The Impact of Large Changes in Asset Prices on Intra-Market Correlations in the Domestic and International Markets

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Abstract

We consider the impact of "large" changes in asset prices on intra-market correlations in domestic and international markets. Assuming normally distributed asset returns, we show that the absolute magnitude of the correlation, conditional on a change greater than or equal to a given absolute size of one of the variables, is monotonically increasing in the magnitude of that absolute change. Empirical tests using domestic and international-market data support this theoretical result. These results have significant implications for portfolio management, hedging interest rate risk, tests of asset pricing models, Roll's concern with asset pricing models' explanatory power, and implementation of Value-at-Risk.

JEL Codes: C10, G12
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1. Introduction

Effective portfolio diversification is one of the fundamental goals of portfolio management, requiring a judicious combination of evaluating expected returns, volatilities and correlations. This paper focuses on that third component, correlations, and the sensitivity of that statistic to "large" changes in asset values.

Specifically, in his 1988 Presidential Address to the American Finance Association, "R^2," Roll (1988) was concerned with the predictive ability of asset pricing models such as the CAPM and the APT:

> Even with hindsight, the ability to explain stock price changes is modest…The average adjusted R^2 s are only about 35% with monthly data and 20% with daily data…The paucity of explanatory power represents a significant challenge to our science.

This paper contributes to the literature by calculating the relevant market-model R^2 s on days of large and small price moves. It is shown that, for domestic and international stock-market returns, on the large-move days when (presumably) one is more concerned with a model's predictive ability, the R^2 s are significantly larger than on days when small changes occur.

Hedging an interest rate book, an options book or an equity book is of critical importance to many and varied financial institutions, including commercial and investment banks, which are exposed to one or more of these risks. In this context, the institutional hedger typically has larger exposure on days of "large" moves in asset prices, up or down, than he or she does on days when asset price movements are "small." Consequently, the hedger has a greater interest in conditional correlation — conditional on the *absolute* magnitude of asset price movements, irrespective of sign — than he or she does in the unconditional correlation.[1]

Thus, consider an institution hedging an interest rate book. The work of Litterman and Scheinkman (1988) considers three factors, "*level*, *steepness* and *curvature* [emphasis in original]." They note that these "explain — at a minimum — 96% of the variability of excess returns of any zero" coupon bond. Consistent with these empirical results, the two-factor interest rate model of Brennan and Schwartz (1982) explicitly allows for a non-perfect correlation between the long- and short-term rates of interest. Similarly, Longstaff and Schwartz (1992) and Fong and Vasicek (1991) consider two-factor models in which interest rates and interest rate volatility are both stochastically changing through time; Heath, Jarrow and Morton (1990, 1992) provide a rigorous theoretical framework for one- and multi-factor no-arbitrage interest rate models. These papers permit a non-perfect correlation between the long- and short-term rates of interest.

Given the importance of level changes in interest rates, and the widely-prevalent use of duration-type measures for hedging in practice, the need for multi-factor hedging remains an open question. Indeed, one of the critical issues of this paper is whether, for practical purposes, such multi-factor hedging is required. This paper demonstrates that a one-factor model of level (but not parallel) shifts adequately describes interest rate movements for large changes. Thus, this paper addresses the following important question: Are time periods of large changes in asset prices different from those of smaller changes?

In an investigation into the return comovements of U. S. and Japanese stock using ADRs, Karolyi and Stulz (1996) focus on "the key role in international finance" played by stock return cross-country covariances. *Inter alia,* they observe that:

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[1] Balyeat and Muthuswamy compare hedging portfolios formed based on unconditional and conditional correlations and find that the unconditional analysis leads to strategies that are not optimal, frequently by more than 10%.

> Comovements [between U. S. and Japanese stocks] are high when contemporaneous absolute returns of national indices are high…Our evidence shows that covariances are high when markets move a lot. This suggests that international diversification does not provide as much diversification against large shocks to national indices as one might have thought.

This paper presents the theoretical result and subsequent empirical verification that demonstrates that correlations conditional on the magnitude of asset price changes differ from unconditional correlations.

The issue of conditional correlations has been analyzed by several papers, especially in the context of the sensitivity of the correlation to downside vs. upside market moves. Ang and Chen (2000) and Longin and Solnik (2001) conclude that correlations tighten in bear but not bull markets. This issue also arises *inter alia* in the discussion of contagion, as in Rigobon (2000) and Forbes and Rigobon (2001), where the authors attempt to separate contagion effects from other statistical (e.g., heteroscedasticity and omitted variables) effects. In contrast, our paper considers both domestic and international equity as well as interest rate markets in quantifying the presence of conditional correlations distinct from their unconditional analogues, and in considering their Value-at-Risk and R^2 implications. We also provide results that quantify Value-at-Risk of a portfolio composed of instruments that depend linearly on two normally distributed random factors, as well as the expected loss beyond Value-at-Risk, with respect to the correlation of the random factors.[2]

The paper is organized as follows. First, we present the framework and the theoretical

[2] Boyer, Gibson and Loretan (1999) and Loretan and English (2000), independently of our work, have also investigated the issue of conditional correlations. While our paper is similar to these papers in their argument regarding the impact conditioning has on correlations, there are differences: Boyer, Gibson and Loretan discuss the impact of conditioning on the correlation and present empirical results for exchange rates; Loretan and English take the theoretical results from that predecessor paper, and present empirical results for the conditional correlation for equity and bond returns, and for exchange rates. In contrast, our paper formalizes the impact of conditioning on R^2 and Value-at-Risk calculations; our empirical section examines the impact of conditioning on factor analysis for bond returns, hedging effectiveness of bond portfolios, Roll's R^2 estimates for domestic equities and implications for the CAPM, and on the R^2 for international indices vs. the S&P 500.

results relating the conditional correlation to the absolute value of the change in asset prices. We also discuss the relation between Value-at-Risk, expected loss beyond Value-at-Risk, and correlation, and outline the implications for various aspects of financial economics. Second, we report empirical results pertaining to the bond market. In this portion, we also demonstrate the interest rate hedging implications of our analysis. Third, we apply the research methodology to stock market data, including a consideration of Roll's R^2 analysis. Fourth, we conduct a similar R^2 analysis for international market returns. Finally, we summarize and discuss the implications of our results.

2. Conditional correlations

The fundamental theoretical result is due to Stambaugh:[3]

Theorem (Stambaugh):

1. Assume x and y are distributed according to a bivariate normal distribution, and $E(y) = 0$.[4]
2. Define the unconditional covariance matrix

$$\text{Cov}\left(\begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x & y \end{bmatrix}\right) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}, \qquad (1)$$

where the parameters σ_x, σ_{xy}, σ_y are assumed constant.

3. Define

[3] Rob Stambaugh presented this result in his discussion of the Karolyi and Stulz (1996) paper at the May 1995 NBER Conference on Financial Risk Assessment and Management.
[4] This assumption can be relaxed without loss of generality. In the case of non-zero means, the following additional translation needs to be performed: $y' \equiv y - E(y)$.

$$\delta \equiv \frac{a^2 - \sigma_x^2}{\sigma_x^2}. \tag{2}$$

Then the correlation between x and y, conditional on $x^2 = a^2$, is

$$\text{Corr}\left(x, y \middle| x^2 = a^2\right) = \left(\frac{1+\delta}{1+\delta\rho^2}\right)^{1/2} \rho . \tag{3}$$

It is easy to verify that this conditional correlation increases (decreases) with a given positive (negative) unconditional correlation. Although the result engenders the intuition, empirical comparisons are not possible due to the lack of multiple observations of a given value for *x*. However, if the conditioning is changed to values (x, y) such that $|x| \geq a$, then it is possible to carry out empirical tests.

We will denote the correlation of x and y conditional on the absolute value of x being greater than a cutoff value, *a*, as:

$$\text{Corr}_a(x, y) \equiv \text{Corr}(x, y : |x| \geq a). \tag{4}$$

Theorem 1:

Under the conditions of the previous theorem, the conditional correlation $\text{Corr}_a(x, y)$ is given by

$$\text{Corr}_a(x, y) = \left(\frac{N(-\xi) + \xi n(\xi)}{N(-\xi) + \rho^2 \xi n(\xi)}\right)^{1/2} \rho , \tag{5}$$

where n and N are, respectively, the standard normal density and the standard normal cumulative density functions, and $\xi = a/\sigma_x$.

Proof: See Appendix A.

It can also be verified that this conditional correlation increases (decreases) for a given cut-off *a*, with positive (negative) unconditional correlation. Two special cases are shown in Figure 1. A way to empirically test for the increase in correlation, independent of estimating variances and covariances, is by splitting any sample into subsets, ordered by the size of moves in one of the variables. We use this technique in our empirical study in Sections 3, 4 and 5. Figure 2 shows the anticipated increase in conditional correlation over unconditional correlation when the sample is split in two equal subsamples.

Insert Figure 1 about here

Theorem 1 makes possible pairwise comparisons and inferences based on these comparisons. These results can be extended for more general, non-normal distributions. In particular, given a set of N observations in two variables, $\{Y_i, X_i\}$, ordered by the absolute distance of one of the variables from its mean, the R^2 of regression analysis decreases as more observations are included.

Theorem 2:

Let the set $S(n)$ contain the $n \leq N$ observations with the greatest values of $\left|Y_i - \overline{Y}\right|$. For $X_{i\in S(N)}$, denote by

$$\begin{Bmatrix} \hat{Y}_i \\ \hat{Y}_{in} \end{Bmatrix} \text{the predicted value of } Y \text{ using} \begin{Bmatrix} S(N) \\ S(n) \end{Bmatrix}. \qquad (6)$$

Further, let $\overline{Y} \equiv \sum Y_i/N$ and $\overline{Y}_n \equiv \sum_{i\in S(n)} Y_i/n$. Assume that the average value of Y is the same whether one uses $S(N)$ or $S(n)$., i.e.,

$$\overline{Y} \cong \overline{Y}_n \equiv \overline{Y}_{i\in S(n)} \ . \qquad (7)$$

Further, assume that the intercept and slope coefficients are identical for the unconditional regression [using $S(N)$] and the conditional regression [using $S(n)$],

$$\widehat{Y}_i \cong \widehat{Y}_{in} \equiv \widehat{Y}_{i \in S(n)}. \tag{8}$$

Then the R^2 of the regression is decreasing in n.

