \boldsymbol{\sigma} = \mathbf{f}(t) - (\mathbf{C}\mathbf{q}_1 + \mathbf{q}_2)\}, \ \mathbf{f}(t) = (\mathbf{C}\mathbf{q}_1^{ref} + \mathbf{q}_2^{ref}) \qquad (6)$$

Equation (6) can be interpreted as a requirement that the control error and its derivative are linearly dependent or $\mathbf{C}(\mathbf{q}_1^{ref} - \mathbf{q}_1) + (\mathbf{q}_2^{ref} - \mathbf{q}_2) = \boldsymbol{\sigma}_1(\mathbf{q}^{ref} - \mathbf{q}) = \boldsymbol{\sigma}_1(\mathbf{q}^{ref}) - \boldsymbol{\sigma}_1(\mathbf{q}) = \mathbf{0}$.

2) Force control in robotic systems The spring-damper model of the reaction force (7) is widely accepted in motion control systems

$$\mathbf{F} = \mathbf{K}_{ep}(\mathbf{q}_{e1} - \mathbf{q}_1) + \mathbf{K}_{ed}(\dot{\mathbf{q}}_{e1} - \mathbf{q}_2) \qquad (7)$$

where $\mathbf{q}_{e1}$ is the generalized coordinate of the contact point of the robot tip with environment, $\mathbf{K} > \mathbf{0}$ is spring coefficient matrix. The force control problem in which the contact force **F** should track its reference $\mathbf{F}^{ref}(t)$ can be specified as a requirement that sliding mode is enforced in the manifold (8)

$$S_f = \{\mathbf{q}_1, \mathbf{q}_2, t : \mathbf{F}^{ref} - (\mathbf{K}_{ep}(\mathbf{q}_{e1} - \mathbf{q}_1) + \mathbf{K}_{ed}(\mathbf{q}_{e2} - \mathbf{q}_2)) = \mathbf{0}\}$$
$$S_f = \{\mathbf{q}_1, \mathbf{q}_2 : \boldsymbol{\sigma}(\mathbf{q}_1, \mathbf{q}_2, t) = -\mathbf{f}(t) + (\mathbf{K}_{ep}\mathbf{q}_1 + \mathbf{K}_{ed}\mathbf{q}_2)\}, \quad (8)$$
$$\mathbf{f}(t) = -(\mathbf{F}^{ref} - \mathbf{K}_{ep}\mathbf{q}_{e1} - \mathbf{K}_{ed}\mathbf{q}_{e2})$$

3) The mechanical impedance control The mechanical impedance control can be formulated in a similar way.

$$S_f = \{\mathbf{q}_1, \mathbf{q}_2, t : \mathbf{F} + (\mathbf{K_e q_1} + \mathbf{B_e \dot{q}_1} + \mathbf{M_e \ddot{q}_1}) = \mathbf{0}\} \quad (9)$$

where $\mathbf{K_e}, \mathbf{B_e}, \mathbf{M_e}$ are desired stiffness, damping and mass matrices. In many cases mass is selected very small or zero thus reducing the impedance control to the force control problem.

The trajectory tracking (6), the force control (8) and mechanical impedance control (9) are mathematically defined is the same way thus all can be solved in the framework of sliding mode control systems by enforcing sliding mode in selected manifolds. Moreover the combination of the tasks seems natural so the questions of the unified control strategy in the sliding mode framework seem natural to look in. Before discussing the unified approach to the above control problems let us first look at the control vector selection.