Proof: See Appendix B.

Insert Figure 2 about here

Another area where large moves will have a disproportionate impact is in the risk management of large portfolios of financial instruments. A common tool for estimating the risk of such portfolios is the calculation of the Value-at-Risk of the portfolio, i.e., the maximum loss of the portfolio value over a fixed time horizon, at a certain confidence level.

Consider a portfolio with $*a* invested in asset 1 and $*b* invested in asset 2. The returns on assets 1 and 2 are described by two random variables, *X* and *Y*, respectively, which are distributed according to the bivariate normal distribution with means equal to zero,[5] standard deviations σ_x, σ_y and correlation ρ.

Theorem 3:

Under the assumptions outlined above, the Value-at-Risk at a given confidence level, α, for any two correlations $\rho_0 < \rho_1$, satisfies

$$\mathrm{VaR}_\alpha(\rho_1) = \mathrm{VaR}_\alpha(\rho_0)\left(\frac{a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho_1 ab\sigma_x\sigma_y}{a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho_0 ab\sigma_x\sigma_y}\right)^{1/2}. \tag{9}$$

The expected loss, beyond Value-at-Risk, is given by

[5] As stated above, this assumption is not crucial and can be relaxed. In the case of non-zero means, an additional translation needs to be performed.

$$E\left(\text{Daily Loss}\middle|\text{Daily Loss} \geq \text{VaR}_\alpha\right)$$
$$=\left(\frac{a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho ab\sigma_x\sigma_y}{2\pi}\right)^{1/2} \exp\left\{-\frac{\text{VaR}_\alpha^2}{2\left(a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho ab\sigma_x\sigma_y\right)}\right\}. \tag{10}$$

Proof: See Appendix C.

Theorem 3 quantifies the magnitude of the expected loss beyond the VaR limits and can also be used as a means of calibrating VaR calculations, based on behavior on the tails, rather than on the overall distribution.

3. Empirical results — Interest rates

3.1 Treasuries

Let r be the on-the-run three-month Treasury Bill rate. We consider the distribution of the variables Δr and $\Delta r/r$, where we are implicitly considering Normal and Lognormal distributions for interest rate changes.[6] We stratify the sample into two sub-samples using the following criterion: those with the 50% smallest, and those with the 50% largest values of $|\Delta r|$ and $|\Delta r/r|$; these identify the SMALL-change and LARGE-change dates.

Let l be a longer-maturity interest rate, where $l \in$ {6 mo., 1 yr., 2 yrs., 3 yrs., 5 yrs., 7 yrs., 10 yrs., 20 yrs.} For the period January 1, 1990 to December 31, 2005, we compute the correlations, $\text{Corr}(\Delta r, \Delta l)$ and $\text{Corr}(\Delta r/r, \Delta l/l)$, for the overall sample, as well as the SMALL-change and the LARGE-change samples.[7]

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[6] Clearly, the one- and two-factor continuous-time interest rate processes that model *dr* or *dr/r* imply that empirical correlations should be computed with respect to changes, and not levels, of interest rates.

[7] An important question that arises here is the requirement of a multi-factor arbitrage-free model of interest rate movements that allows for a constant unconditional correlation between interest rate movements. Candidates for such models include Brennan and Schwartz (1982) or Duffie and Kan (1995).

The results of this analysis are reported in Table 1 Panels A and B.[8] From the table, we observe that the conditional correlations $\text{Corr}(\Delta r, \Delta l)$ and $\text{Corr}(\Delta r/r, \Delta l/l)$ are greater for the LARGE-change relative to the SMALL-change days.

Next, we explicitly test for the differences in the correlations. Since the distribution of Pearson correlation is skewed, we use Fisher's z-transformation that converts the distribution of sampled correlations to a normal distribution:

$$\text{Corr}' = .5\left[\ln(1+\text{Corr}) - \ln(1-\text{Corr})\right], \tag{11}$$

$$\sigma_{\text{Corr}'} = 1/\sqrt{N-3}\text{, and} \tag{12}$$

$$z = \frac{\text{Corr}_1' - \text{Corr}_2'}{\sqrt{1/(N_1-3)+1/(N_2-3)}}, \tag{13}$$

where N is the sample size. In Panels C and D, we report our findings of the tests of correlation differences. We observe that, in general, the differences in correlations between the OVERALL-SMALL and LARGE-SMALL sub-samples are positive and significant, whereas the LARGE-OVERALL comparison is less conclusive. This suggests that unconditional correlations are mostly driven by the co-movements in the LARGE-change days and that the SMALL-change days have little or no effect. This result is further reinforced in tests related to hedging effectiveness presented in Tables 6 and 7.

Insert Table 1 Panel A about here

Insert Table 1 Panel B about here

Insert Table 1 Panel C about here

[8] The distributions for changes and percentage changes in interest rates are skewed (either positively or negatively) and exhibit significant kurtosis, and therefore deviate significantly from normality. We use the method proposed by Tabachnick and Fidell (1996) to estimate standard errors for skewness and kurtosis. By symmetrically stratifying the sample, the methodology we employ does not address the asymmetry in conditional correlations that is discussed in Ang and Chen (2002), and thus underestimates the correlation differences reported in the table in the downside.

Insert Table 1 Panel D about here

In order to further empirically validate the result in Theorem 1, we investigate how the conditional correlation between the change in the three-month rate and the ten-year rate is affected, when the cut-off point, ξ, is varied from 0.5 to 2.5 standard deviations. The resulting conditional correlations for daily and weekly samples are plotted in Figure 3. Figure 3 is directly comparable to the theoretical result portrayed in Figure 1. We observe that the general patterns in Figures 1 and 3 are similar. From this analysis, we conclude that the conditional correlations in the bond markets are, indeed, monotonically increasing in the scaled cut-off ξ.

Insert Figure 3 about here

3.2 Principal component analysis — Treasuries

Another way to investigate the correlation structure of interest rates is to conduct principal component analysis. We can indirectly infer conditional correlations by comparing the principal components (or factors) in the LARGE-change relative to the SMALL-change days.

In Table 2 we present our findings of the principal-component analysis conducted over the entire sample and then the two sub-samples. For brevity, we report the results for three factors. Note that the first factor explains the majority of variation in interest rates in the overall sample as well as the LARGE-change and SMALL-change days. Furthermore, for the LARGE-change days, the variance explained by the first factor is greater than that of the other two samples, indicating the importance of a single factor for such days. We observe that the large-move days provide evidence supportive of the hypothesis that large-move days reflect more of a level shift in interest rates.

Insert Table 2 about here

3.3. Regression analysis — Treasuries

Let l_T be the treasury rate for maturity *T*, where $T \in \{10\,\text{yrs.}, 20\,\text{yrs.}\}$. Similar to our analysis in the section above, we divide the sample into two sub-samples by the magnitude of $|\Delta r|$ or $|\Delta r/r|$, respectively, and estimate the following regressions:

$$\Delta r = \alpha + \beta \Delta l_T \text{, and} \tag{14}$$

$$\Delta r / r = \alpha + \beta (\Delta l_T / l_T) \tag{15}$$

across the overall, SMALL- and LARGE-change sub-samples from 1990 to 2005.

Table 3 reports the result of these regressions. Note that the R^2 for the LARGE-change sub-sample is much higher than that of the SMALL-change sub-sample. Once again, these results confirm the relationship between magnitude of changes and the conditional correlation. Table 3 demonstrates that although level changes are of increasing importance as the magnitude of the change increases, they are not *parallel* changes as implied in duration-type hedging. Rather, a more accurate hedge of these level changes would take into account the estimated coefficient of the long-term bond, which is significantly less than one.

Insert Table 3 about here

3.4. Interest rates — The "exceptional" days

Consider the days for which:

$$|\Delta r| > 2.5 \text{ Std.}(\Delta r) \text{ and} \tag{16}$$

$$\Delta r \Delta l_{20} < 0 . \tag{17}$$

That is, days in which the change in the short-term rate was of significant magnitude, and yet the 20-yr. rate of interest l_{20} moved in the opposite direction.

Table 4 demonstrates that the set of days in which the comovements of short- and long-term rates were not aligned, despite the large magnitude of the short-term rate's change, constitutes days in which significant political or monetary events took place.[9] These days may well warrant further analysis and suggest a pure-hedging rationale for an exotic option: a "range floater" that compensates the holder whenever the *slope* of the term structure changes "significantly." While pricing such a derivative may be non-trivial, it would be a useful tool in hedging an interest-rate book in conjunction with a duration-style hedge of the book's value.

Insert Table 4 about here

3.5. One- and two-factor hedging of interest rate risk

Consider the problem of hedging the change in the yield of the on-the-run par-bond of maturity i, dy_i, for $i \in \{0.25, 0.5, 1, 2, 3, 5, 7, 10, 20\}$. For the one-factor case, assume that the hedge vehicle is the two-year security (i.e., dy_2). For the two-factor case, consider the use of dy_2 and the change in the yield-curve slope $d(y_{10} - y_2) \equiv dy_{10} - dy_2$. As before, we consider the hedging of interest-rate changes for all observations, as well as the upper 50% of the large-move days only. In the one-factor case, we regress dy_i on dy_2, subtract the predicted value $d\hat{y}_i$, and compute the standard deviation of the residual, $\hat{\sigma}(dy_i - d\hat{y}_i)$. In the two-factor case, we regress dy_i on dy_2 and $dy_{10} - dy_2$, and similarly compute the standard deviation of the residual. We employ the two-factor model for the overall sample, and the

[9] Perhaps surprisingly, there were no "exceptional" days in the 2001 - 2005 period, where exceptional is defined as days in which the change in the short-term rate was of significant magnitude; i.e., greater than two and a half standard deviations, and yet the change in the thirty-year rate moved in the opposite direction from the change in the short-term rate.

one-factor model for the large-magnitude sample.[10]

Table 5 reports the results of this analysis for the period January 1, 1990 to December 31, 2005. The previous analyses have highlighted the importance of a single-factor in the LARGE-change days. Consistent with these analyses, one-factor hedging using the two-year maturity in the LARGE-magnitude sample achieved a greater reduction in the volatility of the shorter-maturity issues (0.25, 0.5, one and three-year) than was accomplished by the *two-*factor model for the overall sample.