C. Selection of control input

The design of control inputs for system (1), (12) with sliding mode in manifold (3) may follow a few different approaches. All the approaches have common requirement to derive the control input such that the stability of the solution $\boldsymbol{\sigma}(\mathbf{q}_1, \mathbf{q}_2, t) = \mathbf{0}_{nx1}$ is assured. This could be guarantied if Lyapunov stability conditions are satisfied. The Lyapunov function may be selected as $v = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma}$ with first time derivative being $\dot{v} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}}$. To ensure stability the Lyapunov function derivative may be required to have specific form so to ensure that $\dot{v} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T\boldsymbol{\Psi}(\boldsymbol{\sigma}) < 0$. Then one can derive $\boldsymbol{\sigma}^T(\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma})) = 0$ and consequently control should be selected to satisfy $\dot{\boldsymbol{\sigma}} + \boldsymbol{\Psi}(\boldsymbol{\sigma}) = 0 \Rightarrow \boldsymbol{\tau} = \boldsymbol{\tau}_{eq} - \mathbf{M}\boldsymbol{\Psi}(\boldsymbol{\sigma})$. Obviously control will depend on the selection of the function $\boldsymbol{\Psi}(\boldsymbol{\sigma})$. In the literature this function is most often selected as $\boldsymbol{\Psi}(\boldsymbol{\sigma}) = sign(\boldsymbol{\sigma})$ and the resulting control is discontinuous as $\boldsymbol{\tau} = -\tau_0 sign(\boldsymbol{\sigma}) \Rightarrow \tau_i = -\tau_{0i} sign(\sigma_i),\ i = 1,...,n$ [15]. Each component of the control input undergoes discontinuity by taking values from the set $\{-\tau_{0i}, \tau_{0i}\}$. Direct implementation of this algorithm may result in chattering An approach to reduce the effect of the discontinuous control is to implement it as $\tau_i = \hat{\tau}_{eq}^{est} - \tau_{0i} sign(\sigma_i)$ where $\hat{\tau}_{eq}^{est}$ is estimated torque that may be calculated either from the system's model using available measurement and estimated parameters or from disturbance estimation. Asymptotic observers may be used as a bypass for high frequency component [10,11] and to eliminate chattering. With all of these modification the sliding mode control with discontinuity is hardly and acceptable solution due to the chattering problem and actuators dynamic.

1) Discrete-time sliding mode control For system (1) sliding mode in sliding mode manifold (3) can be enforced selecting control in the form

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{eq} - \mathbf{M}\mathbf{D}\boldsymbol{\sigma}, \mathbf{D} > 0 \quad (10)$$

where $\boldsymbol{\tau}_{eq}$ is solution of the algebraic equation $\dot{\boldsymbol{\sigma}}(\boldsymbol{\tau} = \boldsymbol{\tau}_{eq}) = 0$. With control (10) the Lyapunov function $v = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma}$ has time derivative $\dot{v}_l = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} = -\boldsymbol{\sigma}^T\mathbf{D}\boldsymbol{\sigma},\ \mathbf{D} > 0$ and condition $\dot{\boldsymbol{\sigma}} + \mathbf{D}\boldsymbol{\sigma} = \mathbf{0}$ ensures the asymptotic stability of the solution $\boldsymbol{\sigma} = \mathbf{0}$. Control (10) is continuous but it still requires information on systems parameters and external disturbances for calculation of $\boldsymbol{\tau}_{eq}$.

By applying sample and hold process with sampling interval *T*, the discrete-time realization of control (10) can be approximated as depicted in (11)

$$\boldsymbol{\tau}_{k+1} = sat(\boldsymbol{\tau}_k + \mathbf{M}T^{-1}((1 - \mathbf{D}T)\boldsymbol{\sigma}_k - \boldsymbol{\sigma}_{k-1})), \quad (11)$$

Implementation of algorithm (11) requires information on distance from sliding mode manifold and inertia matrix. Application of control (11) to system (1), (3) leads to the

$$\boldsymbol{\sigma}^T(k)\boldsymbol{\sigma}(k-1) = \boldsymbol{\sigma}^T(k)(\mathbf{I} - T\mathbf{D})\boldsymbol{\sigma}(k) \quad (12)$$

If **D** is diagonal matrix for each of the components in (12), one can write $\sigma_i(k)\sigma_i(k-1) = \sigma_i^2(k)(1 - Td_{ii})$ and d_{ii} may be selected so that $0 < (1 - Td_{ii}) < 1$, which ensures existence of quazi-sliding mode motion.

2) *The robustness in SMC motion control systems* From (5) follows that the ideal sliding mode motion of system (1) in manifold (3) does not depend on the systems disturbance nor on the systems parameters and is fully defined by the design parameter **C**. To realize such a motion the equivalent control should be determined and that poses a problem since all parameters of the system and the disturbance are necessary to determine equivalent control. Application of control (10) for the trajectory tracking leads to the sliding mode equations of motion being defined as $\dot{\boldsymbol{\sigma}} + \mathbf{D}\boldsymbol{\sigma} = \mathbf{0}$ or

$$(\ddot{\mathbf{q}}^{ref} - \ddot{\mathbf{q}}) + (\mathbf{C} + \mathbf{D})(\dot{\mathbf{q}}^{ref} - \dot{\mathbf{q}}) + \mathbf{CD}(\mathbf{q}^{ref} - \mathbf{q}) = \mathbf{0} \quad (13)$$

what represents second order system transient determined by the design parameters **C** and **D**. This show that the robustness of the system is preserved but the transient is now of the second order.

Using approximated control (11) one can find that

$$\dot{\boldsymbol{\sigma}}(k)+\mathbf{D}\boldsymbol{\sigma}(k)=\left(\boldsymbol{\tau}_{dis}(k)-\boldsymbol{\tau}_{dis}(k-1)\right)T \qquad (14)$$

what shows that the motion of the system will have an error of the *o(T)* order, which can be made small enough by selecting sampling interval. The above robustness properties are valid for trajectory tracking, force control and impedance control problems.

III. THE GENERALIZATION OF SMC IN MOTION CONTROL

The sliding mode manifolds for position tracking, force control and impedance control are defined in a similar ways but still they have considerable differences so that some sort of the hybrid control scheme should be used in order to achieve smooth motion of the system in the presence of obstacles. The similarities in the sliding mode manifolds definition and the sliding mode equation (13), which represents the closed loop behavior of the system, are justifying the attempt to search for the definition of the sliding mode manifold in such a way that all three control problems may be solved by the same controller and switching between different modes: the trajectory tracking, the force control and impedance control may be combined so to allow control to be reaction to the present of the contact with environment and the controller parameters the same for all control modes. From trajectory tracking manifold it is apparent that the system configuration could be modified by changing function *f(t)* in (6). On the other hand the impedance control may be easily interpreted from (13). The closed loop behavior of the system (13) can be interpreted in a way that the system under control acts as a mass-spring-damper system with equivalent mass being $\mathbf{M}_e=\mathbf{I}$, Equivalent spring $\mathbf{K}_p=\mathbf{CD}$ and equivalent damper $\mathbf{K}_d=\mathbf{C+D}$ as depicted in Fig. 2

In the contact with environment the controlled system will react as virtual impedance creating a force $\mathbf{F}_e$ due to the contact. In the above framework it is possible to unify all of the above tasks in one and specify the control task as a requirement that control should be selected to enforce the configuration of the system $\dot{\mathbf{q}}+\mathbf{C}\mathbf{q}=\boldsymbol{\sigma}_q$ to track its reference

$$S=\left\{\mathbf{q},\dot{\mathbf{q}}:\left(\boldsymbol{\beta}\boldsymbol{\sigma}_q^{ref}-\boldsymbol{\sigma}_q\right)-\boldsymbol{\alpha}\mathbf{F}_e=\boldsymbol{\sigma}=\mathbf{0}\right\}$$
$$\dot{\mathbf{q}}+\mathbf{C}\mathbf{q}=\boldsymbol{\sigma}_q; \qquad (15)$$
$$\dot{\mathbf{q}}^{ref}+\mathbf{C}\mathbf{q}^{ref}=\boldsymbol{\sigma}_q^{ref}$$

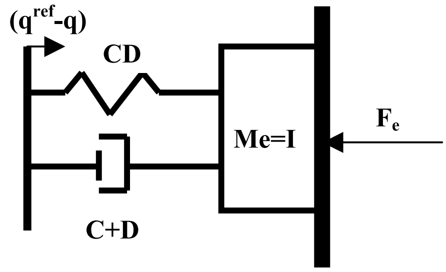

Fig. 2 The interpretation of the SMC closed loop system in contact wiyth environment