Insert Table 5 about here

Letting P_i denote the price of a maturity i bond, we consider a price-based hedge of a five-year par bond using either a two-year bond or a two-year/ten-year portfolio. The analysis below may be considered more purely *ex-ante* than its Table 5 predecessor, as it does not make use of any time-series properties.

In a large-move day, contrast the performance of the one-factor hedge portfolio,

$$d\Pi_5 = dP_5 - \Delta\, dP_2 = 0 \quad \Rightarrow \quad \Delta = \frac{dP_5}{dP_2} = \frac{dP_5}{dy_5}\frac{dP_2}{dy_2} \equiv \frac{\text{Dollar Duration}_5}{\text{Dollar Duration}_2} \tag{18}$$

with the analogous performance of a two-factor hedge portfolio,

$$d\Pi_5 = dP_5 - \Delta_2\, dP_2 - \Delta_{10}\, dP_{10} = \frac{dP_5}{dy_5}dy_5 - \Delta_2 \frac{dP_2}{dy_2}dy_2 - \Delta_{10}\frac{dP_{10}}{dy_{10}}dy_{10} = 0\text{, and} \tag{19}$$

$$dy_5 = w_2\, dy_2 + w_{10}\, dy_{10} \tag{20}$$

for weights w_2, w_{10}, which sum to unity. We now have two equations in two unknowns:

[10]By definition, this process eliminates all residual volatility for the two-year security in the one-factor case, and the two-year and 10-year securities in the two-factor case.

$$\begin{cases} \frac{dP_5}{dy_5} w_2 - \Delta_2 \frac{dP_2}{dy_2} & = & 0 \\ \\ \frac{dP_5}{dy_5} w_{10} - \Delta_{10} \frac{dP_{10}}{dy_{10}} & = & 0 \end{cases} \Rightarrow \begin{cases} \Delta_2 & = & w_2 \frac{\text{Dollar Duration}_5}{\text{Dollar Duration}_2} \\ \\ \Delta_{10} & = & w_{10} \frac{\text{Dollar Duration}_5}{\text{Dollar Duration}_{10}} \end{cases} \tag{21}$$

where in the implementation we set

$$w_2 = \frac{\text{Dollar Duration}_{10} - \text{Dollar Duration}_5}{\text{Dollar Duration}_{10} - \text{Dollar Duration}_2}, \text{ and} \tag{22}$$

$$w_{10} = 1 - w_2. \tag{23}$$

Defining hedging effectiveness by the proportional reduction in residual variance, $\sigma_{d\Pi_i} / \sigma_{dP_i} - 1$, Table 6 reports on the result of the analysis.[11]

As in Table 5, for intermediate maturities of three and five years the hedging effectiveness of the one-factor hedge on the large-move days is close to, or actually outperforms, the two-factor model in the overall sample.

Insert Table 6 about here

4. Stock market

4.1. Stock market returns

We consider the returns R_{it} on the 100 largest market-cap companies traded on the NYSE/ AMEX with complete data for the period January 1, 1990 to December 31, 2005, and define $\bar{R}_t$ to be the 100 stocks' value-weighted average return. We sort the data by the magnitude of

[11] We note that, for the two-year maturity in the one-factor model on the large-move days; and the two- and ten-year maturities for the two-factor model in the overall sample, the hedging effectiveness is by definition -100% since we are taking an offsetting position in one or both of the underlying factors.

$|\bar{R}_t|$ and divide the sample days into two subsets as LARGE-change days and SMALL-change days. On each trading day, we observe the sign of $(\bar{R}_t)$ and the signs of (R_{it}). The percentage number, and the percentage market value of the stocks whose returns have the same sign as $\bar{R}_t$, are recorded for the overall, SMALL- and LARGE-change days in the sample period in Table 7. Figure 4 displays the value weighted proportions of stocks that moved in the same direction as the index as the absolute magnitude of the change in R_t is varied from 0.5 to 2.5 standard deviations. Similar to Figure 3, these results again confirm the increasing comovement of stocks as the magnitude of "market" move increases.

Insert Table 7 about here

Insert Figure 4 about here

4.2. Roll's R^2

As noted in the introduction, Roll (1988) was concerned with the profession's empirical inability to explain cross-sectional stock price movements. This may, however, be attributable to market microstructure effects, which are pervasive on the SMALL-change days. In a partial attempt to address this issue, we extract daily returns for 24 companies (the ones used by Roll) from the CRSP tapes. For each stock, daily returns are regressed on the value weighted index of all the companies listed in the New York and American Stock Exchanges on the entire data set and on the large- and small-move days of this index. The resultant R^2 s are tabulated in Table 8.

Table 8 demonstrates a significant increase in R^2 between the SMALL- and LARGE-change days:[12] The SMALL-change days have R^2 on the order of 1% – 5%, whereas the

[12] We also report the skewness and the kurtosis of the overall stock return distributions in Table 8. Note that out of the 24 companies, three exhibit significant negative skewness and all have significantly leptokurtic return distributions.

LARGE-change days evidence R^2 in the vicinity of 10% to 45%. Clearly, an attempt to address Roll's concerns requires a follow-up to this type of analysis; nevertheless, there may be hope here that CAPM/APT models can explain a larger variation of movement on days "when it matters," than was thought possible.

Insert Table 8 about here

5. International markets

We now consider an analysis analogous to the one in Section 3 for the case of returns in international — both emerging[13] and developed — markets. The results are presented in Table 9. The sample period extends from January 1, 1990 to December 31, 2005.

The use of weekly data is preferred because daily data suffer from frictions that are especially pronounced in an emerging markets setting. Furthermore, daily data would induce noise due to time-zone differences in the countries analyzed. Roll (1988) points out statistically significant lead and lag effects of time-zone differences in the comovement of international equities.

The analysis proceeds in the following stages: First, weekly returns are generated from the weekly values of the dollar denominated country indices. Second, the dataset is split in two subsamples, based on the behavior of the U.S. index: days of LARGE-changes and days of SMALL-changes. Third, for each country, we regress the returns of the country index vs. the U.S. index for (a) the whole dataset, (b) the dataset with the LARGE-changes, (c) the dataset with the SMALL-changes.

[13] Since our sample includes emerging markets, there is a concern that there may be complications with the operation of newly established stock markets such that the data are not reliable or price movements are not indicative of value-relevant information. The newest stock market (Turkey) in our data set is four years old in 1990 at the beginning of our sample period.

The results are reported in Table 9.[14] The regressions for the small days have an R^2 of the order of 0% - 7%, while the regressions for the large days have R^2 between 0% - 40% for the developing markets, and 10% - 40% for the developed markets.

Insert Table 9 about here

6. Summary and implications

We have derived and tested the implications of the correlation between two assets, conditional on the value of one of the assets making a large move. We have shown that the absolute magnitude of the conditional correlation is increasing in the size of the move. The paper confirms the empirical regularity of these results in the bond, stock and international markets. Specifically, bond markets are relatively more influenced by level changes on large-, rather than small-, move days. On the equity side, markets are more likely to move uniformly on high-, vs. low-, volatility days. Also, the market model has greater explanatory power on days when the market has made a substantial move.

The economic interpretation for the equity-market results appears straightforward: on the large-move days, economic factors — political, macroeconomic, and industry-wide, etc. — dominate, whereas on the small-move days, additional factors, including those of market microstructure, prevail.

The implicit assumption of this paper is that only large moves "matter." There are several areas of financial economics where conditional-correlation analysis may have an impact:

1. <u>Hedging interest rate risk:</u> A one-factor model, reflecting level (though not

[14] In Table 9, we also report the skewness and the kurtosis of the overall country return distributions. Note that almost all country returns exhibit significant skewness and kurtosis.

parallel) changes, may be adequate for hedging an interest rate book composed primarily of non-callable bonds. The intuition behind this result is that the effective number of important factors decreases as one conditions on larger moves.

2. Tests of asset pricing models, including CAPM, CCAPM, and such models as the three-factor Fama/French model. Recall that the models of Ingersoll (1975) and Kraus and Litzenberger (1976) emphasized the role of additional moments, such as skewness, in asset pricing models. An explicit modeling of aversion to large asset price changes, incorporating kurtosis, may yield new insights in this area. At the least, it would be interesting to see the efficacy of asset pricing models when tested separately over large- and small-change sample days.

3. Roll's R^2 analysis: The explanatory power of CAPM/APT may usefully be conducted separately over days in which asset price changes are large and small. On large-move days, conditional correlations tighten, and portfolio managers' ability to diversify decreases.

4. Value-at-Risk: Risk measurement in portfolios containing derivatives has become an important research and public-policy issue. The concept of Value-at-Risk depends on historical volatility and (unconditional) correlations for the calculation of confidence intervals for losses. Yet, if one is concerned with regulatory implications for systemic losses, the distinction among conditional and unconditional variances, covariances and correlations is an important one. Conditional correlations indicate more accurately the impact of a large negative shock. The use of unconditional moments may provide an unjustified, downward-

biased degree of confidence in the estimate of losses incurred on large-move days. Value-at-Risk might be more accurately computed by focusing on large move days, rather than the overall sample. We leave these extensions for future work.