Matrix $\boldsymbol{\beta}$ is diagonal with diagonal elements in the directions in which contact force should be controlled should be changed so to maintain the required contact force – the simples and yet efficient solution is to keep these coefficients just proportional to force error. In the directions where forces should not be maintained at the required level but either trajectory tracking or compliance control coefficients of matrix $\boldsymbol{\beta}$ should be kept at 1. Matrix $\boldsymbol{\alpha}$ defines the compliance parameters. This matrix is diagonal with elements being different from zero in the directions in which compliance is to be maintained and being zero in the directions in which either contact force or trajectory tracking should be maintained.

The control is the same as in (11) and the closed loop transient is described by $\dot{\boldsymbol{\sigma}}+\mathbf{D}\boldsymbol{\sigma}=\mathbf{0}$. In the directions in which compliance is maintained the system acts as a damper spring system and the dynamics is defined by $\Delta\dot{\mathbf{q}}+\mathbf{C}\Delta\mathbf{q}-\boldsymbol{\alpha}\mathbf{F}_e=\mathbf{0}$. The structure of such a controller is depicted in Fig. 3

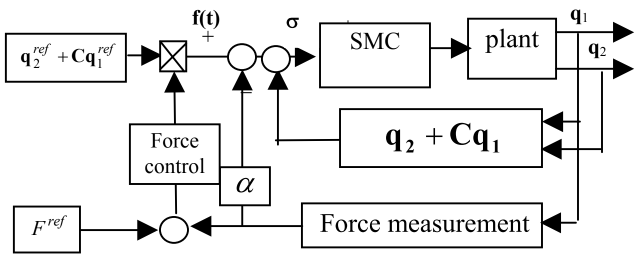

Fig. 3. The structure of the system

IV. EXAMPLES

A. SMC in PZT position tracking control

As an illustrative example of sliding mode control a position control for PZT actuator is discussed in this section. The electromechanical lumped model exhibits large hysterezis as a main nonlinearity of the system [16]. Since the accuracy of the system depends on the disturbance as depicted in (14) the SMC is combined with disturbance observer to see the possible influence on the achievable accuracy. The observer is designed as a position tracking system and it exhibits the second order dynamics.

1) *Experimental results in position tracking* - The experimental setup consists of a Piezomechanik's

PSt150/5/60 stack actuator ($x_{max} = 60\,\mu m$, $F_{max} = 800$ N, $v_{max} = 150$ Volt) connected to SVR150/3 low-voltage, low-power amplifier. The actuator has built-in strain-gages for position measurement. Force measurement is accomplished by a load cell placed against the actuator. The entire setup is connected to DS1103 module hosted in a PC with dSPACE software Control Desk v.2.0. In all experiments the parameters of the sliding mode controllers (both controllers in disturbance observer and in outer loop) are kept as C=800, D=2500, and they are not tuned for best performance. The experimental results are depicted in Fig. 4 and 5 respectively. For the feedback signal filtering a simple filter is used but it was not accounted for in the controller design so all three systems have unmodelled dynamics in the loop.

The behavior of the sliding mode controller (Fig. 4) and sliding mode controller with disturbance observer is similar but error for the system with disturbance observer (Fig. 5) is about 50% smaller. In addition the output noise is much smaller in the system with disturbance observer. This demonstrates the possibility of combining the disturbance observer with SMC control methods.

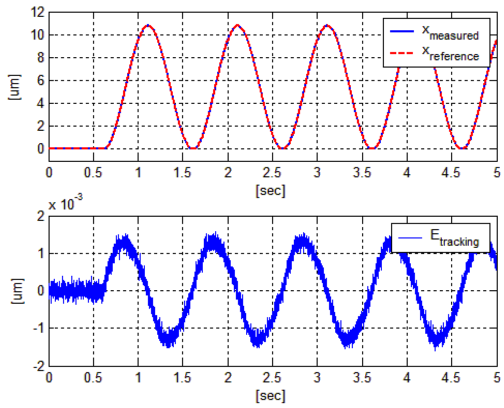

Fig. 4 Tracking of the sinusoidal reference for a PZT actuator. Sliding mode control without disturbance feedback

2) *PZF force control* - For the force observer the modeling of hysteresis is needed so a nonlinear model of the plant constructed as a nominal plant with hysteresis added is used for force estimation. The structure of the force control system is depicted in Fig. 6, while the experimental set-up is as in Fig. 7. The experiments depicted in Fig.7 shows smooth transients in the force control. The transients without any overshoot is most likely due to the fact that actuator is in contact with environment before the force reference is changed so no impact force is present in the system.