A. Proof of Theorem 1

Let (X,Y) be distributed as

$$N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma_x^2 & \rho\sigma_x\sigma_y\\ \rho\sigma_x\sigma_y & \sigma_y^2\end{pmatrix}\right). \tag{A1}$$

Then we have the following result[15]:

$$\text{Var}(X:|X|<a)=\int_{-a}^{a}x^2 f(x:|x|<a)dx=\left(1-\frac{2\xi n(\xi)}{2N(\xi)-1}\right)\sigma_x^2, \tag{A2}$$

where n and N are standard normal density and standard normal cumulative density functions respectively, and

$$\begin{aligned}\text{Var}_a(X)&=\text{Var}(X:|X|\geq a)\\ \text{Var}_a(Y)&=\text{Var}(Y:|X|\geq a)\\ \text{Cov}_a(X,Y)&=\text{Cov}\left(X,Y:|X|\geq a\right)\end{aligned}\quad. \tag{A3}$$

We start with the conditional variance of x:

$$\begin{aligned}\text{Var}_a(X)&=\int_{-\infty}^{-a}x^2 f(x:|x|\geq a)dx+\int_{a}^{\infty}x^2 f(x:|x|\geq a)dx\\ &=2\int_{a}^{\infty}x^2 f(x:|x|\geq a)dx\end{aligned} \tag{A4}$$

where

$$f(x:|x|\geq a)=\begin{cases}\left[2N(-\xi)\right]^{-1}\dfrac{1}{\sqrt{2\pi}\sigma_x}e^{-x^2/2\sigma_x^2} & |x|\geq a\\ 0 & |x|<a\end{cases}. \tag{A5}$$

The unconditional variance is given by:

[15] The interested reader is referred to Johnson, Kotz and Balakrishnan (1994).

$$\begin{aligned}\mathrm{Var}(X) &= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma_x} e^{-x^2/2\sigma_x^2} dx \\ &= \int_{-\infty}^{-a} x^2 \frac{1}{\sqrt{2\pi}\sigma_x} e^{-x^2/2\sigma_x^2} dx + \int_{a}^{\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma_x} e^{-x^2/2\sigma_x^2} dx \\ &+ \int_{-a}^{a} x^2 \frac{1}{\sqrt{2\pi}\sigma_x} e^{-x^2/2\sigma_x^2} dx \\ &= 2N(-\xi)\mathrm{Var}_a(X) + \left[2N(\xi)-1\right]\mathrm{Var}(X :| X |< a).\end{aligned} \tag{A6}$$

Now, we have

$$\begin{aligned}\mathrm{Var}_a(X) &= \frac{\sigma_x^2 - \left(2N(\xi)-1\right)\mathrm{Var}(X :| X |< a)}{2N(-\xi)} \\ &= \frac{\sigma_x^2 - \left(2N(\xi)-1\right)\left(1-\frac{2\xi n(\xi)}{2N(\xi)-1}\right)\sigma_x^2}{2N(-\xi)} \\ &= \frac{\sigma_x^2\left(1-(2N(\xi)-1)\left(1-\frac{2\xi n(\xi)}{2N(\xi)-1}\right)\right)}{2N(-\xi)} \\ &= \frac{\sigma_x^2\left(1-\left(2N(\xi)-1\right)+2\xi n(\xi)\right)}{2N(-\xi)} \\ &= \frac{2\sigma_x^2(1+\xi n(\xi)-N(\xi))}{2N(-\xi)} \\ &= \sigma_x^2 \frac{1+\xi n(\xi)-N(\xi)}{N(-\xi)} \\ &= \sigma_x^2 \frac{\xi n(\xi)+N(-\xi)}{N(-\xi)} \\ &= \sigma_x^2\left(\frac{\xi n(\xi)}{N(-\xi)}+1\right).\end{aligned} \tag{A7}$$

For $\mathrm{Cov}_a(X,Y)$ and $\mathrm{Var}_a(Y)$, define:

$$\begin{aligned} f_a(x,y) &= f(x,y :| x |\geq a), \text{ and} \\ f_a(x) &= f(x :| x |\geq a).\end{aligned} \tag{A8}$$

We have

$$f_a(y \mid x) = \frac{f_a(x,y)}{f_a(x)}, \tag{A9}$$

or

$$f_a(x,y) = f_a(x) f(y \mid x), \tag{A10}$$

and it follows that

$$f_a(y \mid x) = \frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_y} e^{-\frac{\left(y-\rho\frac{\sigma_y}{\sigma_x}x\right)^2}{2(1-\rho^2)\sigma_y^2}}. \tag{A11}$$

Now,

$$\begin{aligned}
\mathrm{Cov}_a(X,Y) &= E\left(XY :|X| \geq a\right) - E\left(X :|X| \geq a\right)E\left(Y :|Y| \geq a\right) \\
&= E\left(XY :|X| \geq a\right) \\
&= 2\int_a^\infty \int_{-\infty}^\infty xyf_a(x,y)dydx \\
&= 2\int_a^\infty \int_{-\infty}^\infty xyf_a(x) f_a(y \mid x)dydx \\
&= 2\int_a^\infty xf_a(x)\int_{-\infty}^\infty yf_a(y \mid x)dydx \\
&= 2\int_a^\infty xf_a(x)\rho\frac{\sigma_y}{\sigma_x}xdx \\
&= 2\rho\frac{\sigma_y}{\sigma_x}\int_a^\infty x^2 f_a(x)dx \\
&= \rho\frac{\sigma_y}{\sigma_x}\mathrm{Var}_a(X) \\
&= \rho\frac{\sigma_y}{\sigma_x}\sigma_x^2\left(\frac{\xi n(\xi)}{N(-\xi)}+1\right) \\
&= \rho\sigma_y\sigma_x\left(\frac{\xi n(\xi)}{N(-\xi)}+1\right),
\end{aligned} \tag{A12}$$

and

$$\begin{aligned}
\mathrm{Var}_a(Y) &= E\left(Y^2 :| X |\geq a\right) - E\left(Y :| X |\geq a\right)^2 \\
&= E\left(Y^2 :| X |\geq a\right) \\
&= 2\int_a^\infty \int_{-\infty}^\infty y^2 f_a(x,y)dydx \qquad . \\
&= 2\int_a^\infty \int_{-\infty}^\infty y^2 f_a(x) f_a(y | x)dydx \\
&= 2\int_a^\infty f_a(x) \int_{-\infty}^\infty y^2 f_a(y | x)dydx.
\end{aligned} \tag{A12}$$

Rewriting the second integral as:

$$\int_{-\infty}^\infty y^2 f_a(y | x)dy = \frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_y} \int_{-\infty}^\infty y^2 e^{-\frac{\left(y-\rho\frac{\sigma_y}{\sigma_x}x\right)^2}{2(1-\rho^2)\sigma_y^2}} dy\ , \tag{A13}$$

and letting $u = y - \rho\frac{\sigma_y}{\sigma_x}x$ we have $du = dy, y^2 = u^2 + 2u\rho\frac{\sigma_y}{\sigma_x}x + \rho^2\frac{\sigma_y^2}{\sigma_x^2}x^2$. After substitutions, the integral becomes:

$$\begin{aligned}
&\frac{1}{2\pi(1-\rho^2)\sigma_y} \int_{-\infty}^\infty \left(u^2 + 2u\rho\frac{\sigma_y}{\sigma_x}x + \rho^2\frac{\sigma_y^2}{\sigma_x^2}x^2\right) e^{-\frac{u^2}{2(1-\rho^2)\sigma_y^2}} du \\
&= (1-\rho^2)\sigma_y^2 + 0 + \rho^2\frac{\sigma_y^2}{\sigma_x^2}x^2.
\end{aligned} \tag{A14}$$

Returning to the original integral,

$$
\begin{aligned}
2\int_a^{\infty} f_a(x)\int_{-\infty}^{\infty} y^2 f_a(y\,|\,x)dydx &= 2\int_a^{\infty} f_a(x)\left((1-\rho^2)\sigma_y^2+\rho^2\frac{\sigma_y^2}{\sigma_x^2}x^2\right)dx \\
&= (1-\rho^2)\sigma_y^2+\rho^2\frac{\sigma_y^2}{\sigma_x^2}\mathrm{Var}_a(X) \\
&= (1-\rho^2)\sigma_y^2+\rho^2\frac{\sigma_y^2}{\sigma_x^2}\sigma_x^2\left(\frac{\xi n(\xi)}{N(-\xi)}+1\right) \\
&= \sigma_y^2\left(1+\rho^2\frac{\xi n(\xi)}{N(-\xi)}\right),
\end{aligned}
\tag{A15}
$$

and finally

$$
\begin{aligned}
\mathrm{Cor}_a(X,Y) &= \frac{\mathrm{Cov}_a(X,Y)}{\left(\mathrm{Var}_a(X)\mathrm{Var}_a(Y)\right)^{1/2}} \\
&= \frac{\rho\sigma_x\sigma_y\left(\frac{\xi n(\xi)}{N(-\xi)}+1\right)}{\left(\sigma_x^2\left(\frac{\xi n(\xi)}{N(-\xi)}+1\right)\sigma_y^2\left(1+\rho^2\frac{\xi n(\xi)}{N(-\xi)}\right)\right)^{1/2}},
\end{aligned}
\tag{A16}
$$

or

$$
\mathrm{Cor}_a(X,Y) = \left(\frac{N(-\xi)+\xi n(\xi)}{N(-\xi)+\rho^2\xi n(\xi)}\right)^{1/2}\rho\,. \tag{A17}
$$

Q.E.D.

B. Proof of Theorem 2

By the definition of $S(n)$ in the statement of the theorem, on average

$$\left|Y_i-\bar{Y}\right|_{i\notin S(n)} \le \left|Y_j-\bar{Y}\right|_{j\in S(n)}, \tag{B1}$$

which implies

$$\sum\nolimits_{i\notin S(n)}\left(Y_i-\bar{Y}\right)^2(N-n) \le \sum\nolimits_{i\in S(n)}\left(Y_i-\bar{Y}_n\right)^2 n, \tag{B2}$$

and so

$$\begin{aligned}\sum_i\left(Y_i-\bar{Y}\right)^2 &= \sum_{i\in S(n)}\left(Y_i-\bar{Y}\right)^2+\sum_{i\notin S(n)}\left(Y_i-\bar{Y}\right)^2\\ &\le \frac{N}{n}\sum_{i\in S(n)}\left(Y_i-\bar{Y}\right)^2 \cong \frac{N}{n}\sum_{i\in S(n)}\left(Y_i-\bar{Y}_n\right)^2\\ &\hat{Y}_i \cong \hat{Y}_{in} \equiv \hat{Y}_{i\in S(n)}.\end{aligned} \tag{B3}$$

Furthermore, under the assumption of homoscedasticity, on average,

$$\left|Y_i-\hat{Y}_i\right|_{i\notin S(n)} \cong \left|Y_j-\hat{Y}_j\right|_{j\in S(n)}. \tag{B4}$$

Thus,

$$\begin{aligned}\sum_i\left(Y_i-\hat{Y}_i\right)^2 &= \sum_{i\in S(n)}\left(Y_i-\hat{Y}_i\right)^2+\sum_{i\notin S(n)}\left(Y_i-\hat{Y}_i\right)^2\\ &\cong \frac{N}{n}\sum_{i\in S(n)}\left(Y_i-\hat{Y}_i\right)^2 \cong \frac{N}{n}\sum_{i\in S(n)}\left(Y_i-\hat{Y}_{in}\right)^2.\end{aligned} \tag{B5}$$