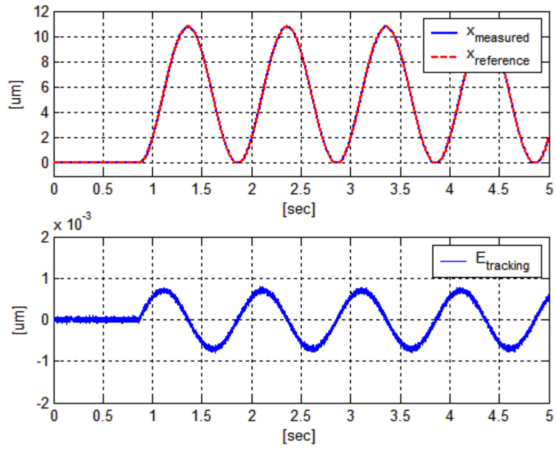

Fig. 10 Tracking of the sinusoidal reference for a PZT actuator. Sliding mode control without disturbance feedback

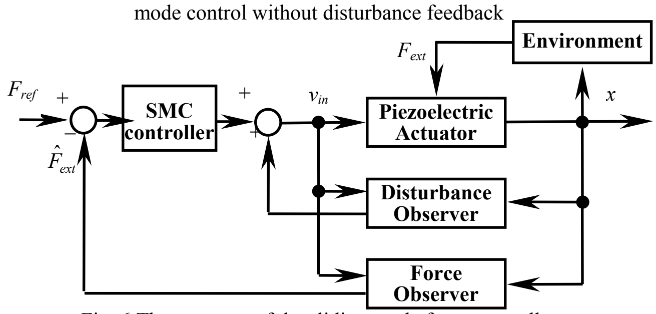

Fig. 6 The structure of the sliding mode force controller

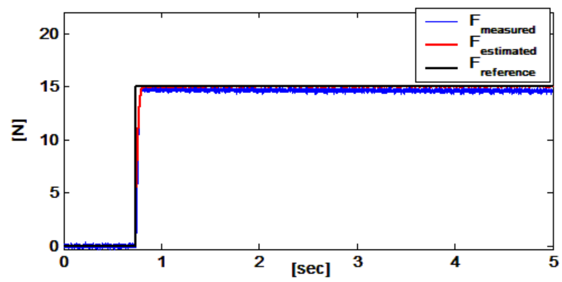

Fig. 7. 15N Step force response with estimated force as feedback signal

B. *Generalized SMC control – simulation results*

The behavior of the generalized system with a sliding mode manifold (15) and control (11) is simulated for single axis system described by:

$$\dot{x} = v \qquad m\dot{v} = K_t i - F_{dis} \tag{16}$$

$$m = m_0\left(1 + a\sin\left(f_m t\right)\right), a < \qquad K_t = K_{t0}\left(1 + b\sin\left(f_{kt} t\right)\right), b < 1 \qquad F_{dis} = F_0 + F_1\sin\left(f_F t\right) + F_2\sin\left(3 f_F t\right) + F_{friction} \tag{17}$$

A moving obstacle $x_o = x_{o0}\left(1 + c\sin\left(f_o t\right)\right)$ had been introduced to the system. All frequencies are selected different within a range of (0.5-5) Hz. Parameters of the controller have been selected C=200 and D=250 and are kept constant for all experiments. The contact force with environment has been simulated as

$F_e = K_{ep}(x - x_o) + K_{ed}(\dot{x} - \dot{x}_o)$ with K_{ep}=1000 and K_{ed}=50. The reference trajectory is selected to be sinusoidal with frequency 1 Hz.