In conclusion, we have

$$\sum_i\left(Y_i-\bar{Y}\right)^2 \le \frac{N}{n}\sum_{i\in S(n)}\left(Y_i-\bar{Y}_n\right)^2, \text{ and} \tag{B6}$$

$$\sum_i\left(Y_i-\hat{Y}_i\right)^2 \cong \frac{N}{n}\sum_{i\in S(n)}\left(Y_i-\hat{Y}_{in}\right)^2. \tag{B7}$$

Now consider the explanatory power of the overall regression R_N^2, which, by definition, is

$$R_N^2 \equiv 1 - \frac{\sum\left(Y_i - \widehat{Y}_i\right)^2}{\sum\left(Y_i - \overline{Y}\right)^2} \;. \tag{B8}$$

Using equations (B6) and (B7),

$$R_N^2 = -\frac{\sum\left(Y_i - \widehat{Y}_i\right)^2}{\sum\left(Y_i - \overline{Y}\right)^2} \leq 1 - \frac{\frac{N}{n}\sum_{i\in S(n)}\left(Y_i - \widehat{Y}_{in}\right)^2}{\frac{N}{n}\sum_{i\in S(n)}\left(Y_i - \overline{Y}_n\right)^2} = 1 - \frac{\sum_{i\in S(n)}\left(Y_i - \widehat{Y}_{in}\right)^2}{\sum_{i\in S(n)}\left(Y_i - \overline{Y}_n\right)^2} \equiv R_n^2. \tag{B9}$$

Q.E.D.

C. Proof of Theorem 3

Under the conditions in the theorem, the density of the distribution is given by

$$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left[-\frac{\left(\frac{x}{\sigma_x}\right)^2-2\rho\frac{x}{\sigma_x}\frac{y}{\sigma_y}+\left(\frac{y}{\sigma_y}\right)^2}{2(1-\rho^2)}\right] \tag{C1}$$

The Value-at-Risk VaR_α of the portfolio is defined as

$$\mathrm{Prob}(\mathrm{Dailyloss}\geq \mathrm{VaR}_\alpha)=\alpha \tag{C2}$$

for α a constant number, signifying the probability of losses greater than VaR_α over one day (typical values of α are 5%, 1% and 0.1%). The daily change of the value of the portfolio is given by

$$aX+bY \tag{C3}$$

and VaR_α can be implicitly determined from

$$\int_{-\infty}^{\infty}\int_{\frac{\mathrm{VaR}_\alpha-ax}{b}}^{\infty} f(x,y)dydx=\alpha \tag{C4}$$

In order to identify the dependence of VaR_α on the correlation level, equation (C4) can be transformed to a standard form through a series of changes of variables. Consider

$$\begin{aligned} x_1&=\frac{\sqrt{2}}{2}\frac{x}{\sigma_x}+\frac{\sqrt{2}}{2}\frac{y}{\sigma_y}, \text{ and}\\ y_1&=-\frac{\sqrt{2}}{2}\frac{x}{\sigma_x}+\frac{\sqrt{2}}{2}\frac{y}{\sigma_y}. \end{aligned} \tag{C5}$$

Then, equation (C4) becomes

$$\int_{-\infty}^{\infty}\int_{\frac{\mathrm{VaR}_\alpha-\frac{\sqrt{2}}{2}(a\sigma_x+b\sigma_y)x_1}{\frac{\sqrt{2}}{2}(b\sigma_y-a\sigma_x)}}^{\infty} f_1(x_1,y_1)dy_1dx_1=\alpha \tag{C6}$$

where

$$f_1(x_1, y_1) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{x_1^2}{2(1+\rho)} - \frac{y_1^2}{2(1-\rho)} \right]. \tag{C7}$$

A second change of variables (rescaling), with

$$\begin{aligned} x_2 &= x_1\sqrt{1+\rho} \\ y_2 &= y_1\sqrt{1-\rho} \end{aligned}, \tag{C8}$$

transforms (C6) into

$$\int_{-\infty}^{\infty} \int_{\frac{\mathrm{VaR}_\alpha - \frac{\sqrt{2}}{2}\sqrt{1+\rho}\left(a\sigma_x + b\sigma_y\right)x_2}{\frac{\sqrt{2}}{2}\sqrt{1-\rho}\left(b\sigma_y - a\sigma_x\right)}}^{\infty} f_2(x_2, y_2)dy_2dx_2 = \alpha . \tag{C9}$$

where

$$f_2(x_2, y_2) = \frac{1}{2\pi} \exp\left[-\frac{x_2^2 + y_2^2}{2} \right]. \tag{C10}$$

An additional rotation by an angle ϕ with

$$\tan\phi = \frac{b\sigma_y - a\sigma_x}{b\sigma_y + a\sigma_x}\sqrt{\frac{1-\rho}{1+\rho}} \tag{C11}$$

brings equation (C9) to its canonical form,

$$\int_{-\infty}^{\infty} \int_{\frac{\mathrm{VaR}_\alpha}{\left(a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho ab\sigma_x\sigma_y\right)^{1/2}}}^{\infty} f_2(x_2, y_2)dy_2dx_2 = \alpha . \tag{C12}$$

Conceptually, the changes of variables performed can be seen to affect the level sets of the joint probability distribution in the following way:

- The first change of variables corresponds to a rotation of the level sets of f by $\pi/4$ and a rescaling of the coordinates by σ_x and σ_y. The level sets of f_1 are ellipses, with semi-axes parallel to the x_1, y_1 axes.
- The second change of variables transforms the level sets of f_1 to circles.
- The last rotation does not affect the level sets of f_2, but rotates the line of

interest $ax + by = \mathrm{VaR}_\alpha$, so that it is perpendicular to the x_2 axis.

From the final form of the integral that defines Value-at-Risk we can deduce that, if $\mathrm{VaR}_\alpha(\rho_0)$ is known, then, for a different correlation ρ_1

$$\mathrm{VaR}_\alpha(\rho_1) = \mathrm{VaR}_\alpha(\rho_0)\left(\frac{a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho_1 ab\sigma_x\sigma_y}{a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho_0 ab\sigma_x\sigma_y}\right)^{1/2}. \quad \text{(C13)}$$

Another way to interpret the above result is to notice that, for normally distributed variables, VaR is scaled by the standard deviation corresponding to the distribution of the value of the portfolio.

For increasing correlations, $\rho_1 > \rho_0 \geq 0$, we have

$$\mathrm{VaR}_\alpha(\rho_1) > \mathrm{VaR}_\alpha(\rho_0). \quad \text{(C13)}$$

To calculate the expected losses conditional on the losses exceeding the Value-at-Risk, we have

$$\begin{aligned} E(\text{DailyLoss} \mid \text{DailyLoss} \geq \mathrm{VaR}_\alpha) &= \int_{-\infty}^{\infty}\int_{\frac{\mathrm{VaR}_\alpha - ax}{b}}^{\infty} (ax+by) f(x,y)\,dy\,dx \\ &= \cdots = \\ &= \left(a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho ab\sigma_x\sigma_y\right)^{1/2} I, \end{aligned} \quad \text{(C14)}$$

where

$$\begin{aligned} I &= \int_{-\infty}^{\infty}\int_{\frac{\mathrm{VaR}_\alpha}{\left(a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho ab\sigma_x\sigma_y\right)^{1/2}}}^{\infty} x_2 f_2(x_2, y_2)\,dy_2\,dx_2 \\ &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mathrm{VaR}_\alpha^2}{2(a^2\sigma_x^2 + b^2\sigma_y^2 + 2\rho ab\sigma_x\sigma_y)}\right). \end{aligned} \quad \text{(C15)}$$

Q.E.D.

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Figures

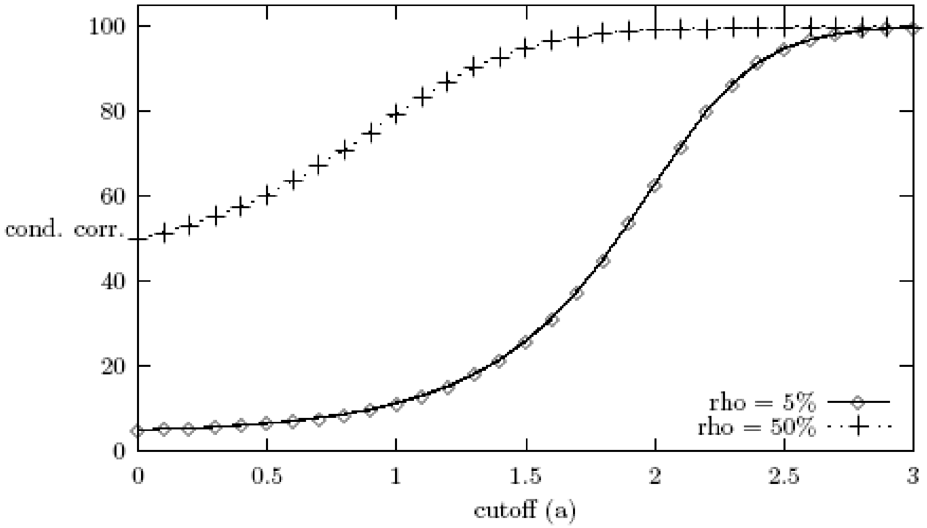

Figure 1

Conditional correlation as a function of the cutoff

Figure 1 shows the parametric plots of conditional correlation as a function of the cutoff value for two values of the unconditional correlation, 0.05 and 0.5.