In order to illustrate the validity of the approach the system motion had been simulated in different tasks. The trajectory tracking, the force control and the impedance control on the manifold (6), (16) and (9) with controllers as defined by (11) (note that different definition of σ is used in each case) were analyzed and work of each structure separately had been confirmed. The simulation of the generalized structure as defined by (15) is depicted in Fig. 8 and 9.

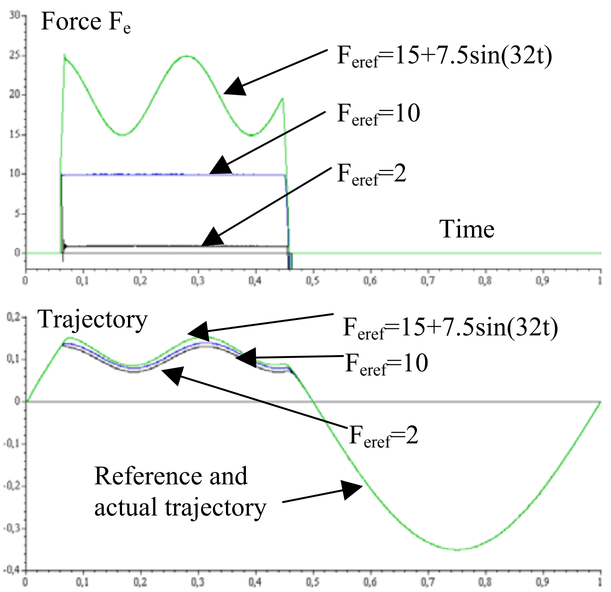

Fig. 8 The trajectory tracking combined with force control in contact with an unknown obstacle. Obstacle defined as xa=0.1(1+0.3sin(25t)), force in contact with environment is calculated as Fe=1000(x-xa)+50(v-va)

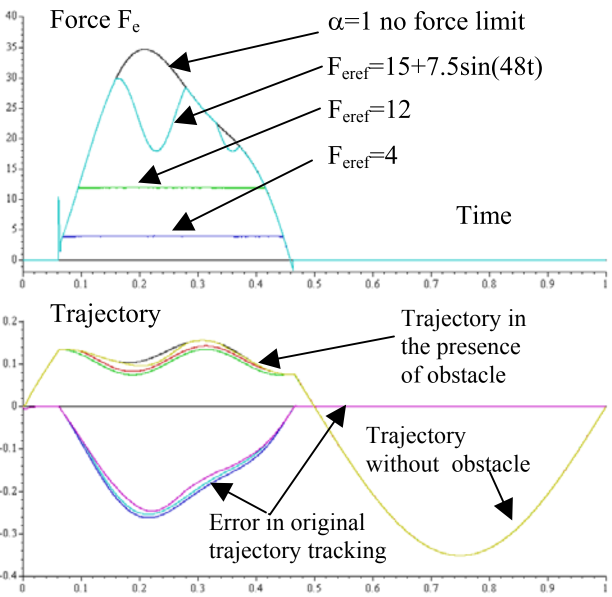

Fig 9. The trajectory tracking and force control in contact with unknown obstacle and α=1 in (15). Obstacle position and force as in Fig. 8.

V. CONCLUSIONS

In this paper the sliding mode control framework in motion control systems is discussed. The emphasis is put on the robustness of the system motion in sliding mode and the general solution for tracking, force control and the compliance in motion control systems. It has been shown that in the ideal sliding mode the motion is defined by design parameters only and it does not depend on the plant parameters or the external disturbances. The equivalency with acceleration control is shown. The realization of the sliding mode control in discrete-time framework is discussed. As an example a PZT actuator position tracking and force controls tasks are discussed. It has been shown that both tasks can be effectively solved in the sliding mode framework. The general framework for the combination of the trajectory tracking with compliance and the force control in fully actuated systems is proposed and it has been shown that such framework allows natural behavior of the system in the presence of the contact with environment if such a contact results in the reaction force in some of the manipulator's joints.

VI. REFERENCES

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