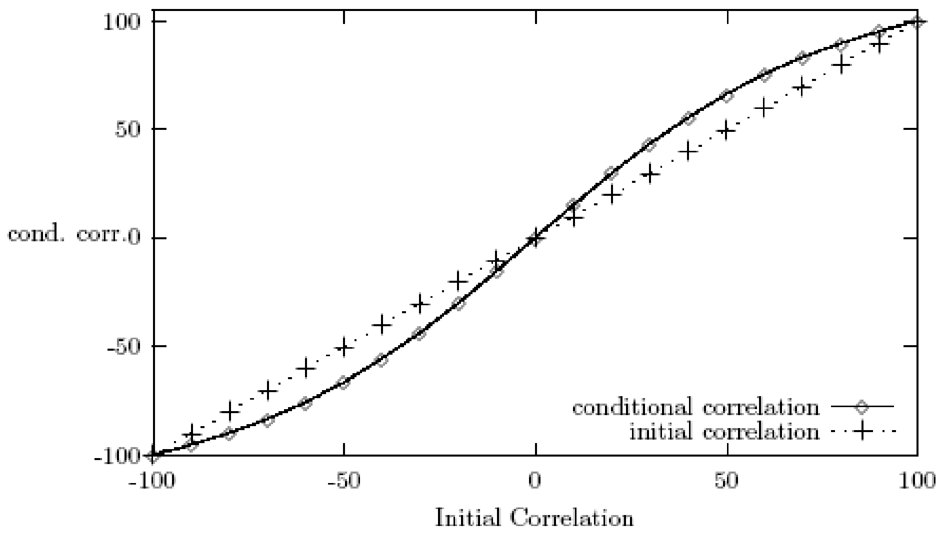

Figure 2

Conditional correlation as a function of the initial correlation

Figure 2 shows the anticipated change in conditional correlation over unconditional correlation when the sample is split into two equal-sized subsamples after being ranked. Conditional correlation plot is for the “LARGE”-change subsample.

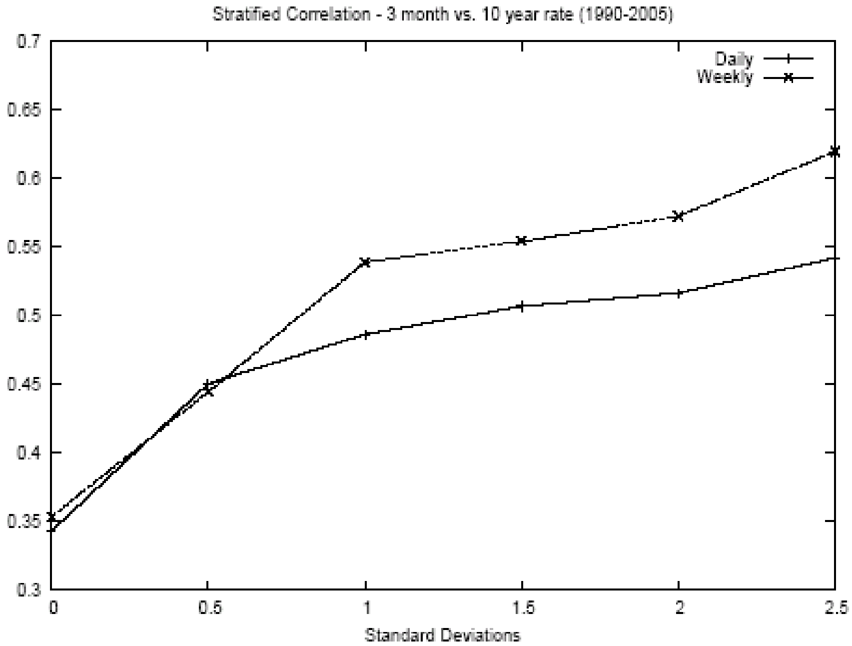

Figure 3

Correlations conditional to large moves for interest rate data

Figure 3 provides an emprical validation of the result in Theorem 1. Using daily and weekly samples, we compute the conditional correlation between the change in the three-month rate and the ten-year rate, when the cut-off point is varied from 0.5 to 2.5 standard deviations.

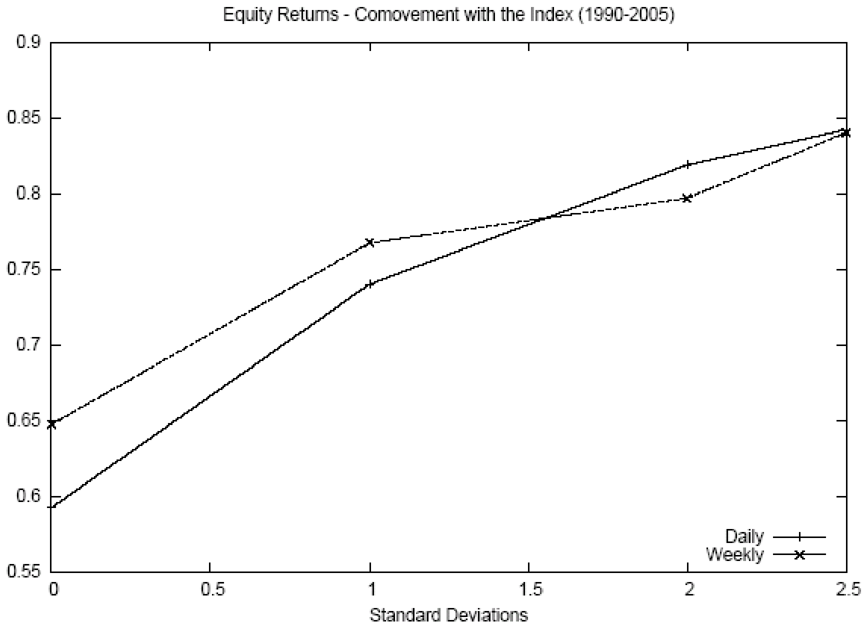

Figure 4

Value-weighted percentage of stocks that move in tandem with the index, conditional on large index moves

Figure 4 displays the value weighted proportions of the 100 largest market capitalization stocks traded in the NYSE/AMEX that moved in the same direction as a value-weighted index of the same stocks, as the absolute magnitude of the change in index value is varied from 0.5 to 2.5 standard deviations. The results shown are with daily and weekly samples.

Tables

Table 1

Conditional correlations

Panel A. Correlations on OVERALL, SMALL-, and LARGE-change samples, 1990-2005

We report the unconditional (OVERALL sample) and conditional (SMALL- and LARGE-change samples) correlations after stratifying the sample into two sub-samples based on change in the on-the-run three-month Treasury-Bill rate. Skewness and Kurtosis parameter estimates are also reported for the OVERALL sample. The standard error for skewness is given by $\sqrt{6/\#\text{ of observations}}$, while the standard error for kurtosis is given by $\sqrt{8/\#\text{ of observations}}$.

OVERALL (N=3936 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	20-yr.
3-mo.	1								
6-mo.	0.708	1							
1-yr.	0.462	0.725	1						
2-yr.	0.329	0.614	0.845	1					
3-yr.	0.305	0.588	0.821	0.936	1				
5-yr.	0.274	0.545	0.765	0.873	0.926	1			
7-yr.	0.233	0.475	0.683	0.798	0.854	0.925	1		
10-yr.	0.208	0.441	0.647	0.76	0.822	0.903	0.949	1	
20-yr.	0.143	0.344	0.535	0.635	0.696	0.789	0.871	0.886	1
Skewness	-0.496***	-0.753***	-0.492***	-0.470***	0.125***	0.202***	0.327***	0.351***	0.334***
Kurtosis	20.033***	10.548***	6.833***	3.794***	3.161***	2.281***	2.057***	2.016***	1.560***

SMALL (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	20-yr.
3-mo.	1								
6-mo.	0.154	1							
1-yr.	0.100	0.519	1						
2-yr.	0.072	0.446	0.721	1					
3-yr.	0.049	0.39	0.687	0.861	1				
5-yr.	0.036	0.372	0.638	0.795	0.874	1			
7-yr.	0.049	0.321	0.555	0.700	0.79	0.893	1		
10-yr.	0.039	0.304	0.519	0.645	0.742	0.848	0.919	1	
20-yr.	0.029	0.249	0.415	0.497	0.577	0.699	0.79	0.827	1

LARGE (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	20-yr.
3-mo.	1								
6-mo.	0.715	1							
1-yr.	0.497	0.774	1						
2-yr.	0.359	0.658	0.865	1					
3-yr.	0.327	0.628	0.834	0.937	1				
5-yr.	0.296	0.578	0.773	0.877	0.929	1			
7-yr.	0.246	0.500	0.685	0.793	0.860	0.929	1		
10-yr.	0.221	0.46	0.639	0.744	0.817	0.903	0.954	1	
20-yr.	0.138	0.346	0.506	0.587	0.657	0.759	0.838	0.874	1

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.

Panel B. Correlations on OVERALL, SMALL and LARGE-change samples, 1990-2005

We report the unconditional (OVERALL sample) and conditional (SMALL- and LARGE-change samples) correlations after stratifying the sample into two sub-samples based on the percentage change in the on-the-run three-month Treasury-Bill rate. Skewness and Kurtosis parameter estimates are also reported for the OVERALL sample. The standard error for skewness is given by $\sqrt{6/\#\ \text{of observations}}$, while the standard error for kurtosis is given by $\sqrt{8/\#\ \text{of observations}}$.

OVERALL (N=3936 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	20-yr.
3-mo.	1								
6-mo.	0.761	1							
1-yr.	0.540	0.784	1						
2-yr.	0.429	0.688	0.87	1					
3-yr.	0.407	0.667	0.849	0.949	1				
5-yr.	0.361	0.602	0.771	0.859	0.916	1			
7-yr.	0.298	0.518	0.683	0.777	0.838	0.926	1		
10-yr.	0.256	0.467	0.631	0.725	0.792	0.896	0.948	1	
20-yr.	0.173	0.355	0.509	0.589	0.655	0.771	0.864	0.88	1
Skewness	-0.739***	-0.483***	0.290***	0.539***	0.600***	0.534***	0.549***	0.490***	0.307***
Kurtosis	19.321***	15.325***	11.822***	9.295***	8.101***	5.457***	3.878***	3.060***	2.410***

SMALL-change (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	20-yr.
3-mo.	1								
6-mo.	0.241	1							
1-yr.	0.202	0.589	1						
2-yr.	0.198	0.528	0.767	1					
3-yr.	0.170	0.477	0.733	0.888	1				
5-yr.	0.116	0.420	0.648	0.790	0.870	1			
7-yr.	0.116	0.369	0.562	0.691	0.783	0.900	1		
10-yr.	0.097	0.348	0.518	0.638	0.731	0.855	0.922	1	
20-yr.	0.045	0.266	0.39	0.473	0.552	0.694	0.787	0.824	1

LARGE-change (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	20-yr.
3-mo.	1								
6-mo.	0.781	1							
1-yr.	0.594	0.824	1						
2-yr.	0.483	0.726	0.884	1					
3-yr.	0.457	0.703	0.859	0.951	1				
5-yr.	0.410	0.639	0.782	0.870	0.925	1			
7-yr.	0.330	0.544	0.686	0.780	0.846	0.930	1		
10-yr.	0.282	0.482	0.622	0.716	0.791	0.898	0.955	1	
20-yr.	0.152	0.325	0.446	0.524	0.599	0.728	0.822	0.859	1

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.

Panel C. Significance of correlation differences, 1990-2005

We test for the differences in correlations reported in Table 1 Panel A. Since the distribution of Pearson correlation is skewed, we use Fisher's z-transformation that converts the distribution of sampled correlations to a normal distribution, where: $\text{Corr}' = .5\left[\ln(1+\text{Corr}) - \ln(1-\text{Corr})\right]$, $\sigma_{\text{Corr}'} = 1/\sqrt{N-3}$, and

$$z = \frac{\text{Corr}_1' - \text{Corr}_2'}{\sqrt{1/(N_1-3)+1/(N_2-3)}}.$$

OVERALL-SMALL (N=3936 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.
6-mo.	0.554***							
1-yr.	0.362***	0.206***						
2-yr.	0.257***	0.168***	0.124***					
3-yr.	0.256***	0.198***	0.134***	0.075***				
5-yr.	0.238***	0.173***	0.127***	0.078***	0.052*			
7-yr.	0.184***	0.154***	0.128***	0.098***	0.064**	0.032		
10-yr.	0.169***	0.137***	0.128***	0.115***	0.080***	0.055**	0.030	
20-yr.	0.114***	0.095***	0.120***	0.138***	0.119***	0.090***	0.081***	0.059**

LARGE-OVERALL (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.
6-mo.	0.007							
1-yr.	0.035	0.049						
2-yr.	0.030	0.044*	0.020					
3-yr.	0.022	0.040	0.013	0.001				
5-yr.	0.022	0.033	0.008	0.004	0.003			
7-yr.	0.013	0.025	0.002	-0.005	0.006	0.004		
10-yr.	0.013	0.019	-0.008	-0.016	-0.005	0.000	0.005	
20-yr.	-0.005	0.002	-0.029	-0.048*	-0.039	-0.030	-0.033	-0.012

LARGE-SMALL (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.
6-mo.	0.561***							
1-yr.	0.397***	0.255***						
2-yr.	0.287***	0.212***	0.144***					
3-yr.	0.278***	0.238***	0.147***	0.076**				
5-yr.	0.260***	0.206***	0.135***	0.082**	0.055*			
7-yr.	0.197***	0.179***	0.130***	0.093***	0.070**	0.036		
10-yr.	0.182***	0.156***	0.120***	0.099***	0.075**	0.055*	0.035	
20-yr.	0.109***	0.097***	0.091***	0.090***	0.080**	0.060*	0.048	0.047

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.

Panel D. Significance of correlation differences, 1990-2005

We test for the differences in correlations reported in Table 1 Panel B. Since the distribution of Pearson correlation is skewed, we use Fisher's z-transformation that converts the distribution of sampled correlations to a normal distribution, where: $\text{Corr}' = .5\left[\ln(1+\text{Corr}) - \ln(1-\text{Corr})\right]$, $\sigma_{\text{Corr}'} = 1/\sqrt{N-3}$, and

$$z = \frac{\text{Corr}_1' - \text{Corr}_2'}{\sqrt{1/(N_1-3)+1/(N_2-3)}}.$$

OVERALL-SMALL (N=3936 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.
6-mo.	0.520***							
1-yr.	0.338***	0.195***						
2-yr.	0.231***	0.160***	0.103***					
3-yr.	0.237***	0.190***	0.116***	0.061**				
5-yr.	0.245***	0.182***	0.123***	0.069**	0.046*			
7-yr.	0.182***	0.149***	0.121***	0.086***	0.055**	0.026		
10-yr.	0.159***	0.119***	0.113***	0.087***	0.061**	0.041	0.026	
20-yr.	0.128***	0.089***	0.119***	0.116***	0.103***	0.077***	0.077***	0.056**

LARGE-OVERALL (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.
6-mo.	0.020							
1-yr.	0.054*	0.040						
2-yr.	0.054*	0.038	0.014					
3-yr.	0.050*	0.036	0.010	0.002				
5-yr.	0.049*	0.037	0.011	0.011	0.009			
7-yr.	0.032	0.026	0.003	0.003	0.008	0.004		
10-yr.	0.026	0.015	-0.009	-0.009	-0.001	0.002	0.007	
20-yr.	-0.021	-0.030	-0.063**	-0.065**	-0.056**	-0.043	-0.042	-0.021

LARGE-SMALL (N=1968 days)	3-mo.	6-mo.	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.
6-mo.	0.540***							
1-yr.	0.392***	0.235***						
2-yr.	0.285***	0.198***	0.117***					
3-yr.	0.287***	0.226***	0.126***	0.063**				
5-yr.	0.294***	0.219***	0.134***	0.080**	0.055*			
7-yr.	0.214***	0.175***	0.124***	0.089***	0.063**	0.030		
10-yr.	0.185***	0.134***	0.104***	0.078**	0.060*	0.043	0.033	
20-yr.	0.107***	0.059*	0.056*	0.051	0.047	0.034	0.035	0.035

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.

Table 2

Principal components factor analysis, 1990-2005

We report the results of Principal Components Factor Analysis of the Treasury rates in the OVERALL, SMALL-, and LARGE-change samples. LARGE- and SMALL-change subsamples are based on the change in the three-month Treasury rate.

OVERALL (N= 3936days)	Factor1	Factor2	Factor3	Commonality
3-mo.	0.752	0.646	0.122	0.997
6-mo.	0.831	0.553	0.056	0.998
1-yr.	0.918	0.392	0.004	0.996
2-yr.	0.974	0.203	-0.091	0.998
3-yr.	0.993	0.071	-0.095	0.999
5-yr.	0.985	-0.145	-0.084	0.998
7-yr.	0.966	-0.24	-0.079	0.997
10-yr.	0.927	-0.372	0.008	0.997
20-yr.	0.845	-0.508	0.158	0.997
Variance	8.26	1.636	0.078	8.641

SMALL (N= 1968days)	Factor1	Factor2	Factor3	Commonality
3-mo.	0.873	-0.464	0.142	0.999
6-mo.	0.897	-0.436	0.066	0.999
1-yr.	0.93	-0.362	-0.026	0.997
2-yr.	0.978	-0.184	-0.097	0.999
3-yr.	0.994	-0.045	-0.099	1
5-yr.	0.976	0.205	-0.063	0.999
7-yr.	0.949	0.309	-0.026	0.997
10-yr.	0.898	0.438	0.028	0.999
20-yr.	0.794	0.593	0.12	0.997
Variance	7.668	1.253	0.064	8.221

LARGE (N= 1968days)	Factor1	Factor2	Factor3	Commonality
3-mo.	0.776	0.619	0.101	0.997
6-mo.	0.85	0.523	0.046	0.998
1-yr.	0.927	0.37	0.006	0.997
2-yr.	0.978	0.19	-0.08	0.999
3-yr.	0.994	0.063	-0.084	0.999
5-yr.	0.986	-0.147	-0.073	0.998
7-yr.	0.968	-0.235	-0.072	0.998
10-yr.	0.934	-0.354	0.005	0.998
20-yr.	0.869	-0.471	0.144	0.998
Variance	8.449	1.468	0.061	8.752

Table 3

Regression of long-term Treasuries on 3-month Treasury, Data period for 20-year, 10/1993 – 12/2005; Data period for 10-year, 1990 – 2005

We report the R^2s from regressions of the long-term rate on the three-month Treasury rate in the OVERALL, SMALL, and LARGE-change samples. In separate regressions, we use 20-year and 10-year Treasury rates as independent variables. SMALL- and LARGE-change subsamples are based on the change and the percentage change in the three-month Treasury rate. Two tailed t-values are in parentheses.

20-year Treasury and 3-month Treasury

Difference	R^2	Constant	Coefficient	N(Days)
OVERALL	0.0704	-0.0006	0.3031	3062
		(0.63)	(15.23)***	
SMALL	0.0315	0.0004	1.1862	1532
		(0.32)	(6.58)***	
LARGE	0.1078	-0.0011	0.2929	1532
		(0.89)	(14.44)***	

Per. change	R^2	Constant	Coefficient	N(Days)
OVERALL	0.0756	-0.0001	0.1818	3062
		(0.68)	(15.82)***	
SMALL	0.0360	0.0000	0.4252	1532
		(0.14)	(7.05)***	
LARGE	0.1147	-0.0002	0.1724	1532
		(0.85)	(14.96)***	

10-year Treasury and 3-month Treasury

Difference	R^2	Constant	Coefficient	N(Days)
OVERALL	0.1180	-0.0005	0.4333	4001
		(0.55)	(23.13)***	
SMALL	0.0353	0.0012	1.2856	2000
		(0.91)	(7.87)***	
LARGE	0.1699	-0.0014	0.4230	2001
		(1.18)	(21.71)***	

Per. change	R^2	Constant	Coefficient	N(Days)
OVERALL	0.1217	-0.0001	0.2882	4001
		(0.62)	(23.54)***	
SMALL	0.0409	0.0002	0.5675	2000
		(0.78)	(8.50)***	
LARGE	0.1825	-0.0003	0.2782	2001
		(1.32)	(22.68)***	

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.

Table 4

Interest Rates — The "exceptional" days, 1990-2005

We report the significant news that appeared on the days in which the change in the three-month Treasury rate was more than 2.5 standard deviations, and the short-term and the 20-yr. rates of interest moved in opposite directions. Note that, perhaps surprisingly, there were no "exceptional" days in the 2001-2005 period satisfying the above criterion.

Date	WSJ article
Jan. 9, 1991	Flight to quality, as Secretary of State Baker reports "nothing that suggested any Iraqi flexibility"
Aug. 21, 1991	Flight from quality, as "investors hailed collapse of coup in the Soviet Union"
Feb. 10, 1992	"Short-term rates dropped in reaction to a surprisingly gloomy employment report, … but bond prices plummet[ed] later in the day after the Federal Reserve took no immediate action to cut rates"
Nov. 16, 1994	Reaction to Federal Reserve tightening 0.75 percentage points, "sending bond prices whipsawing wildly in the final 90 minutes of trading"
Dec. 8, 1994	Flight to quality, as "investors grappled with fallout from the Orange County bankruptcy-law filing"
Dec. 29, 1995	Thin trading and some weak economic data pushed longer-term treasuries higher.
May 21, 1997	"Longer-term treasuries fell amid rumors of hedge fund selling and renewed concerns about inflation and the strength of the economy"
Feb. 4, 1998	"Longer-term treasuries were marginally higher as the Fed, as expected, left rates unchanged. The bond market was stronger early in the day as the Treasury Dept. weighed using the budget surplus to repurchase bonds."
Oct. 8, 1998	"Speculation that Japan would undertake an economic stimulus package created a flight to yen. Japanese investors were pulling money from the U.S. and some hedge funds were rumored to be unwinding leveraged bets against the yen"
Aug. 12, 1999	"The Beige Book report relieved some inflation fears that had prompted economists to advance the prospect of a rate hike in August"
May 17, 2000	"A falling stock market drove some investors into shorter term securities while long rates rose slightly in anticipation of continued Fed tightening. The Fed raised both the discount and fed-funds rate by 50 bps on 5/16/00"
May 18, 2000	"Long rates rose on the concern that the Fed would continue to raise rates"
May 22, 2000	"Continued weakness in the stock market prompted some flight to quality. Long rates were slightly lower on speculation that Fed would leave rates unchanged at next meeting"
June 2, 2000	"Long rates fell on economic news suggesting the Fed may become less aggressive in raising rates"
Jul. 3, 2000	"Long rates fell marginally as the NAPM release showed only modest growth. Pre-holiday trading was very thin"
Jan. 3, 2001	"The Fed unexpectedly cut the fed-funds rate by 50 bps. and the discount rate by 25 bps. The stock market rallied and money moved from longer-term treasuries to stocks"

Table 5

Test of hedging effectiveness, 1990-2005

Hedging effectiveness is measured as the residual volatility from two different models: 1) One-factor model, where the factor is the change in two-year Treasury rate, and 2) Two-factor model, where the factors are the change in the two-year rate and the change in the slope of the yield curve, which is defined as the change in the difference between the 10-year and two-year rates. Reported results are based on the OVERALL and LARGE-change samples.

Move magnitude (1)	Maturity years (2)	Standard deviation (3)	One-factor residual volatility (4)	Percentage change (5) = (4)/(3) – 1	Two-factor residual volatility (6)	Percentage change (7) = (6)/(3) – 1
OVERALL	0.25	3.69%	–	–	3.48%	-5.76%
LARGE	0.25	4.28%	3.97%	-7.22%	–	–
OVERALL	0.5	3.30%	–	–	2.77%	-16.11%
LARGE	0.5	3.98%	3.27%	-17.78%	–	–
OVERALL	1	3.63%	–	–	2.12%	-41.62%
LARGE	1	4.14%	2.47%	-40.31%	–	–
OVERALL	2	4.23%	–	–	0%	100%
LARGE	2	4.19%	0%	100%	–	–
OVERALL	3	4.31%	–	–	1.41%	-67.32%
LARGE	3	4.52%	1.84%	-59.38%	–	–
OVERALL	5	4.31%	–	–	1.41%	-67.35%
LARGE	5	4.59%	2.56%	-44.12%	–	–
OVERALL	7	4.17%	–		1.22%	-70.77%
LARGE	7	4.55%	3.06%	-32.75%	–	–
OVERALL	10	3.94%	–	–	0%	-100%
LARGE	10	4.33%	3.13%	-27.60%	–	–
OVERALL	20	3.57%	–	–	1.54%	-56.89%
LARGE	20	3.97%	3.18%	-19.93%	–	–

Table 6

Analysis of ex-ante hedging effectiveness, 1990-2005

Ex-ante hedging effectiveness is defined as the proportional reduction in residual variance of the hedge portfolio. The one-factor hedge uses the two-year Treasury instrument. The two-factor hedge uses the two-year and ten-year instruments. Reported results are for the OVERALL and LARGE-change samples.

Maturity (years)	LARGE-change days 1-factor: P_2	OVERALL 2-factors: P_2, P_{10}
0.25	−15.7%	24.1%
0.5	−19.2%	−6.2%
1	−56.3%	−47.6%
2	−100.0%	−100.0%
3	−83.9%	−81.5%
5	−70.4%	−74.4%
7	−48.0%	−82.7%
10	−37.9%	−100.0%
20	79.6%	−55.6%

Table 7

Percentage number and value of stocks positively co-moving with the index, 1990-2005

We report the percentage number and the percentage market capitalization of the 100 largest stocks traded in the NYSE/AMEX that moved in the same direction as a value-weighted index of the same stocks in the OVERALL, SMALL- , and LARGE-change samples.

Percentage number	Mean	Std. dev.	No. of days
OVERALL	57.00%	9.94%	4091
SMALL	51.24%	6.63%	2046
LARGE	62.77%	13.32%	2045

Percentage Value	Mean	Std. dev.	No. of days
OVERALL	63.00%	4.54%	4091
SMALL	55.57%	2.68%	2046
LARGE	71.27%	6.78%	2045

Table 8

Roll's stocks: R^2 on the OVERALL, SMALL- and LARGE-change samples, 1990-2005

We report the R^2 from the market-model regression of daily returns for a sample of stocks used by Roll (1988) in the OVERALL, SMALL-, and LARGE-change samples. The market is defined as the value-weighted index of all the stocks listed in the NYSE/AMEX. The standard error for skewness is given by $\sqrt{6/\#\text{ of observations}}$, while the standard error for kurtosis is given by $\sqrt{8/\#\text{ of observations}}$.

	R^2				
	OVERALL	SMALL	LARGE	Skewness	Kurtosis
Miscellaneous Companies					
BOEING	15.88%	1.32%	25.96%	-0.282***	6.094***
GENERAL MILLS	6.30%	1.01%	10.18%	0.111***	3.441***
GOODYEAR TIRE	14.20%	1.19%	23.60%	0.145***	4.622***
INTERNATIONAL PAPER	15.28%	2.31%	25.30%	0.335***	2.934***
ITT	12.64%	2.19%	29.19%	0.349***	4.302***
UNION CARBIDE	8.60%	1.12%	13.61%	0.817***	6.096***
Oil Companies					
EXXON	9.11%	2.67%	10.61%	0.210***	3.237***
PENNZOIL	5.77%	1.09%	7.40%	2.598***	47.032***
Transportation - Railroad and Airlines					
CSX	15.13%	2.21%	24.51%	0.134***	4.749***
DELTA AIRLINES	16.24%	1.43%	25.04%	-0.399***	23.051***
NORFOLK SOUTHERN	15.97%	2.48%	28.31%	0.380***	4.457***
UNION PACIFIC	12.15%	1.77%	23.68%	0.091**	2.121***
Utilities					
AMERICAN ELECTRIC POWER	9.88%	2.14%	12.46%	-0.027	31.680***
COMMONWEALTH EDISON	6.94%	1.88%	11.30%	0.053	3.542***
GTE CORP	12.82%	1.56%	16.77%	0.218***	2.471***
PACIFIC GAS & ELECTRIC	5.78%	1.02%	7.21%	-0.609***	62.900***
SOUTHERN CALIFORNIA EDISON	4.51%	0.74%	6.28%	0.440***	61.573***
Retailers					
JC PENNEY	15.50%	2.38%	20.86%	0.592***	4.886***
SEARS	16.21%	1.78%	27.57%	0.010	17.059***
Financial Services					
BANK OF AMERICA CORP	3.82%	0.50%	5.60%	0.004	2.941***
CIGNA COR	13.29%	1.79%	19.05%	0.575***	9.229***
GENERAL RE CORP	2.25%	0.25%	40.17%	1.114***	13.612***
JP MORGAN	35.30%	4.92%	43.19%	0.371***	4.503***
Average R^2	11.89%	1.73%	19.91%		

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.

Table 9

International markets: R^2 on the OVERALL, SMALL and LARGE-change samples, 1990-2005

We report the R^2 from the market-model regression of weekly returns for a sample of international stock market indices in the OVERALL, SMALL-, and LARGE-change samples. The market is defined as the value-weighted index of all the stocks listed in the NYSE/AMEX. The standard error for skewness is given by $\sqrt{6/\#\ \text{of observations}}$, while the standard error for kurtosis is given by $\sqrt{8/\#\ \text{of observations}}$.

	R^2				
Developing Markets	OVERALL	SMALL	LARGE	Skewness	Kurtosis
ARGENTINA	4.19%	2.29%	7.11%	0.672***	7.536***
BRAZIL	9.99%	1.25%	17.12%	-0.317***	5.894***
CHILE	11.27%	1.29%	17.72%	-0.039	1.118***
JORDAN	0.04%	0.07%	0.17%	0.564***	2.107***
KOREA	7.96%	0.91%	11.94%	0.473***	2.945***
MALAYSIA	4.33%	1.45%	5.91%	2.475***	35.228***
MEXICO	25.17%	2.67%	40.00%	-0.308***	4.329***
PHILIPPINES	5.17%	0.63%	7.94%	0.642***	6.565***
TAIWAN	6.01%	2.07%	9.43%	-0.020	2.446***
THAILAND	5.62%	0.40%	7.68%	1.902***	16.754***
TURKEY	2.06%	0.24%	3.04%	0.311***	2.315***
PORTUGAL	7.19%	1.52%	11.53%	0.123	1.289***
Average R^2	7.42%	1.23%	11.63%		

	R^2				
Developed Markets	OVERALL	SMALL	LARGE	Skewness	Kurtosis
AUSTRALIA	18.41%	2.40%	28.60%	-0.199***	0.915***
GERMANY	33.44%	6.94%	44.03%	-0.145	2.205***
JAPAN	6.28%	2.69%	10.57%	0.403***	2.734***
SINGAPORE	14.23%	5.93%	18.78%	-0.115	9.515***
SWITZERLAND	27.12%	6.25%	38.18%	-0.324***	2.041***
UK	29.07%	6.72%	42.90%	0.000	2.322***
Average R^2	21.43%	5.16%	30.51%		

***, ** and * indicate statistical significance at the 0.01, 0.05 and 0.1 levels.